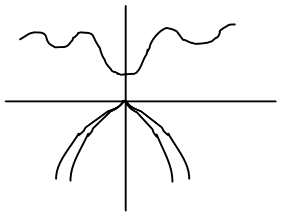
半導体の有効質量

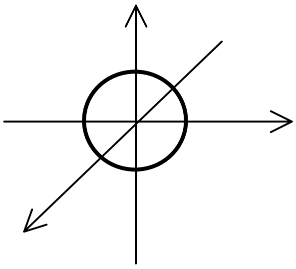


直接遷移型半導体の電子の有効質量

k_x, k_y, k_z 全ての方向に同じ有効質量

等エネルギー面はk空間でΓ点（原点）を中心に

球状 $$E-E_C=\frac{\hbar^2(k_x^2+k_y^2+k_z^2)}{2m_e^*}$$

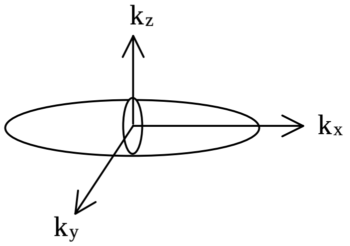


間接遷移型半導体の有効質量

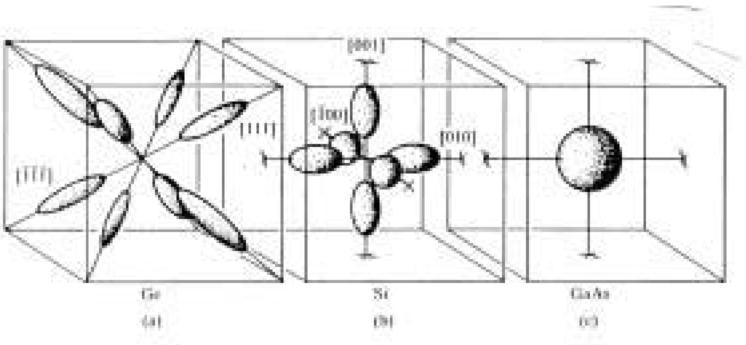
Si, Ge の場合

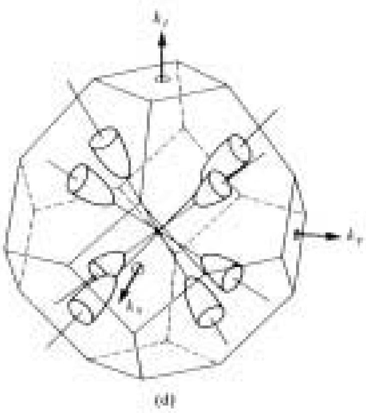
$$E-E_c=\frac{\hbar^2}{2m_\ell^*}k_x^2+\frac{\hbar^2}{2m_t^*}(k_y^2+k_z^2)$$

$$\frac{k_x^2}{\alpha^2}+\frac{k_y^2+k_z^2}{\beta^2}=1$$



$$\alpha=\sqrt{\frac{2m_\ell^*(E-E_c)}{\hbar^2}}\qquad\beta=\sqrt{\frac{2m_t^*(E-E_c)}{\hbar^2}}$$





$$N_{el}\left(\frac{4}{3}\pi\alpha\beta^2\right) = \frac{4}{3}\pi k_{eff}^3 \Rightarrow N_{el}(m_l^* m_t^{*2}) = (m_e^*)^{\frac{3}{2}}$$

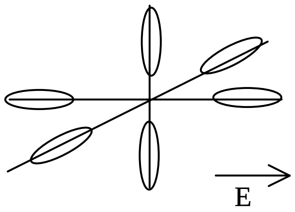
$\Downarrow$

ブリリュアンゾーン中の
葉巻の数 Si で６、Ge で４

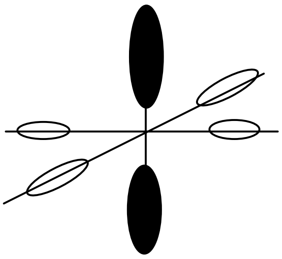
$$\begin{cases} m_e^* = 6^{\frac{2}{3}}(m_l^* m_t^{*2})^{\frac{1}{3}} & \text{が Si} \\ m_e^* = 4^{\frac{2}{3}}(m_l^* m_t^{*2})^{\frac{1}{3}} & \text{が Ge} \end{cases}$$

→ 電子の状態密度有効質量　m_{de}^*　（状態の数）

電子の電気伝導度有効質量　m_{ce}^*　は間接遷移では異方的



４つの軽い質量と２つの重い質量



$$\frac{1}{m_{ce}^*} \equiv \frac{1}{3m_l^*} + \frac{2}{3m_t^*}$$

等エネルギー面を
変化させてm*を
かえて移動度をあげる

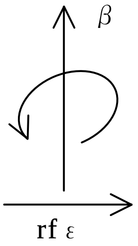
正孔の有効質量　　m_h^*

$$g_V(E) = \frac{\sqrt{2}m_h^{*\frac{3}{2}}(E_V - E)^{\frac{1}{2}}}{\pi^3\hbar^3} = \frac{\sqrt{2}m_{hh}^{*\frac{3}{2}}(E_V - E)^{\frac{1}{2}}}{\pi^3\hbar^3} + \frac{\sqrt{2}m_{lh}^{*\frac{3}{2}}(E_V - E)^{\frac{1}{2}}}{\pi^3\hbar^3}$$

よって　$(m_h^*)^{\frac{3}{2}} = (m_{hh}^*)^{\frac{3}{2}} + (m_{lh}^*)^{\frac{3}{2}}$

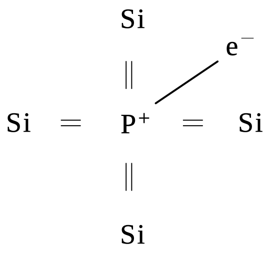
$$m_h^* = \left[(m_{hh}^*)^{\frac{3}{2}} + (m_{lh}^*)^{\frac{3}{2}}\right]^{\frac{2}{3}}$$

有効質量の測定法　サイクロトロン共鳴



有効質量	Ge	Si	GaAs
m_l^*/m_0	1.588	0.19163	—
m_t^*/m_0	0.08152	0.1905	—
m_e^*/m_0	—	—	0.067
m_{hh}^*/m_0	0.347	0.537	0.51
m_{lh}^*/m_0	0.0429	0.153	0.082
m_{so}^*/m_0	0.077	0.234	0.154

不純物半導体

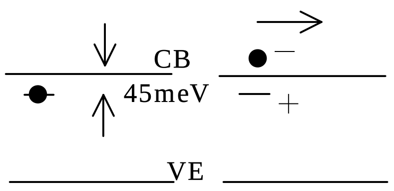


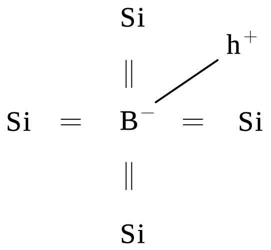
結合エネルギー（ｲｵﾝ化ｴﾈﾙｷﾞｰ）

45meV << Eg=1.1eV

n 型不純物

ドナー不純物



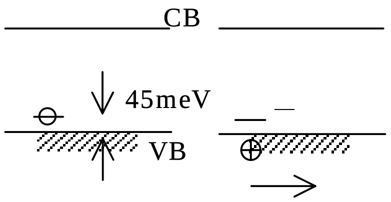


イオン化エネルギー

45meV << Eg=1.1eV

P 型不純物

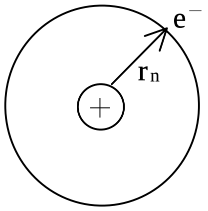
アクセプタ不純物



GaAs 中の C は？

- ボーア原子（浅い不純物）

角運動量＝$m^*vr_n = n\hbar$‥‥‥①



SI 単位系で $\frac{m^*v^2}{r_n} = \frac{e^2}{4\pi\varepsilon\varepsilon_0 r_n^2}$ ‥‥‥②

$\hookrightarrow$ 8. 85×10^{-12}

①－②より $r_n = \frac{4\pi\varepsilon\varepsilon_0 (n\hbar)^2}{m^* e^2}$ ‥‥‥n=1　ボーア半径

‥‥‥③

Ge	r_1～8nm
Si	r_1～3nm
GaAs	r_1～12nm

電子の運動エネルギー(E_k)とポテンシャルエネルギー(U)は

$$E_k = \frac{1}{2}m^*v^2 = \frac{1}{2}\left(\frac{e^2}{4\pi\varepsilon\varepsilon_0 r_n}\right)$$

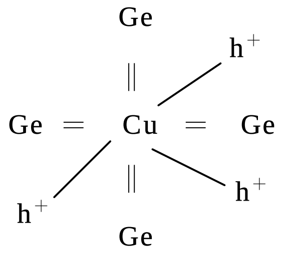
$$U = -\frac{e^2}{4\pi\varepsilon\varepsilon_0 r_n}$$　（ r ＝∞で U＝０）

よって、$E_n = E_k + U = -\frac{1}{2}\left(\frac{e^2}{4\pi\varepsilon\varepsilon_0 r_n}\right)$‥‥‥④

③→④　$E_n = -\frac{m^* e^4}{2(4\pi\varepsilon\varepsilon_0 n\hbar)^2} = \frac{13.6}{\varepsilon^2}\frac{m^*}{m_0}$　(eV)

大体の半導体で ε～10　$m^* \le m_0 \Rightarrow E_n \le 0.1eV$

深い準位



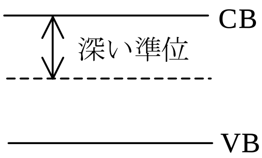
Cu^0 (中性の Cu)は 3 個のホールを有する		
Cu^{-1}	は 2 個	〃
Cu^{-2}	は 1 個	〃
Cu^{-3}	は 0 個	〃

イオン化エネルギー：$E^{0/-} < E^{-1/-2} < E^{-2/-3} >> E_1(Ge) \simeq 10meV$

33meV　　浅い準位

実空間で局在している

ΔX が小さい

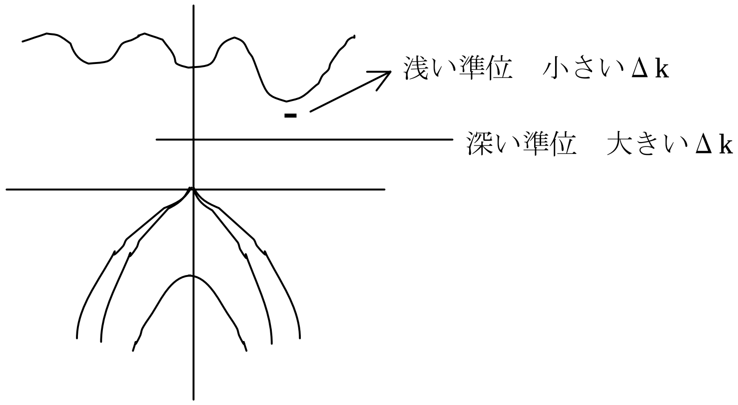


E―k のバンド図では？

ΔpΔx〜h

ℏ ΔkΔx〜h

ΔkΔx〜2π



Si = Si = Si

‖　‖　‖　　Au は Si 内の深い準位

Si = Si = Si

‖　‖ Au ‖

Si = Si = Si

$n_i^2 = np$　　　中性の条件　　　→pn 接合

↓　　　$p - n + N_D^+ - N_A^- = 0$ →トタンジスター

縮退　　　→ホール効果　　　<u>SiC の例</u>

フェルミレベルの位置

真性半導体　$n = p \rightarrow N_C e^{(E_F - E_C)/kT} = N_V e^{(E_V - E_F)/kT}$

ここで$E_F = E_i$とおくと

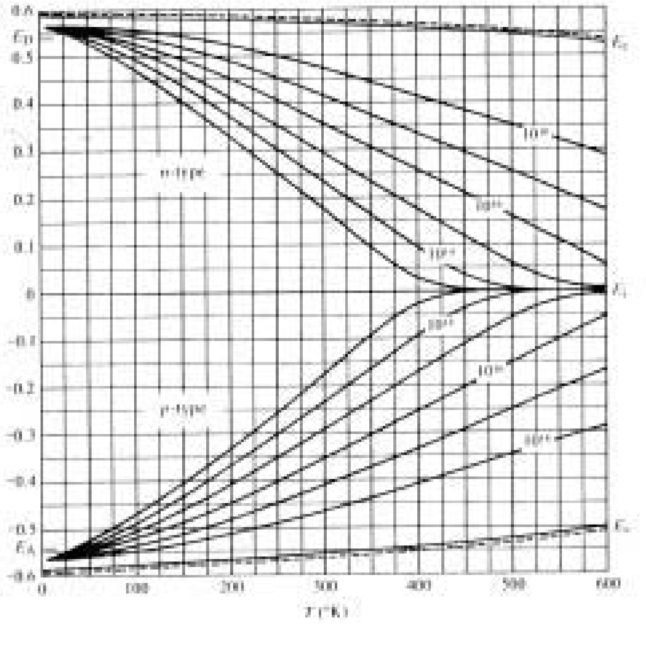
$$E_i = \frac{E_C + E_V}{2} + \frac{kT}{2}\underline{\ln\left(\frac{N_V}{N_C}\right)} \nearrow \quad \frac{N_V}{N_C} = \left(\frac{m_h^*}{m_e^*}\right)^{\frac{3}{2}}$$

$$E_i = \frac{E_C + E_V}{2} + \frac{3}{4}kT\ln\left(\frac{m_h^*}{m_e^*}\right)$$

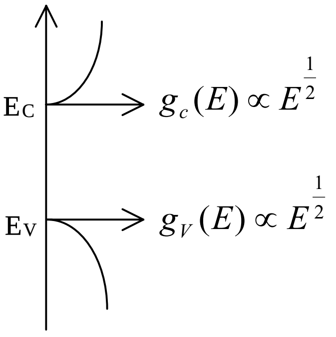
不純物半導体　$kT > E_D, E_A$　（ドナー、アクセプタのイオン化エネルギー）
$n = N_D$（ドナー濃度）又は　$p = N_A$（アクセプタ濃度）

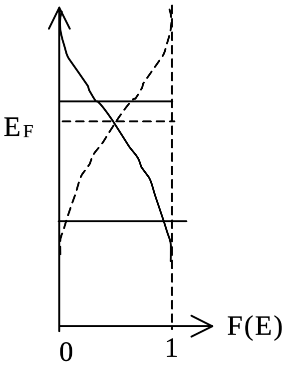
$$E_F - E_i = kT\ln\left(\frac{n}{n_i}\right) = -kT\ln\left(\frac{P}{n_i}\right)$$

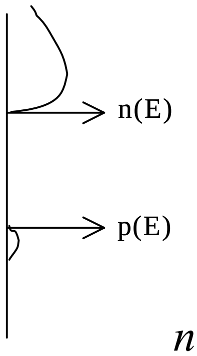
$$\begin{cases} E_F - E_i = kT\ln\left(\dfrac{N_D}{n_i}\right), & N_D >> N_A, N_D >> n_i \\[2em] E_i - E_F = kT\ln\left(\dfrac{N_A}{n_i}\right), & N_A >> N_D, N_A >> n_i \end{cases}$$



n 型

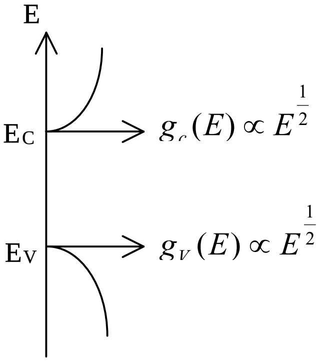


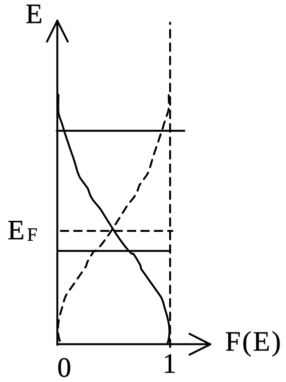


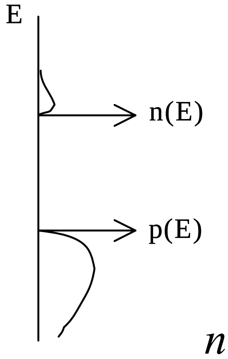


$$n_i^2 = np$$

p 型



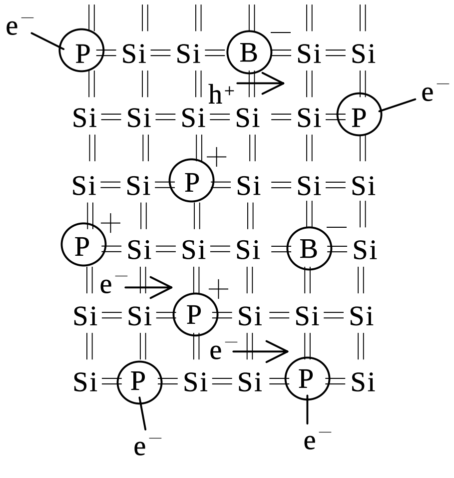




$$n_i^2 = np$$

電荷中性の法則

浅い準位の例で考える



$N_D = 7$

$N_A = 2$

左の例では $N_D^+ = 3$， $N_A^- = 2$

n=2 ， p=1

マイナスの数　$n + N_A^- = 4$

プラスの数　$p + N_D^+ = 4$

等しくなるので中性

$$p - N + N_D^+ - N_A^- = 0$$

pn 接合

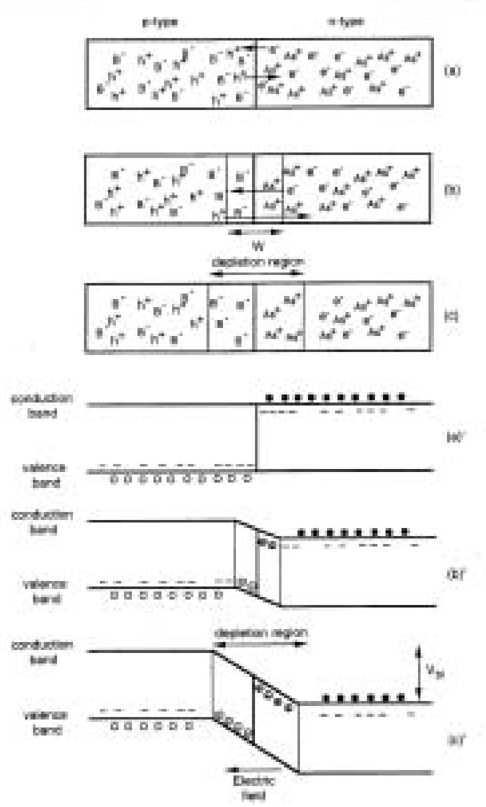
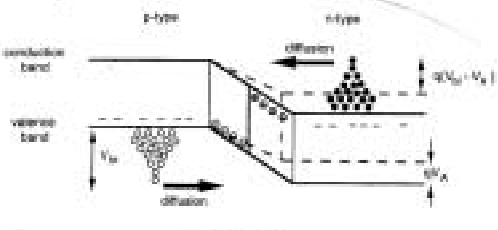
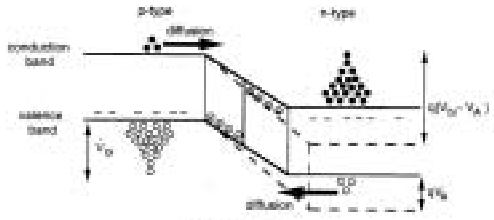


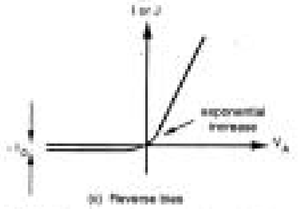
Fig. 2 pn-junctions and their band diagrams



(a) Forward bias



(b) Reverse bias



(c) Reverse bias

Fig. 3 Current flow mechanisms in pn-junctions

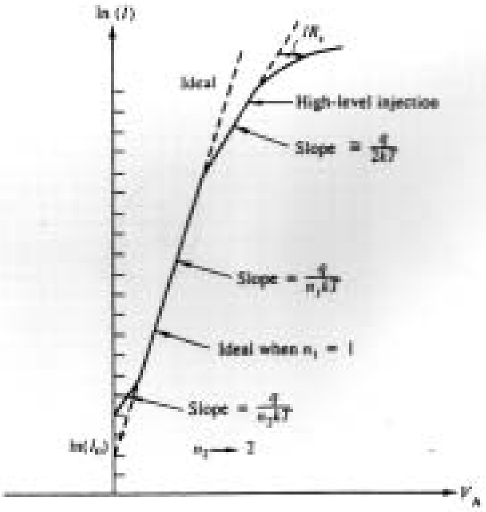


Fig. 4 順バイアスにおける電流・電圧特性（文献１より転写）

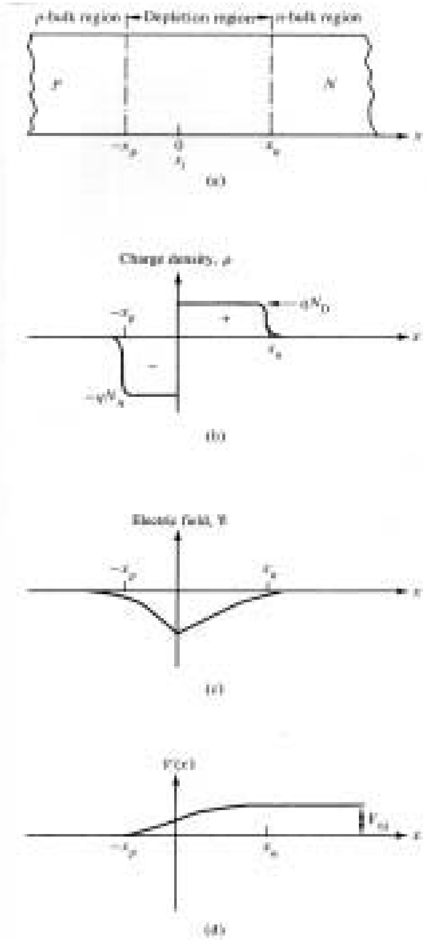


Fig. 2.3 Depletion region electrostatics.

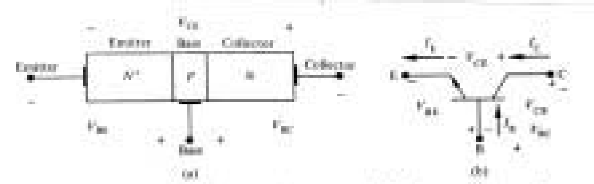


Fig. 1.2 npn bipolar transistor: (a) semiconductor types, and (b) voltage and current reference polarities.

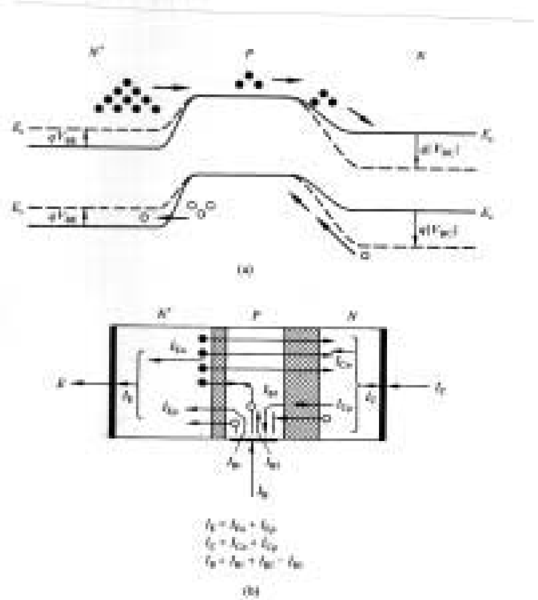


Fig. 1.7 n^+pn in the active region: (a) energy band diagram (—— thermal equilibrium, - - - active region); (b) current components and carrier flux.

JFET

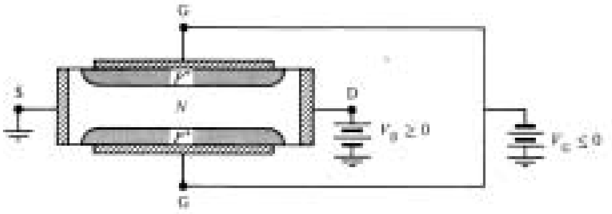


Fig. 1.4 Specification of the device structure and biasing conditions assumed in the qualitative analysis.

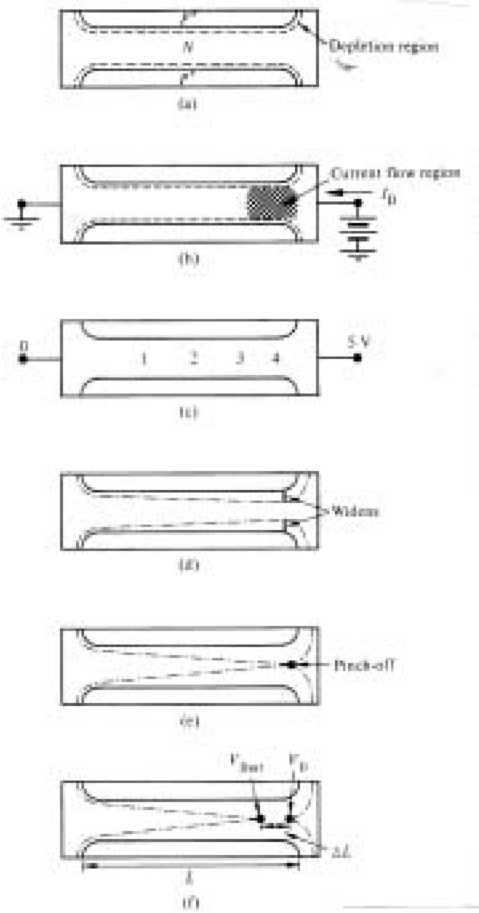


Fig. 1.5 Visualization of various phases of $V_G = 0$ J-FET operation. (a) Equilibrium ($V_D = 0$, $V_G = 0$); (b) small V_D biasing; (c) voltage drop down the channel for an arbitrarily assumed $V_D = 5V$; (d) channel narrowing under moderate V_D biasing; (e) pinch-off; (f) postpinch-off ($V_D > V_{Dsat}$).

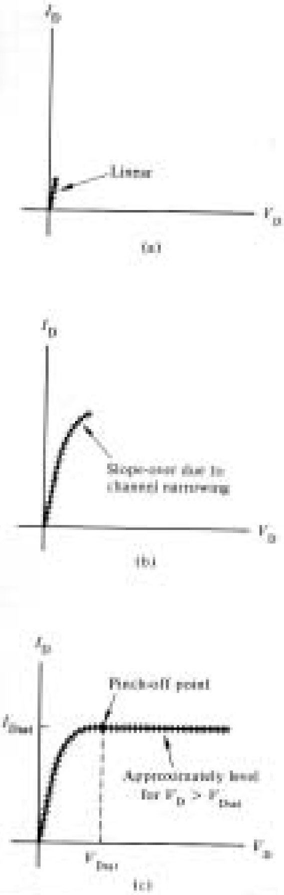


Fig. 1.6 General form of the I_D–V_D characteristics. (a) Linear, simple resistor, variation for very small drain voltages. (b) Slope-over at moderate drain biases due to channel narrowing. (c) Pinch-off and saturation for drain voltages in excess of V_{Dsat}.

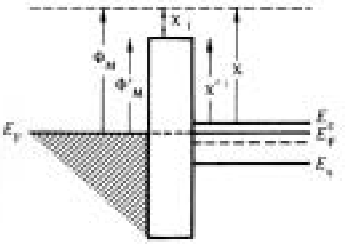


Fig. 2.3 Equilibrium energy band diagram for an ideal MOS structure.

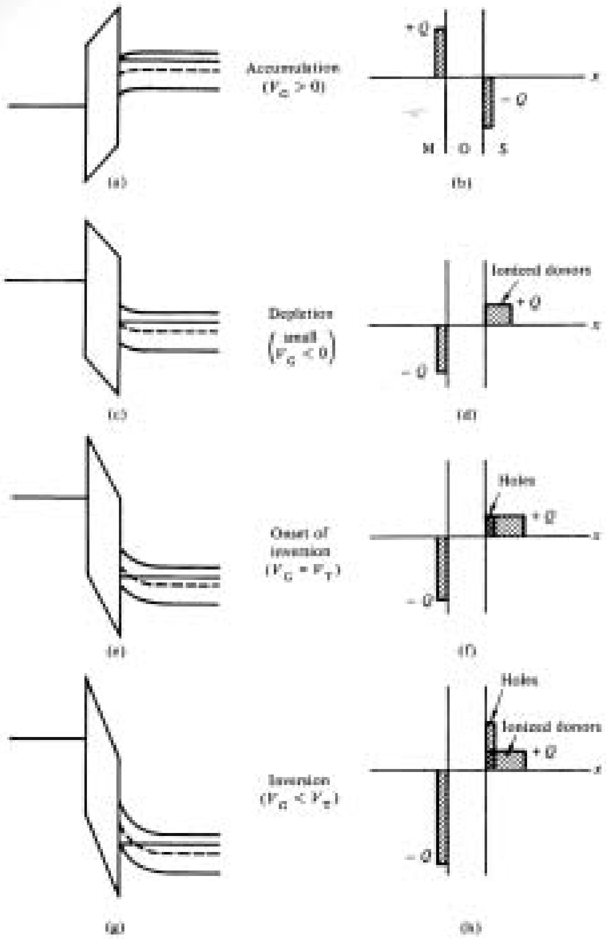


Fig. 2.5 Energy band diagrams and corresponding block charge diagrams describing the static state in an ideal *n*-type MOS-capacitor.

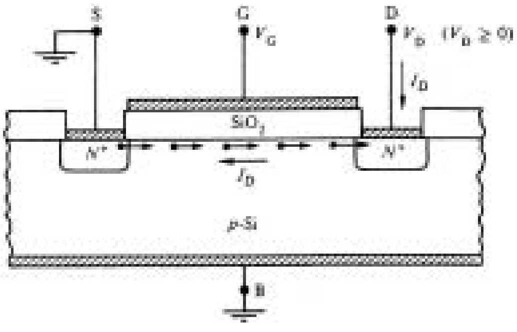


Fig. 3.1 Cross-sectional view of the basic *p*-bulk (*n*-channel) MOSFET structure showing the terminal designations, carrier and current flow directions, and standard biasing conditions.

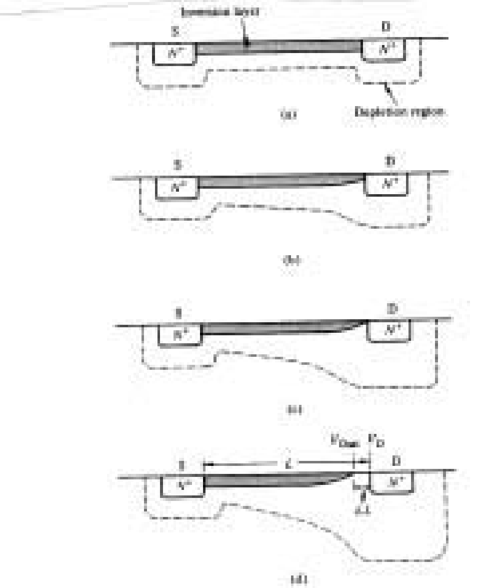


Fig. 3.3 Visualization of various phases of $V_G > V_T$ MOSFET operation. (a) $V_D = 0$; (b) channel (inversion layer) narrowing under moderate V_D biasing; (c) pinch-off; and (d) postpinch-off ($V_D > V_{Dsat}$) operation. (Note that the inversion layer widths, depletion widths, etc. are not drawn to scale.)

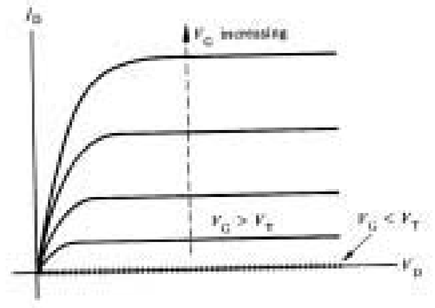
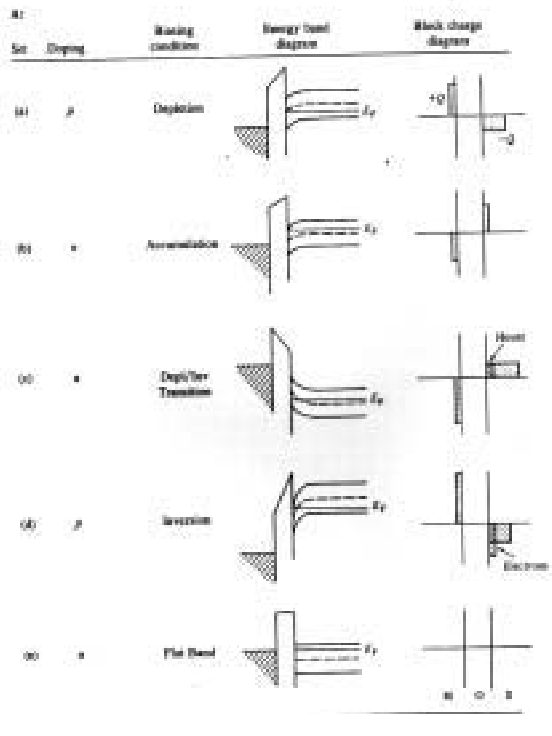
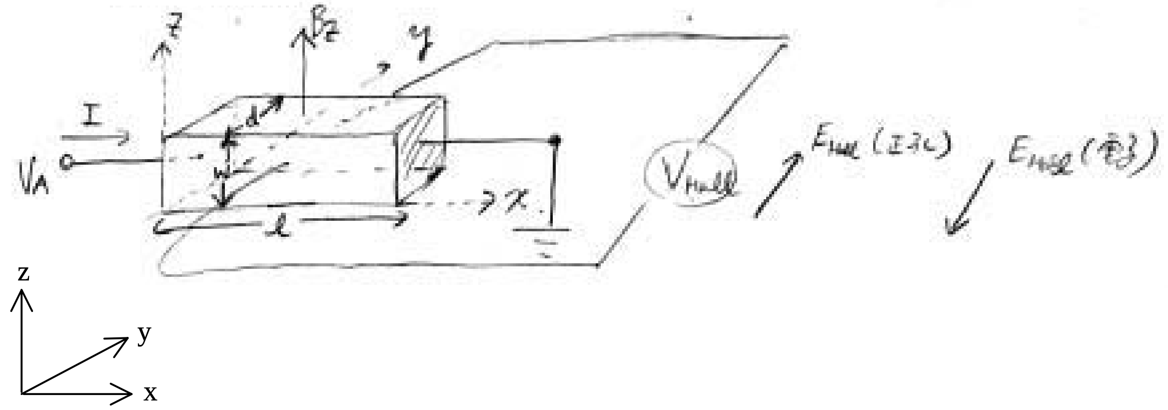


Fig. 3.4 General form of the I_D–V_D characteristics expected from a long channel ($\Delta L \ll L$) MOSFET.



ホール効果



$$\vec{F} = -e\vec{v} \times \vec{B} = -e\mu\vec{E} \times \vec{B}$$
$$F_y = -e\mu E_x B_z$$

平衡状態では　$-eE_{Hall} = -F_y = e\mu E_x B_z$

電子の場合　$E_{Hall} = -\mu_x E_x B_z$

ホール係数

$$\boxed{\begin{aligned} R_H &= \frac{E_{Hall}}{J_x B_z} = \frac{V_{Hall}/d}{(I/wd)B} = \frac{V_H W}{BI} \\ &= \frac{E_{Hall}}{\sigma_x E_x B_z} = \frac{-\mu B_z E_x}{\underbrace{\mu_x en}_{\sigma_x} E_x B_z} = -\frac{1}{en} \end{aligned}}$$

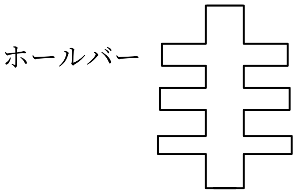
$$R_H = \frac{V_{Hall} W}{BI} \begin{cases} = -\dfrac{1}{en} (\text{電子}) \\ = \dfrac{1}{en} (\text{正孔、ホール}) \end{cases}$$

さらに　$\sigma = \begin{cases} en\mu \\ ep\mu \end{cases}$

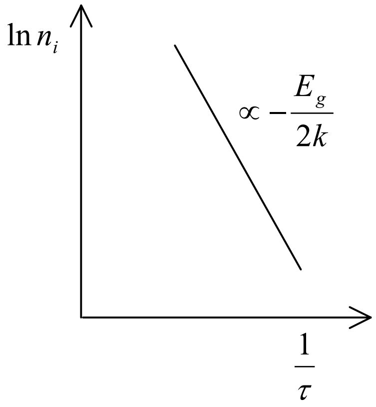
だから　抵抗測定から　σ　を求め

ホール効果から　n

移動度 μ も計算できる。



真性半導体のホール効果測定効果



必要なエネルギーは E_g

$$\begin{tikzcd} Si^0 \arrow[r, shift left] & Si^+ \arrow[l, shift left] \arrow[d] & + e^- \arrow[d] \\ & p & n \end{tikzcd}$$

$$\frac{nP}{[Si]} = C\exp\left(-\frac{E_g}{kT}\right)$$

$$n_i^2 = C'\exp\left(-\frac{E_g}{kT}\right)$$

$$n_i = C'\exp\left(-\frac{E_g}{2kT}\right)$$

では不純物半導体では？

ドナー濃度 $N_D >> 0$ で考える。　アクセプタ濃度 $N_A = 0$

$$\begin{tikzcd} D^0 \arrow[r] \arrow[d] & D^+ \arrow[d] & + e^- \arrow[d] \\ N_D^0 & N_D^+ & n \end{tikzcd}$$

$$\therefore \quad \frac{N_D^+ n}{N_D^0} = C_D \exp\left(-\frac{E_D}{kT}\right)$$

ここで　$N_D^+ = n$　　$N_D^0 = N_D - n$

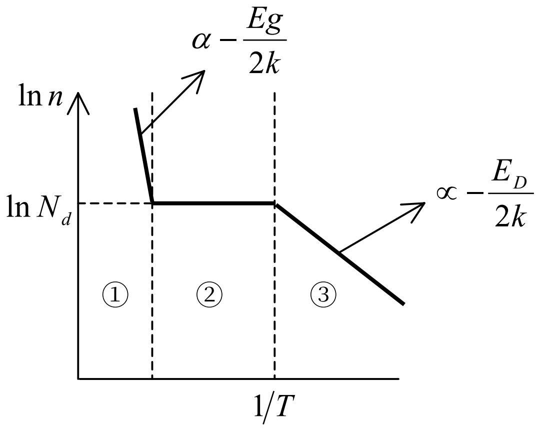
$$\therefore \quad \frac{n^2}{N_D - n} = C_D \exp\left(-\frac{E_D}{kT}\right)$$

①高温領域　$kT >> E_g >> E_D \rightarrow n_i >> N_D$　　$n \propto \exp\left(-\frac{E_g}{2kT}\right)$

②中温領域　$E_g >> kT >> E_D \rightarrow N_D \approx N_D^+$　　$n \approx N_D$

③低温領域　$E_D > kT$　$N_D >> n$　よって

$$\frac{n^2}{N_D - \cancel{n}} = C_D \exp\left(-\frac{E_D}{kT}\right) \Rightarrow n = C_D' \exp\left(-\frac{E_D}{2kT}\right)$$



補償のある半導体では？

N_DとN_Aが共存している。ただし、$N_D >> N_A$（n型）を仮定すると

$$D^0 \to D^+ + e^-$$

↓　↓　↓　　は同じ

N_D^0　N_D^+　n

$$\frac{N_D^+ n}{N_D^0} = C_{DA}\exp\left(-\frac{E_D}{kT}\right)$$

ここで
$$\begin{cases} N_D^+ = n + N_A \\ N_D^0 = N_D - N_D^+ = N_D - n - N_A \end{cases}$$

→ すべてのアクセプタが－にイオン化$N_A^- = N_A$
同じ量のドナーが＋にイオン化$N_D^+ = N_A$

$$\frac{(n + N_A)n}{N_D - n - N_A} = C_{DA}\exp\left(-\frac{E_D}{kT}\right)$$

①高温領域　$kT >> E_g >> E_D \to n_i >> N_D$　　$n \propto \exp\left(-\frac{E_g}{kT}\right)$

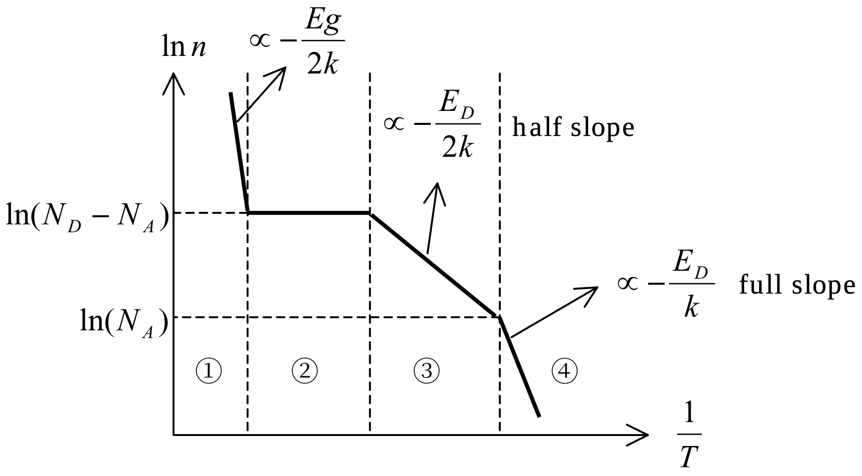
②中温領域　$E_g >> kT >> E_D \to n = N_D - N_D$　一定

③低温領域　$E_D >> kT$　　$N_D >> n >> N_A$

$$\frac{(n + \cancel{N_A})n}{N_D - \cancel{n} - \cancel{N_A}} = \frac{n^2}{N_D} = C_{DA}\exp\left(-\frac{E_D}{kT}\right) \Rightarrow n = C'_{DA} - \exp\left(-\frac{E_D}{2kT}\right)$$

④超低温領域　$E_D >> kT$　　$N_D >> N_A >> n$

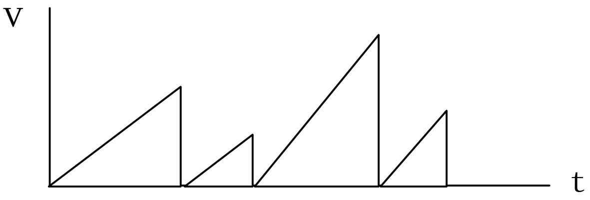
$$\frac{(n + N_A)n}{N_D - n - N_A} = \frac{nN_A}{N_D} = C_{DA}\exp\left(-\frac{E_D}{kT}\right) \Rightarrow n = C''_{DA}\exp\left(-\frac{E_D}{kT}\right)$$



移動度の変化は？

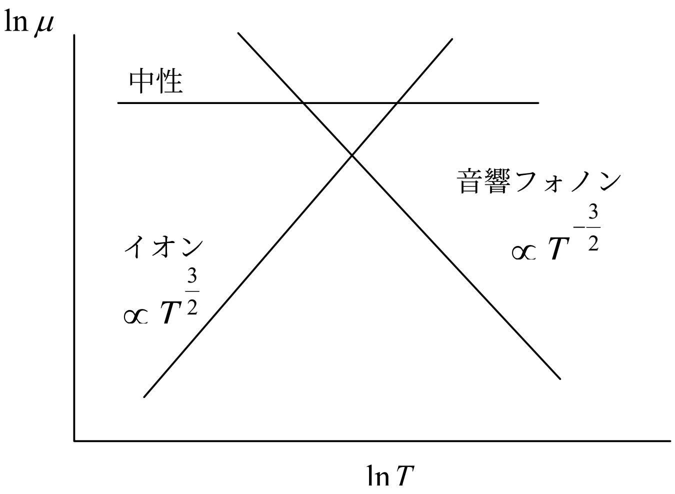
$$\mu = \frac{e\langle\tau\rangle}{m^*}$$

$\langle\tau\rangle$：平均緩和時間



散乱の要因

- 音響フォノン散乱　$\langle\tau\rangle \propto T^{-\frac{3}{2}}$
- イオン化不純物散乱　$\langle\tau\rangle \propto T^{\frac{3}{2}}$
- 中性不純物散乱　$\langle\tau\rangle$はTに依存しない
- 光学フォノン散乱・インターバレー散乱　$\langle\tau\rangle \propto T^{-2.3}$位



トータルな移動度 μ_{tot}

$$\frac{1}{\mu_{tot}} = \frac{1}{\mu_{\text{音響フォノン}}} + \frac{1}{\mu_{\text{イオン}}} + \frac{1}{\mu_{\text{中性}}}$$