Ann. Phys. (Leipzig) 8 (1999) 7 – 9, 631 – 637

Scaling analysis of the low temperature conductivity in neutron-transmutation-doped ^{70}Ge:Ga

K. M. Itoh[1,2], M. Watanabe[1], Y. Ootuka[3], and E. E. Haller[4]

[1]Department of Applied Physics and Physico-Informatics, Keio University, Yokohama 223-8522, Japan
[2]PRESTO-JST
[3]Institute of Physics, University of Tsukuba, Ibaraki 305-8571, Japan
[4]UC Berkeley and Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA
kitoh@appi.keio.ac.jp

Received 29 July 1999, accepted 1 August 1999 by M. Schreiber

Abstract. We report on the scaling analysis of low temperature electron transport properties of nominally uncompensated neutron-transmutation-doped ^{70}Ge:Ga samples in the critical regime for the metal-insulator transition. Ga concentration (N) and temperature (T) dependent conductivities $\sigma(N,T)$ are shown to collapse onto a single universal curve using finite temperature scaling of a form $\sigma(N,T) \propto T^x f(|N/N_c - 1|/T^y)$ with $x \approx 0.38$ and $y \approx 0.32$ for the very small region of $N = N_c \pm 0.004N_c$. The conductivity critical exponent $\mu = x/y = 1.2 \pm 0.2$ found from this analysis is significantly larger than $\mu \approx 0.5$ found from the analysis we performed previously on the same series of samples covering the much larger region of the concentration $N_c < N < 1.4N_c$. Determination of the true critical region, either $N = N_c \pm 0.4\%$ or $N = N_c \pm 40\%$, is necessary in the future for the reliable determination of μ in Ge:Ga.

Keywords: disordered solids, metal-insulator transitions, doped semiconductor
PACS: 61.43.-j, 71.30.+h, 73.40.Hm

1 Introduction

The metal-insulator (MI) transition in the presence of both disorder and electron-electron interaction turns out to be one of the most challenging subjects in condensed-matter physics. Despite many decades of theoretical [1–5] and experimental efforts [6], researchers are yet to agree upon an unified description of the phenomena. [7] The doping-induced MI transition in single crystalline semiconductors is the best example of disorder and interaction induced transition that has been studied extensively via measurements of physical quantities such as electrical conductivity, dielectric constant, and heat capacity. In particular the critical behavior of the electrical conductivity at zero temperature $\sigma(0)$ has been evaluated as a function of a parameter t that describes the degree of the disorder and interaction;

$$\sigma(0) \propto |t/t_c - 1|^{\mu} \tag{1}$$

where μ is the conductivity critical exponent and t_c is the critical value of t that separates the insulating and metallic phases. The MI transition in semiconductors has been investigated as a function of the doping concentration (N), externally applied magnetic field (B), and externally applied uniaxial stress (S), i.e., $t \equiv N, B, S$, and $t_c \equiv N_c, B_c, S_c$, respectively, in eq. (1). In this paper we consider the MI transition in the uncompensated system Ge:Ga when N is taken as a variable.

Since the classic experiment by Rosenbaum *et al.* that showed $\mu \approx 0.5$ for stress(S)-tuned Si:P [8], a wide variety of experiments has been performed on nominally uncompensated semiconductors. $\mu \approx 0.5$ has been found for doping(N)-tuned Si:As [9] and Ge:Ga [10, 11] while $\mu \approx 0.65$ for N-tuned Si:B. [12] $\mu \approx 0.5$ is puzzling from a theoretical point of view since it violates Chayes *et al.*'s inequality $\mu > 2/3$[13] assuming $\mu = \nu$ [14] where ν is the critical exponent for the localization length ξ. [15] More recently, Löhneysen and co-workers questioned the use of a relatively wide range of N and S that can be fitted with $\mu \approx 0.5$ (typically up to $t = 1.2t_c$ or more) and obtained $\mu \approx 1.3$ on N-tuned Si:P by limiting the critical region to $N_c < N < 1.07N_c$. [16] Rosenbaum *et al.* immediately argued that $\mu \approx 1.3$ region analyzed in Ref. [16] was an artifact due to an inhomogeneous dopant distribution [17]. The question raised "How wide is the critical region?" has, therefore, become one of the most important issues along with the homogeneity of the dopant distribution in the samples.

In all of the above mentioned experiments, the zero temperature conductivity $\sigma(0)$ has been obtained from extrapolation of the finite temperature conductivity $\sigma(T)$ to $T = 0$ assuming a particular temperature dependence of $\sigma(T)$; typically $\propto T^{1/2}$ or $\propto T^{1/3}$. In this work finite temperature scaling of the form; [5]

$$\sigma(t,T) \propto T^x f(|t/t_c - 1|/T^y) \tag{2}$$

has been employed for the analysis of the low temperature conductivity. Here $y = 1/z\nu$ where z is the dynamical scaling exponent. The critical exponent is given by $\mu = x/y$. Eq. (2) has two advantages over the conventional analysis involving eq. (1). Firstly, eq. (2) allows us to use values of $\sigma(t,T)$ taken at finite temperatures, i.e., the conventional extrapolation to $T = 0$ can be avoided. Secondly, eq. (2) allows us to evaluate $\sigma(t,T)$ taken on the both sides of the transition ($t < t_c$ and $t_c < t$). The application of eq. (1), on the other hand, has been limited to the analysis of $\sigma(t,T)$ on the metallic side only.

Very recently, strong evidence has been presented for stress-tuned Si:P [18] and Si:B [19] that $\sigma(S,T)$ scales with eq. (2) on both sides of the transition with $\mu = x/y \approx 1.0$ and 1.6 for Si:P and Si:B, respectively. It was demonstrated graphically that plots of $\sigma(S,T)/T^x$ vs. $|S/S_c - 1|/T^y$ collapse to form a single scaling curve on each side of the transition. [19] Unfortunately, an exact form of the mathematical expression $f(|S/S_c - 1|/T^y)$ in eq. (2) was not obtained for Si:P or Si:B, i.e., it was not clear whether the same $f(|S/S_c - 1|/T^y)$ works on both sides of the transition. Nevertheless, the fact is that the values of $\mu = x/y \approx 1.0$ and 1.6 for Si:P and Si:B, respectively, are significantly larger than the typical values of μ previously obtained by the conventional extrapolation analysis on the metallic samples only, e.g., $\mu \approx 0.5$ for the S-tuned Si:P [8] and $\mu \approx 0.65$ for the N-tuned Si:B. [12]

The present work demonstrates finite temperature scaling of the critical conductivity tuned by impurity concentration (N) in a nominally uncompensated semiconductor, though the range is limited to $N = N_c \pm 0.004N_c$ The exact form of the function $f(|t/t_c - 1|/T^y)$ has been obtained in terms of a third-order non-linear equation, and it describes very well the critical behavior of $\sigma(t,T)$ on both sides of the MI transition. The success of this work is due mostly to the high quality of the samples prepared for this study. Our sample fabrication technique, neutron-transmutation-doping (NTD) of isotopically enriched ^{70}Ge single crystals, leads to a completely random impurity distribution down to the atomic-level. [10, 11] The situation is very different in the melt-doped samples that have been employed in most of the previous studies, [8, 9, 12, 16, 19, 18] in which the spatial fluctuation of N due to dopant striations and segregation can easily be on the order of 1% across a typical sample for four-point conductivity measurements.[20] The present study differs from the previous ones since it analyzes $\sigma(N,T)$ taken from the $|N/N_c - 1| < 1\%$ region using truly homogeneous Ge:Ga samples.

2 Experimental

Eight NTD ^{70}Ge:Ga samples employed in this study have Ga concentrations N in the range $0.994N_c < N < 1.028N_c$. Chemically pure, isotopically enriched ^{70}Ge single crystals of isotopic composition [^{70}Ge]=96.2 at.% and [^{72}Ge]=3.8 at.% were grown and irradiated with thermal neutrons at University of Missouri Research Reactor. Upon capturing a thermal neutron ^{70}Ge becomes a ^{71}Ga acceptor via electron capture, while ^{72}Ge becomes ^{73}Ge which is stable. Therefore the crystal is doped exclusively with Ga with a compensation ratio less than 0.001. The concentration N of Ga after NTD is proportional to the neutron fluence n with the relation,

$$N(\mathrm{cm}^{-3}) = 0.1155 \times n(\mathrm{cm}^{-2}) \tag{3}$$

for our irradiation condition. The following irradiation sequence was conducted in order to minimize the absolute error of N in each sample. The eight samples were doped together in a single irradiation run up to $N = 0.994N_c$. At this point one sample was taken out of the reactor, then the irradiation continued with the remaining seven samples. Another sample was taken out after a short duration of the neutron irradiation corresponding to $\delta N \approx 0.001N_c$, then the irradiation continued with the remaining six samples. This cycle was repeated until the last sample was irradiated up to $N = 1.028N_c$. The relative error in N between samples near N_c is estimated to be less than 0.005%. All the samples were annealed at 650°C for 10 sec in order to remove the structural defects that may have been introduced by the unavoidable flux of fast neutrons. The electrical conductivity measurements were performed in the temperature range 20mK to 1K using a dilution refrigerator.

3 Results and Discussions

Fig. 1 shows the temperature dependence of the electrical conductivity $\sigma(N,T)$ of the eight samples in the temperature range $T = 0.02 - 1$ K. The horizontal axis is proportional to $T^{1/3}$ since this dependence of $\sigma(N,T)$ was firmly established for the same series of the Ge:Ga system when $|N/N_c - 1| < 0.2\%$.[11] The possible origin of the $T^{1/3}$ dependence near the transition has been proposed by several theories.[21,

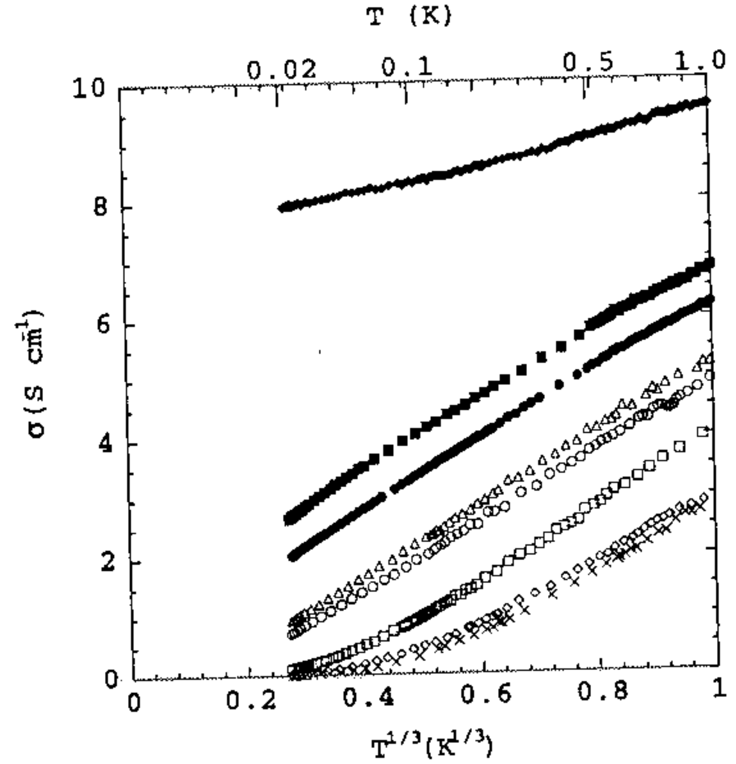

Fig. 1 Electrical conductivity vs. $T^{1/3}$ for the eight samples used in this study. From bottom to top in units of $10^{17}\mathrm{cm}^{-3}$, the concentrations N are 1.848, 1.850, 1.853, 1.856, 1.858, 1.861, 1.863, and 1.912, respectively.

15] $\sigma(N,T) \propto T^{1/3}$ at $N \sim N_c$ immediately implies $x \sim 1/3$ in eq. (2) since the contribution to the temperature dependence from $f(|N/N_c - 1|/T^y)$ becomes negligibly small for the region $N \sim N_c$. It was also shown for the same series of metallic Ge:Ga samples having a wide range of the concentration $N_c < N < 1.4N_c$ that $\mu = 0.5$ when the conventional zero temperature analysis was performed with $N_c = 1.860 \times 10^{17}\mathrm{cm}^{-3}$. [10, 11] Thus $x = 1/3$ and $y = x/\mu = 2/3$ in eq. (2) are expected. Fig. 2 (a) shows the finite temperature scaling plot of $\sigma(N,T)$ using eq. (2) with $x = 1/3$, $y = 2/3$, and $N_c = 1.860 \times 10^{17}\mathrm{cm}^{-3}$ as used above. [10, 11] $\sigma(N,T)$ taken between $T = 20$ and 750mK was used for the analysis in Fig. 2. Fairly good scaling was obtained on the metallic side as expected, while scaling on the insulating side is clearly unsatisfactory. In order to find a better set of x, y, and N_c, a numerical fitting has been performed using the following non-linear equation;

$$\sigma(t,T) = T^x \left[a_0 + a_1 \frac{N/N_c - 1}{T^y} + a_2 \left(\frac{N/N_c - 1}{T^y}\right)^2 + a_3 \left(\frac{N/N_c - 1}{T^y}\right)^3\right]. \quad (4)$$

Fig. 2(b) shows the result of the fitting analysis when the critical region was limited to $N = N_c \pm 0.004N_c$, i.e., only the data from three insulating and two metallic samples closest to the transition are fitted. Unfortunately, we could not achieve any reasonable fit when data from all the eight samples were included. The solid curve in Fig.2(b) is the fit with $x = 0.38$, $y = 0.32$, $N_c = 1.8590 \times 10^{17}\mathrm{cm}^{-3}$, $a_0 = 5.75$, $a_1 = 580$, $a_2 = 1.97 \times 10^4$, and $a_3 = 3.15 \times 10^6$ in eq. (4). A similar fit with a fourth-order

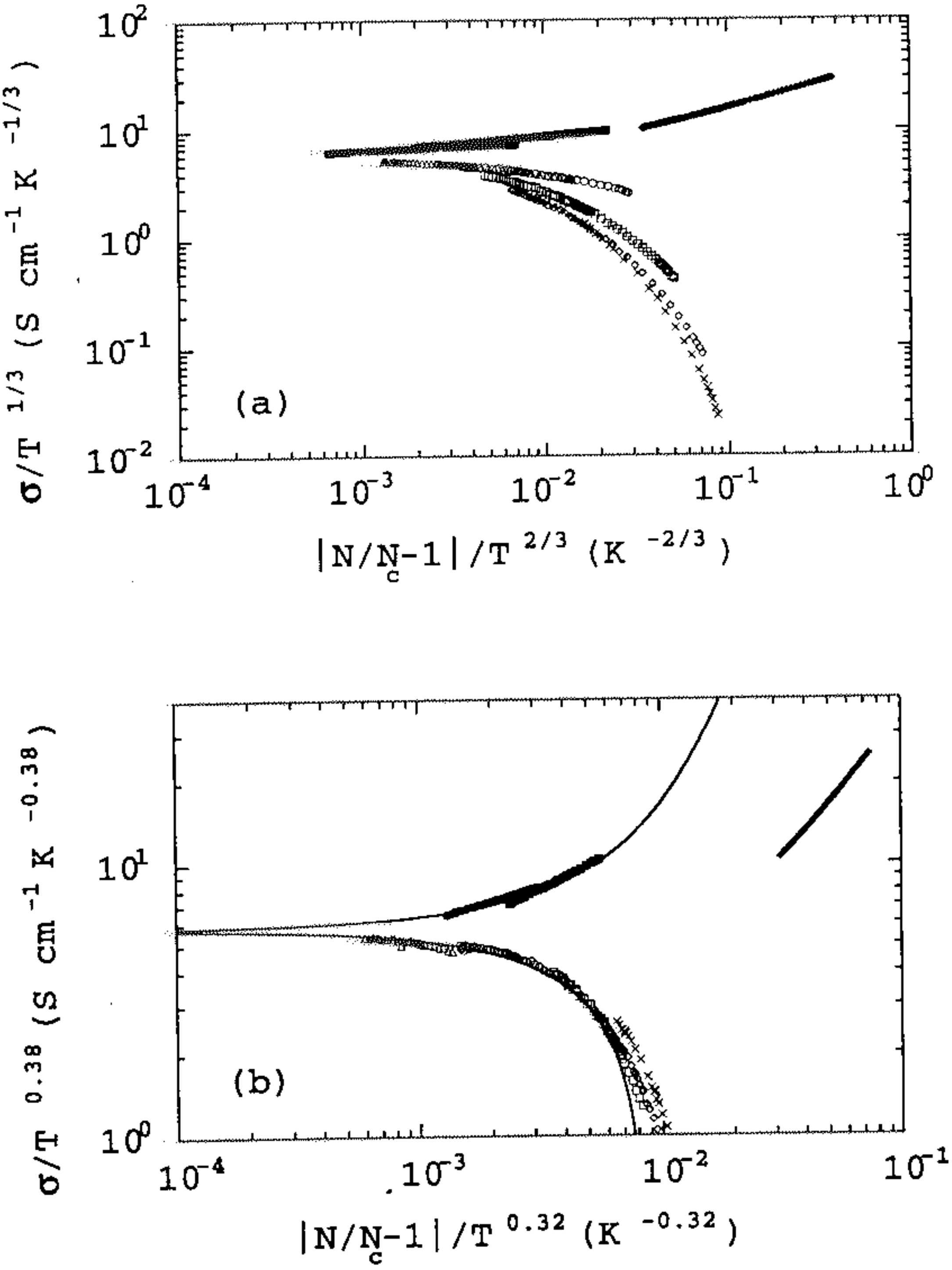

Fig. 2 Finite temperature scaling analysis of $\sigma(N,T)$ using eq. (2) with $x = 1/3$, $y = 2/3$, and $N_c = 1.860 \times 10^{17}\text{cm}^{-3}$, and (b) $x = 0.38$, $y = 0.32$, and $N_c = 1.8590 \times 10^{17}\text{cm}^{-3}$. The solid curve in (b) is the best fit to the data. The symbol for each sample is same as the one in Fig.1.

equation lead to practically the same set of parameters with the absolute value of the fourth-order term being much smaller that that of the lower-order terms, i.e., eq. (4) of the third-order is sufficient for a good fit. The major consequence of this analysis is $\mu = x/y = 1.2 \pm 0.2$ which satisfies the Chayes *et al.*'s inequality $\mu = \nu > 2/3$.[13] Although the range of N we chose for scaling may appear to be too small, we emphasize again that the range is the only part which can be scaled with eq. (2) and that the five samples included are the only ones having $\sigma(N,T) \propto T^{1/3}$ with approximately the same slope $d\sigma/dT^{1/3}$ (see Fig.1). The recent finite temperature scaling of S-tuned Si:P also showed $\mu \approx 1$ only when the data with $\sigma(N,T) \propto T^{1/3}$ are included in the analysis. [18] Assuming Wegner's scaling law $\mu = \nu$ for $d = 3$ [14], we find $z = 3.2$ from the relation $y = 1/z\nu$. This value is consistent with $z \approx 3.2$ found from the relation $x = 0.38 = (z-2)/z$ expected for $d = 3$ when the analytic-term associated with the density of the states is considered. [5] Interestingly, very similar values $x \approx 1/3$ and $\mu \approx 1$ have been reported in the recent S-tuning experiment of Si:P. [18] It was also demonstrated clearly in Ref. [18] that the temperature dependence of the conductivity changes significantly between N- and S-tuning of the identical system Si:P. The finite temperature scaling of N-tuned semiconductors, including Si:P, across N_c has never been reported before possibly because of an inherently small critical region that cannot be studied with the standard doping technique. The experimental study of such a small N region is made possible only by our sample fabrication technique, neutron-transmutation-doping of isotopically enriched semiconductors.

Since we have found two different values of μ, 0.5 and 1.2, depending on the range of the concentration that we analize, further studies which allow for the reliable determination of the width of the critical region are clearly necessary. Recently $\sigma(N,T)$ of Ge:As has been reported to maintain the same $d\sigma/dT^{1/3}$ at low temperatures up to $N = 1.15N_c$. [22] The true test for our approach will be to prepare Ge:As by neutron-transmutation-doping of isotopically enriched ^{74}Ge in order to see if the finite temperature scaling works up to such a large N. If it does, the critical regime may be defined by the samples having a similar $d\sigma/dT^{\alpha}$ around N_c as was proposed by a number of groups. [5, 16]

4 Summary

We have demonstrated finite temperature scaling of the critical conductivity across the MI transition in doping tuned, nominally uncompensated Ge:Ga. The conductivity critical exponent of $\mu = 1.2 \pm 0.2$ has been obtained for the very small region of $N = N_c \pm 0.004N_c$. The values of $\mu \approx 1$ and $z \approx 3$ are consistent with Wegner's scaling law. [14] The exact size of the critical region in terms of doping concentration N should be determined in the future using a variety of homogeneously doped semiconductors.

We acknowledge T. Ohtsuki, K. Slevin, R. N. Bhatt, T. G. Castner, M. P. Sarachik, S. Bogdanovich, J. C. Phillips, and T. Kawarabayashi for valuable discussions. The work at Keio was supported in part by the Grant-in-Aid from the Ministry of Education, Science, Sports, and Culture. One of the authors (M. W.) is a (post-doctoral) Research Fellow of the Japan Society for the Promotion of Science (JSPS). The work at Berkeley was supported in part by the Director, Office of Energy Research, Office of Basic Energy Science, Materials

Science Division of the U.S. Department of Energy under Contract No.DE-AC03-76SF00098 and in part by U.S. NSF Grant No. DMR-9732707.

References

[1] N. F. Mott, *Metal-Insulator Transitions*, 2nd ed. Taylor&Francis, London, 1990
[2] E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42** (1979) 673
[3] P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57** (1985) 287
[4] A. Kawabata, Prog. Theor. Phys. Suppl. **84** (1985) 16
[5] D. Belitz and T. R. Kirkpatrick, Rev. Mod. Phys. **66** (1994) 261
[6] M. P. Sarachik, in *Metal-Insulator Transitions Revisited*, edited by P. P. Edwards and C. N. Rao, Taylor&Francis, London, 1995
[7] E. Abrahams and G. Kotliar, Science **274** (1996) 1853.
[8] T. F. Rosenbaum, K. Andres, G. A. Thomas, and R. N. Bhatt, Phys. Rev. Lett. **45** (1980) 1723
[9] P. F. Newman and D. F. Holocomb, Phys. Rev. B **28** (1983) 638; W. N. Shfarman, D. W. Koon, and T. G. Castner, Phys. Rev. B **40** (1989) 1216
[10] K. M. Itoh, E. E. Haller, J. W. Beeman, W. L. Hansen, J. Emes, L. A. Reichertz, E. Kreysa, T. Shutt, A. Cummings, W. Stockwell, B. Sadoulet, J. Muto, J. W. Farmer, and V. I. Ozhogin, Phys. Rev. Lett. **77** (1996) 4058
[11] M. Watanabe, Y. Ootuka, K. M. Itoh, and E. E. Haller Phys. Rev. B. **58** (1998) 9851
[12] P. Dai, Y. Zhang, and M. P. Sarachik, Phys. Rev. Lett. **66** (1991) 1914
[13] J. Chayes, L. Chayes, D. S. Fisher, and T. Spencer, Phys. Rev. Lett. **57** (1986) 2999
[14] F. Wegner, Z. Phys. B **25** (1976) 327
[15] There are theory that have attempted to explain $\mu \approx 0.5$ in uncompensated semiconductors. Example is: J. C. Phillips, J. Phys. Soc. Jpn. **67** (1999) 3346
[16] H. Stupp, M. Hornung, M. Lakner, O. Madel, and H. v. Löhneysen, Phys. Rev. Lett. **71** (1993) 2634
[17] T. F. Rosenbaum, G. A. Thomas, and M. A. Paalanen: Phys. Rev. Lett. **72** (1994) 2121
[18] S. Waffenschmidt, C. Pfleiderer, and H. v. Löhneysen, cond-mat/9905297, submitted to Phys. Rev. Lett.
[19] S. Bogdanovich, M. P. Sarachik, and R. N. Bhatt: Phys. Rev. Lett. **82** (1999) 137
[20] See, for example, F. Shimura, *Semiconductor Silicon Crystal Technology* , Academic Press, San Diego, 1988, p. 159-161
[21] T. Ohtsuki and T. Kawarabayashi, J. Phys. Soc. Jpn. **66** (1997) 314
[22] I. Shlimak, M. Kaveh, R. Ussyshkin, V. Ginodman, and L. Resnick: Phys. Rev. Lett. **77** (1996) 1103
[23] M. Watanabe, K. M. Itoh, Y. Ootuka, and E. E. Haller, submitted to Phys. Rev. B.