Metal-insulator transition of isotopically enriched neutron-transmutation-doped ^{70}Ge:Ga in magnetic fields

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We have investigated the temperature dependence of the electrical conductivity $\sigma(N,B,T)$ of nominally uncompensated, neutron-transmutation-doped ^{70}Ge:Ga samples in magnetic fields up to $B=8$ T at low temperatures ($T=0.05-0.5$ K). In our earlier studies at $B=0$, the critical exponent $\mu=0.5$ defined by $\sigma(N,0,0)\propto(N-N_c)^{\mu}$ has been determined for the same series of ^{70}Ge:Ga samples with the doping concentration N ranging from 1.861×10^{17} cm^{-3} to 2.434×10^{17} cm^{-3}. In magnetic fields, the motion of carriers loses time-reversal symmetry, the universality class may change and with it the value of μ. In this work, we show that magnetic fields indeed affect the value of μ (μ changes from 0.5 at $B=0$ to 1.1 at $B\geqslant4$ T). The same exponent $\mu'=1.1$ is also found in the magnetic-field-induced MIT for three different ^{70}Ge: Ga samples, i.e., $\sigma(N,B,0)\propto[B_c(N)-B]^{\mu'}$ where $B_c(N)$ is the concentration-dependent critical magnetic induction. We show that $\sigma(N,B,0)$ obeys a simple scaling rule on the (N,B) plane. Based on this finding, we derive from a simple mathematical argument that $\mu=\mu'$ as has been observed in our experiment. [S0163-1829(99)10447-8]

I. INTRODUCTION

Semiconductors with a random distribution of doping impurities have been studied extensively over the past three decades in order to probe the nature of the metal-insulator transition (MIT) in disordered electronic systems.[1,2] The value of the critical exponent μ of the conductivity for the metallic side of the transition, however, still remains controversial. The exponent μ is defined by

$$\sigma(0)=\sigma^*(N/N_c-1)^{\mu} \tag{1}$$

in the critical regime of the MIT ($0<N/N_c-1\ll1$). Here, $\sigma(0)$ is the zero-temperature conductivity, σ^* is a prefactor, N is the impurity concentration, and N_c is the critical concentration for the MIT. An exponent of $\mu\approx0.5$ has been found in a number of nominally uncompensated semiconductors [Si:P,[3] Si:As,[4] Si:Sb,[5] Ge:As,[6] and ^{70}Ge:Ga (Refs. 7 and 8)]. This value is considerably smaller than the results of numerical calculations ($\mu=1.2-1.6$; e.g., Refs. 9 and 10) for the Anderson transition purely driven by disorder. Therefore, electron-electron interaction, which is undoubtedly present in doped semiconductors, must be relevant to the nature of the MIT, at least when impurity compensation is absent. A conclusion that has been reached over the years is that one has to deal simultaneously with disorder and electron-electron interaction in order to understand the MIT in doped semiconductors.

According to theories[2] on the MIT which take into account both disorder and electron-electron interaction, the critical exponent μ does not depend on the details of the system, but depends only on the universality class to which the system belongs. Moreover, there is an inequality $\nu\geqslant2/3$ for the critical exponent ν of the correlation length.[11] The inequality is expected to apply generally to disordered systems, irrespective of the presence of electron-electron interaction.[12] Hence, if one assumes the Wegner relation[13] $\mu=\nu$, which is derived for systems *without* electron-electron interaction, $\mu\approx0.5$ violates the inequality. This discrepancy has been known as the conductivity critical exponent puzzle. Kirkpatrick and Belitz[14] have claimed that there are logarithmic corrections to scaling in universality classes with time-reversal symmetry, i.e., when the external magnetic field is zero, and that $\mu\approx0.5$, found at $B=0$, should be interpreted as an ''effective'' exponent which is different from a ''real'' exponent satisfying $\mu\geqslant2/3$. Therefore, comparison of μ with and without the time-reversal symmetry, i.e., with and without external magnetic fields becomes important. Experimentally, $\mu\approx1$ has been found for magnetic inductions B on the order of one tesla for nominally uncompensated semiconductors: Ge:Sb,[15,16] Si:B,[17] and Si:P (Ref. 18). Since these systems result in different values of μ ranging from 0.5 to 1.0 at $B=0$, the applied magnetic field changes the value of μ for certain systems (Si:B and Si:P), while it does not[16] or does only change little[15] for the other (Ge:Sb). In this work,

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TABLE I. List of ^{70}Ge:Ga samples employed in this study. N is the concentration of gallium determined from the irradiation time and the flux of thermal neutron [the critical concentration for the metal-insulator transition (MIT) is 1.860×10^{17} cm^{-3} (Ref. 8)]; $\sigma(0,0)$ is the zero-temperature conductivity at $B=0$; n is the normalized concentration defined by Eq. (12); m is the temperature coefficient of the conductivity at $B=0$ given by Eq. (3); B_c is the critical magnetic induction for the magnetic-field-induced MIT; $\sigma(4\text{ T},0)$ is the zero-temperature conductivity at $B=4$ T; $m_B(4\text{ T})$ is the temperature coefficient at $B=4$ T which is similar to m.

Sample	N (10^{17} cm^{-3})	$\sigma(0,0)$ (S/cm)	n	m (S cm^{-1} K$^{-1/2}$)	B_c (T)	$\sigma(4\text{ T},0)$ (S/cm)	$m_B(4\text{ T})$ (S cm^{-1} K$^{-1/2}$)
A1	1.861	0.6	0.00	>0	≈0.3	Insulator	
A2	1.863	1.6	0.00	>0	1	Insulator	
A3	1.912	7.7	0.04	1.7	4.1	0.1	7.5
A4	2.210	15.5	0.15	-2.3	7	5.7	6.4
A5	2.232	19.1	0.23	-2.4	8	9.6	5.2
B1	1.933	7.8	0.04	1.4	4	0.2	7.4
B2	2.004	11.9	0.09	-1.2	5.5	2.3	7.8
B3	2.076	12.0	0.09	-1.3	6	2.6	7.4
B4	2.219	18.5	0.22	-2.6	8.0	8.9	5.5
B5	2.290	19.8	0.25	-2.7	8	10.4	4.9
B6	2.362	19.8	0.25	-2.6	8	10.4	5.0
B7	2.434	22	0.32	-2.3	9	13.8	3.9

we aim to achieve a complete understanding of the effect of magnetic fields on the MIT in uncompensated semiconductors by studying the critical behavior of the zero-temperature conductivity as a function of both N (doping-induced MIT) and B (magnetic-field-induced MIT) in magnetic fields up to 8 T for ^{70}Ge:Ga system. To our knowledge, the MIT in Si or Ge has not been analyzed as a function of B. Concerning the critical point,

$$N_c(B)-N_c(0)\propto B^{\beta} \tag{2}$$

with $\beta=0.5$ was obtained for Ge:Sb,[16] while $\beta=1.7\pm0.4$ for Si:B.[19] The exponent β characterizes the phase (metal or insulator) diagram on the (N,B) plane, and provides information on the nature of the MIT in magnetic fields. The above experimental results, however, imply that β could be completely different even though μ in magnetic fields is the same. Hence, a determination of β for various systems is important in order to probe the effect of magnetic fields.

In our earlier studies,[7,8] we obtained $\mu=0.5$ at $B=0$ for ^{70}Ge:Ga. This result was obtained from precisely doped samples with a perfectly random distribution of impurities; our ^{70}Ge:Ga samples were prepared by neutron-transmutation doping (NTD), in which an ideally random distribution of dopants is inherently guaranteed down to the atomic level.[20–23] For the case of melt- (or metallurgically) doped samples that have been employed in most of the previous studies,[3–5,15–19] the spatial fluctuation of N due to dopant striations and segregation can easily be on the order of 1% or more across a typical sample for the four-point resistance measurement (length of ~5 mm or larger),[24] and hence, it will not be meaningful to discuss physical properties in the critical regime (e.g., $|N/N_c-1|<0.01$), unless one evaluates the macroscopic inhomogeneity in the samples and its influence on the results. A homogeneous distribution of impurities is important also for experiments in magnetic fields. Using the same series of ^{70}Ge:Ga samples that was employed in our previous study,[8] we show here that the critical exponent of the conductivity is 1.1 in magnetic fields for both the doping-induced MIT and the magnetic-field-induced MIT. The phase diagram on the (N,B) plane is successfully constructed, and $\beta=2.5$ is obtained for ^{70}Ge:Ga.

II. EXPERIMENT

All of the ^{70}Ge:Ga samples were prepared by neutron-transmutation doping (NTD) of isotopically enriched ^{70}Ge single crystals. We use the NTD process since it is known to produce the most homogeneous, perfectly random dopant distribution down to the atomic level.[20–23] The concentration N of Ga acceptors is determined from the time of irradiation with thermal neutron. The concentration N is proportional to the irradiation time as long as the same irradiation site and the same power of a nuclear reactor are employed. Details of the sample preparation and characterization are described elsewhere.[8] In this work 12 samples that are metallic in zero magnetic field are studied. (See Table I.) The conductivity of the samples in zero magnetic field has been reported in Refs. 7 and 8.

We determined the electrical conductivity of the samples at low temperatures between 0.05 K and 0.5 K using a ^{3}He-^{4}He dilution refrigerator. Magnetic fields up to 8 T were applied in the direction perpendicular to the current flow by means of a superconducting solenoid.

III. RESULTS

A. Temperature dependence of conductivity

Figure 1 shows the temperature dependence of the conductivity of Sample B2 for several values of the magnetic induction B. Application of the magnetic field decreases the conductivity and eventually drives the sample into the insu-

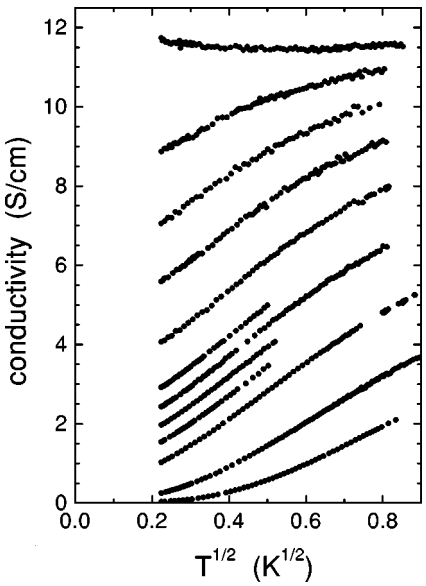

FIG. 1. Conductivity of Sample B2 as a function of $T^{1/2}$ at several magnetic fields. The values of the magnetic induction from top to bottom in units of tesla are 0.0, 1.0, 2.0, 3.0, 4.0, 4.7, 5.0, 5.3, 5.6, 6.0, 7.0, and 8.0, respectively.

lating phase. This property can be understood in terms of the shrinkage of the wave function due to the magnetic field.

The temperature variation of the conductivity $\sigma(N,B,T)$ of a disordered metal at low temperatures is governed mainly by electron-electron interaction,[1] and can be written in zero magnetic field as

$$\sigma(N,0,T)=\sigma(N,0,0)+m(N,0)\,T^{1/2}. \tag{3}$$

When $g\mu_B B \gg k_B T$, i.e., in strong magnetic fields at low temperatures, the conductivity shows another $T^{1/2}$ dependence

$$\sigma(N,B,T)=\sigma(N,B,0)+m_B(N,B)\,T^{1/2}. \tag{4}$$

Here, one should note that these equations are valid only in the limits of $[\sigma(N,0,T)-\sigma(N,0,0)] \ll \sigma(N,0,0)$ or $[\sigma(N,B,T)-\sigma(N,B,0)] \ll \sigma(N,B,0)$. It is for this reason that we have observed a $T^{1/3}$ dependence rather than the $T^{1/2}$ dependence at $B=0$ in ^{70}Ge:Ga as the critical point $[\sigma(N,0,0)=0]$ is approached from the metallic side.[8] However, Fig. 1 shows that the $T^{1/2}$ dependence holds when $B \neq 0$ even around the critical point. Hence, we use Eq. (4) to evaluate the zero-temperature conductivity $\sigma(N,B,0)$ in magnetic fields.

According to an interaction theory for a disordered metal,[1] m and m_B are given by

$$m=\frac{e^2}{\hbar}\frac{1}{4\pi^2}\frac{1.3}{\sqrt{2}}\left(\frac{4}{3}-\frac{3}{2}\tilde{F}\right)\sqrt{\frac{k_B}{\hbar D}} \tag{5}$$

and

$$m_B=\frac{e^2}{\hbar}\frac{1}{4\pi^2}\frac{1.3}{\sqrt{2}}\left(\frac{4}{3}-\frac{1}{2}\tilde{F}\right)\sqrt{\frac{k_B}{\hbar D}}, \tag{6}$$

respectively, where D is the diffusion constant and $\tilde{F}$ is a dimensionless parameter characterizing the Hartree interaction. Since m_B is independent of B, the conductivity for various values of B plotted against $T^{1/2}$ should appear as a group of parallel lines. This is approximately the case as seen in Fig. 1 at low temperatures (e.g., $T<0.25$ K). Values of m (for $B=0$) and m_B at $B=4$ T differ from each other considerably for all the samples as listed in Table I. This implies that $\tilde{F}$ is of the order of unity according to Eqs. (5) and (6). In order to support this finding, we shall estimate $\tilde{F}$ within the context of the Thomas-Fermi approximation.

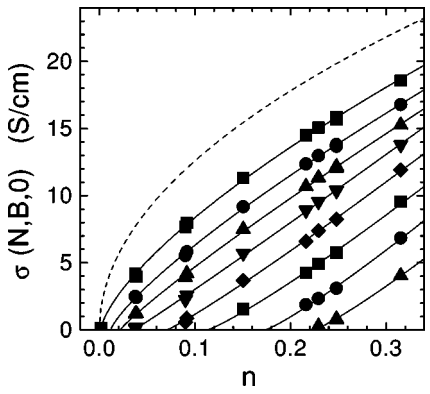

FIG. 2. Zero-temperature conductivity $\sigma(N,B,0)$ vs normalized concentration $n \equiv [\sigma(N,0,0)/\sigma^*(0)]^{2.0} = N/N_c(0)-1$, where $\sigma(N,0,0)$ is the zero-temperature conductivity and $\sigma^*(0)$ is the prefactor both at $B=0$. From top to bottom the magnetic induction increases from 1 T to 8 T in steps of 1 T. The dashed curve at the top is for $B=0$. The solid curves represent fits of $\sigma(N,B,0) \propto [n/n_c(B)-1]^{\mu(B)}$. For $B \geqslant 6$ T, we assume $\mu=1.15$.

The parameter $\tilde{F}$ is related to the average F of the screened Coulomb interaction on the Fermi surface as

$$\tilde{F}=-\frac{32}{3}\left[\frac{1+3F/4-(1+F/2)^{3/2}}{F}\right]. \tag{7}$$

The Thomas-Fermi approximation gives

$$F=\frac{\ln(1+x)}{x}, \tag{8}$$

where

$$x=(2k_F/\kappa)^2, \tag{9}$$

with the Fermi wave vector

$$k_F=(3\pi^2 N)^{1/3}, \tag{10}$$

and the screening wave vector in SI units

$$\kappa=\sqrt{3e^2 N m^*/(\epsilon\epsilon_0\hbar^2 k_F^2)}. \tag{11}$$

For Ge, the relative dielectric constant ϵ is 15.8 and the effective mass m^* of a heavy hole is $0.34m_e$, where m_e is the electron rest mass.[25] Hence $x=1.1\tilde{N}^{1/3}$, where $\tilde{N}$ is in units of 10^{17} cm^{-3}. Thus, in the concentration range covered by the samples, the Thomas-Fermi approximation gives $0.48<\tilde{F}<0.55$, which is consistent with the experimental finding that $\tilde{F}$ is of the order of unity.

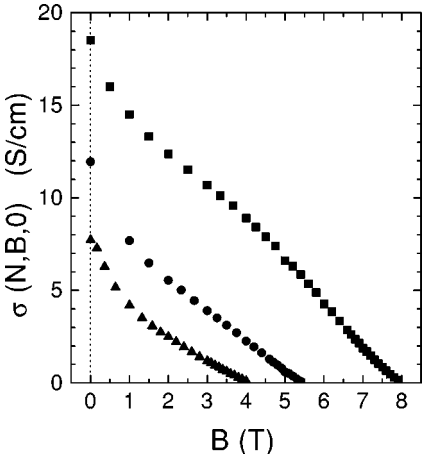

FIG. 3. Zero-temperature conductivity $\sigma(N,B,0)$ of Samples A3, B2, and B4 vs magnetic induction B.

B. Doping-induced metal-insulator transition

The zero-temperature conductivity $\sigma(N,B,0)$ of the ^{70}Ge:Ga samples in various magnetic fields obtained by extrapolation of $\sigma(N,B,T)$ to $T=0$ based on Eq. (4) is shown in Fig. 2. Here, $\sigma(N,B,0)$ is plotted as a function of the normalized concentration:

$$n \equiv [\sigma(N,0,0)/\sigma^*(0)]^{2.0}. \tag{12}$$

Since the relation between N and $\sigma(N,0,0)$ was established for ^{70}Ge:Ga in Ref. 8 as $\sigma(N,0,0)=\sigma^*(0)[N/N_c(0)-1]^{0.50}$ where $N_c(0)=1.861\times10^{17}\,\mathrm{cm}^{-3}$, n is equivalent to $N/N_c(0)-1$. Henceforth, we will use n instead of N because employing n reduces the scattering of the data caused by several experimental uncertainties, and it further helps us concentrate on observing how $\sigma(N,B,0)$ varies as B is increased. Similar evaluations of the concentration have been used by various groups. In their approach, the ratio of the resistance at 4.2 K to that at 300 K is used to determine the concentration.[17] The dashed curve in Fig. 2 is for $B=0$, which merely expresses Eq. (12), and the solid curves represent fits of

$$\sigma(N,B,0)=\sigma_0(B)[n/n_c(B)-1]^{\mu(B)}. \tag{13}$$

The exponent $\mu(B)$ increases from 0.5 with increasing B and reaches a value close to unity at $B\geqslant4$ T. For example, $\mu=1.03\pm0.03$ at $B=4$ T and $\mu=1.09\pm0.05$ at $B=5$ T. When $B\geqslant6$ T, three-parameter [$\sigma_0(B)$, $n_c(B)$, and $\mu(B)$] fits no longer give reasonable results because the number of samples available for the fit decreases with increasing B. Hence, we give the solid curves for $B\geqslant6$ T assuming $\mu(B)=1.15$.

C. Magnetic-field-induced metal-insulator transition

We show $\sigma(N,B,0)$ as a function of B in Fig. 3 for three different samples. When the magnetic field is weak, i.e., the correction $\Delta\sigma_B(N,B,0)\equiv\sigma(N,B,0)-\sigma(N,0,0)$ due to B is small compared with $\sigma(N,0,0)$, the field dependence of $\Delta\sigma_B(N,B,0)$ looks consistent with the prediction by the interaction theory,[1]

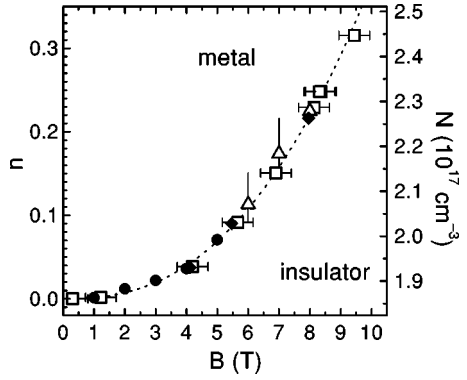

FIG. 4. Phase diagram of ^{70}Ge:Ga at $T=0$. The solid circles and the open triangles represent the critical concentrations n_c, and the solid diamonds and the open boxes the critical magnetic induction B_c.

$$\Delta\sigma_B(N,B,0)=-\frac{e^2}{\hbar}\frac{\tilde{F}}{4\pi^2}\sqrt{\frac{g\mu_B B}{2\hbar D}}\propto\sqrt{B}. \tag{14}$$

In larger magnetic fields, $\sigma(N,B,0)$ deviates from Eq. (14) and eventually vanishes at some magnetic induction B_c. For the samples in Fig. 3, we tuned the magnetic induction to the MIT in a resolution of 0.1 T. We fit an equation similar to Eq. (13),

$$\sigma(N,B,0)=\sigma_0'(n)[1-B/B_c(n)]^{\mu'(n)}, \tag{15}$$

to the data close to the critical point. As a result we obtain $\mu'=1.1\pm0.1$ for all of the three samples. The value of μ' depends on the choice of the magnetic-field range to be used for the fitting, and this fact leads to the error of ±0.1 in the determination of μ'.

D. Phase diagram in magnetic fields

From the critical points $n_c(B)$ and $B_c(n)$, the phase diagram at $T=0$ is constructed on the (N,B) plane as shown in Fig. 4. Here, $n_c(B)$ for $B\geqslant6$ T shown by triangles are obtained by assuming $\mu=1.15$. The vertical solid lines associated with the triangles represent the range of values over which $n_c(B)$ have to exist, i.e., between the highest n in the insulating phase and the lowest n in the metallic phase. Solid diamonds represent B_c for the three samples in which we have studied the magnetic-field-induced MIT in the preceding subsection. Estimations of B_c for the other samples are also shown by open boxes with error bars.

The boundary between metallic phase and insulating phase is expressed by a power-law relation:

$$n=C\,B^{\beta}. \tag{16}$$

From the eight data points denoted by the solid symbols, we obtain $C=(1.33\pm0.17)\times10^{-3}$ $\mathrm{T}^{-\beta}$ and $\beta=2.45\pm0.09$ as shown by the dotted curve.

IV. DISCUSSION

A. Scaling of zero-temperature conductivity in magnetic fields

Now we shall consider the relationship between the two critical exponents: μ for the doping-induced MIT and μ' for

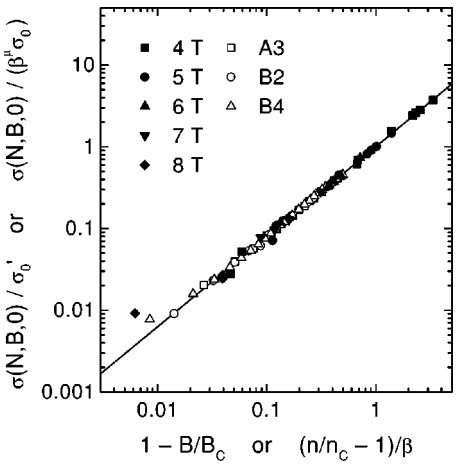

FIG. 5. Normalized zero-temperature conductivity $\sigma(N,B,0)/\sigma_0'(n)$ and $\sigma(N,B,0)/[\beta^\mu \sigma_0(B)]$ as functions of $[1-B/B_c(n)]$ and $[n/n_c(B)-1]/\beta$, respectively, where $\beta=2.5$ and $\mu=1.1$. The solid line denotes a power-law behavior with the exponent of 1.1. The open and solid symbols represent the results of the magnetic-field-induced metal-insulator transition (MIT) in the range $(1-B/B_c)<0.5$ for three different samples (A3, B2, and B4) and the doping-induced MIT in constant magnetic fields (4, 5, 6, 7, and 8 T), respectively.

the magnetic-field-induced MIT. Suppose that a sample with normalized concentration n has a zero-temperature conductivity σ at $B\neq 0$ and that $[n/n_c(B)-1]\ll 1$ or $[1-B/B_c(n)]\ll 1$. From Eqs. (13) and (15), we have two expressions for σ:

$$\sigma=\sigma_0\,(n/n_c-1)^\mu \tag{17}$$

and

$$\sigma=\sigma_0'\,(1-B/B_c)^{\mu'}. \tag{18}$$

On the other hand, we have from Eq. (16)

$$n/n_c=(B/B_c)^{-\beta}=[1-(1-B/B_c)]^{-\beta}\approx 1+\beta(1-B/B_c) \tag{19}$$

in the limit of $(1-B/B_c)\ll 1$. This equation can be rewritten as

$$(n/n_c-1)/\beta\approx(1-B/B_c). \tag{20}$$

Using Eqs. (17), (18), and (20), we obtain

$$\sigma_0'(1-B/B_c)^{\mu'}\approx\beta^\mu\sigma_0(1-B/B_c)^\mu. \tag{21}$$

Since Eq. (21) has to hold for arbitrary B, the following relations

$$\sigma_0'=\beta^\mu\sigma_0 \tag{22}$$

and

$$\mu'=\mu \tag{23}$$

are derived.

In Fig. 5, we see how well Eq. (23) holds for the present system. In Sec. III C, we have already shown that $\mu'=1.1\pm 0.1$ is practically independent of n. Concerning the exponent μ, however, its dependence on B has not been ruled out completely even for the highest B we used in the experiments. This is mainly because the number of available data points at large B is not sufficient for a precise determination of μ. In Fig. 5, the results of the doping-induced MIT for $B\geqslant 4$ T (solid symbols) and the magnetic-field-induced MIT for three different samples (open symbols) are plotted. Here, we plot $\sigma(N,B,0)/[\beta^\mu\sigma_0(B)]$ vs $[n/n_c(B)-1]/\beta$ with $\beta=2.5$ and $\mu=1.1$ for the doping-induced MIT, and $\sigma(N,B,0)/\sigma_0(B)'$ vs $[1-B/B_c(n)]$ for the magnetic-field-induced MIT. Figure 5 clearly shows that the data points align exceptionally well along a single line describing a single exponent $\mu=\mu'=1.1$.

We saw in Fig. 2 that μ apparently takes smaller values in $B\leqslant 3$ T, which seemingly contradicts the above consideration. We can understand this as follows. We find that the critical exponent μ in zero magnetic field is 0.5 which is different from the values of μ in magnetic fields. Hence, one should note whether the system under consideration belongs to the ''magnetic-field regime'' or not. In systems where the MIT occurs, there are several characteristic length scales: the correlation length, the thermal diffusion length, the inelastic scattering length, the spin scattering length, the spin-orbit scattering length, etc. As for the magnetic field, it is characterized by the magnetic length $\lambda\equiv\sqrt{\hbar/eB}$. When λ is smaller than the other length scales, the system is in the ''magnetic-field regime.'' As the correlation length ξ diverges at the MIT, $\lambda<\xi$ holds near the critical point, no matter how weak the magnetic field is. When the field is not sufficiently large, the ''magnetic-field regime'' where we assume $\mu=1.1$ to hold, is restricted to a narrow region of concentration. Outside the region, the system crosses over to the ''zero-field regime'' where $\mu=0.5$ is expected. This is what is seen in Fig. 2.

The constant critical exponent in $B\neq 0$ yields a scaling of the form

$$\sigma(N,B,0)=\tilde{\sigma}(n,B)f(n/B^\beta), \tag{24}$$

where $\tilde{\sigma}(n,B)$ is a prefactor which is irrelevant to the transition. The values of the prefactor are listed in Table II. Here, we list σ^* in Eq. (1) instead of σ_0; σ^* in $B\neq 0$ is calculated from σ_0 as

$$\sigma^*=(1+n_c^{-1})^\mu\sigma_0 \tag{25}$$

because the relation between n/n_c-1 and N/N_c-1 is given by

$$(n/n_c-1)=(1+n_c^{-1})(N/N_c-1). \tag{26}$$

The values of σ^* for zero field and for other doped semiconductors are also given in Table II. The values are normalized to Mott's minimum metallic conductivity defined by

$$\sigma_{\min}\equiv C_{\rm M}(e^2/\hbar)N_c^{1/3}, \tag{27}$$

where we assume $C_{\rm M}=0.05$ as Rosenbaum *et al.*[3] did. The prefactor σ^* can be also defined for the magnetic-field-induced MIT. We proposed to define it based on Eqs. (22) and (23) as

$$\sigma^*\equiv[(1+n^{-1})/\beta]^{\mu'}\sigma_0'. \tag{28}$$

TABLE II. Critical exponent μ and prefactor σ^* for the metal-insulator transition. Values of σ^* normalized to Mott's minimum metallic conductivity defined by Eq. (27) is also listed.

System	Ref.	Magnetic induction (T)	μ	σ^* (10^2 S/cm)	$\sigma^*/\sigma_{\min}$
^{70}Ge:Ga	8	0	0.50 ± 0.04	0.4	6
Ge:Sb	15	0	≈ 0.9	0.6	9
Si:B	17	0	$0.65^{+0.05}_{-0.14}$	1.5	8
Si:P	3	0	≈ 0.48–0.55	3	≈ 13
Si:P	18	0	0.58 ± 0.08	3	14
^{70}Ge:Ga	this work	4	1.1 ± 0.1	0.6	8
^{70}Ge:Ga	this work	5	1.1 ± 0.1	0.6	8
^{70}Ge:Ga	this work	6	1.1 ± 0.1	0.6	9
^{70}Ge:Ga	this work	7	1.1 ± 0.1	0.7	10
^{70}Ge:Ga	this work	8	1.1 ± 0.1	0.7	10
Ge:Sb	15	4	≈ 1.0	0.6	8
Si:B	17	7.5	$1.0^{+0.10}_{-0.20}$	1.7	9
Si:P	18	8	0.86 ± 0.15	3	15

Using this definition, we calculated σ^* for the three ^{70}Ge:Ga samples in which we studied the magnetic-field-induced MIT. The ratios $\sigma^*/\sigma_{\min}$ are 10, 8, and 6 for Samples A3, B2, and B4, respectively. It is reasonable that the ratios $\sigma^*/\sigma_{\min}$ for the three samples and those in Table II are of the same order of magnitude. Note that Mott's minimum metallic conductivity $\sigma_{\min}$ depends on both B and the system through the critical concentration N_c. [See Eq. (27).]

A similar scaling form was studied theoretically by Khmel'nitskii and Larkin.[26] They considered a noninteracting electron system starting from

$$\sigma(N,B,0)\approx \frac{e^2}{\hbar\xi}\ f(B^\alpha \xi), \tag{29}$$

where ξ is the correlation length. They claimed that the argument of the function f should be a power of the magnetic flux through a region with dimension ξ. This means

$$\sigma(N,B,0)\approx \frac{e^2}{\hbar\xi}\ f(\xi/\lambda), \tag{30}$$

where $\lambda\equiv\sqrt{\hbar/eB}$ is the magnetic length, and hence, $\alpha=1/2$. In order to discuss the shift of the MIT due to the magnetic field, they rewrote Eq. (30) as

$$\sigma(N,B,0)\approx \frac{e^2}{\hbar\lambda}\ \phi(t\lambda^{1/\nu}), \tag{31}$$

based on the relation in zero magnetic field

$$\xi\propto t^{-\nu}. \tag{32}$$

Here, t is a measure of distance from the critical point in zero field, e.g.,

$$t\equiv[N/N_c(0)-1]. \tag{33}$$

The zero point of the function ϕ gives the MIT, and the shift of the critical point for the MIT equals

$$N_c(B)-N_c(0)\propto B^{1/2\nu}. \tag{34}$$

Thus, $\beta=1/(2\nu)$ results. Rosenbaum, Field, and Bhatt[16] reported $\beta=0.5$ and $\mu=1$ in Ge:Sb, which satisfies this relation, when one assumes the Wegner relation[13] $\mu=\nu$. In the present system, however, this relation does not hold, as long as we assume the Wegner relation. Experimentally, we find $\beta=2.5$, while $1/(2\nu)=1/(2\mu)=1$ for ^{70}Ge:Ga at $B=0$. The relation does not hold in Si:B, either ($\beta=1.7$ while $\mu=0.65$ at $B=0$).[19]

B. Critical exponents

Finally, we shall discuss the possible origin for the crossover of $\mu=0.5$ at $B=0$ to $\mu=1.1$ at $B\neq0$. According to theories[2] for the MIT dealing with both disorder and electron-electron interaction, systems can be categorized into four universality classes by the symmetry they have as listed in Table III and the value of the critical exponent depends only on the universality class. For classes MF, MI, and SO, the nonlinear sigma model receives some restrictions from the symmetry breaker and the critical exponent ν for the correlation length is calculated for $d=2+\varepsilon$ dimensions as

$$\nu=\frac{1}{\varepsilon}[1+O(\varepsilon)]. \tag{35}$$

Assuming the Wegner relation[13]

$$\mu=\nu(d-2), \tag{36}$$

TABLE III. Universality classes for the metal-insulator transition and values of the critical exponent ν for the correlation length according to Ref. 2. The values are given for $d=2+\varepsilon$ dimensions except for class G, where an approximate value for $d=3$ is given.

Symbol	Symmetry breaker	ν
MF	Magnetic field	$[1+O(\varepsilon)]/\varepsilon$
MI	Magnetic impurities	$[1+O(\varepsilon)]/\varepsilon$
SO	Spin-orbit scattering	$[1+O(\varepsilon)]/\varepsilon$
G	None	$\cong 0.75$

Eq. (35) yields

$$\mu = 1 + O(\varepsilon), \tag{37}$$

which means $\mu \approx 1$ when $\varepsilon \ll 1$. For $d=3$ $(\varepsilon=1)$, however, the theoretical result tells us very little about the value of μ. A calculation for class G is difficult because there is no restriction for the nonlinear sigma model. The value in Table III for $d=3$ is merely an *approximate* one. We believe that ''$\nu \cong 0.75$'' should be treated as less accurate than ''$\nu = 1 + O(1)$'' for the other classes. So, we conclude that values of μ can only be determined at this time by experimental measurements.

Ruling out an ambiguity due to an inhomogeneous distribution of impurities, we have established $\mu = 0.5$ at $B=0$ and $\mu = 1.1$ at $B \neq 0$ for ^{70}Ge:Ga. Since ^{70}Ge:Ga is a *p*-type semiconductor, it is most likely categorized as class SO. Electrical transport properties of *p*-type semiconductors are governed by holes at the top of the valence band, where spin-orbit coupling partially removes the degeneracy and shifts the split-off band down in energy by 0.29 eV in Ge and 0.044 eV in Si.[25] External magnetic fields change the universality class to which a system belongs by breaking the time-reversal symmetry. According to Ref. 2, a system in class SO at $B=0$ belongs to class MI at $B \neq 0$ even if it contains no magnetic impurities. The change of $\mu = 0.5$ to $\mu = 1.1$ observed in ^{70}Ge:Ga due to the application of a magnetic field should be understood in such a context. A similar phenomenon was also found in Si:B.[17] (See Table II.)

V. CONCLUSION

We have measured the electrical conductivity of NTD ^{70}Ge:Ga samples in magnetic fields up to $B=8$ T in order to study the doping-induced MIT (in magnetic fields) and the magnetic-field-induced MIT. For both of the MIT, the critical exponent of the conductivity is 1.1, which is different from the value 0.5 at $B=0$. The change of the critical exponent caused by the applied magnetic fields supports a picture in which μ varies depending on the universality class to which the system belongs. The phase diagram has been determined in magnetic fields for the ^{70}Ge:Ga system.

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