All-Silicon Quantum Computer

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Quantum computation with spins

1. Initialization
   - qubit #1
   - qubit #2
   - qubit #3
   - qubit #4
   - qubit #5
   - qubit #6

# of qubits

2. Operation
   - Rotation
   - Controlled NOT

# of operation

3. Read-out
   - Spin-polarized probe

Spin quantum bits

Classical bit with spins
   - 0
   - 1
   - 0 nor 1 (error)

Quantum bit with spins
   - $|0\rangle + |1\rangle$
   - $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

Quantum spin allows for equal probability of 0 and 1.
### Quantum parallelism

<table>
<thead>
<tr>
<th>Binary numbers</th>
<th>Decimal numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0 0</td>
<td>0</td>
</tr>
<tr>
<td>0 0 1 1</td>
<td>1</td>
</tr>
<tr>
<td>0 1 0 2</td>
<td>2</td>
</tr>
<tr>
<td>0 1 1 3</td>
<td>3</td>
</tr>
<tr>
<td>1 0 0 4</td>
<td>4</td>
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<tr>
<td>1 0 1 5</td>
<td>5</td>
</tr>
<tr>
<td>1 1 0 6</td>
<td>6</td>
</tr>
<tr>
<td>1 1 1 7</td>
<td>7</td>
</tr>
</tbody>
</table>

Simultaneous processing of $2^n$ numbers

### Candidates for qubits

1. Nuclear spin
2. Electronic spin

Long decoherence time needed
Figures of merits

1. Scalable # of qubits (n) \(\rightarrow 2^n\) states

2. Large # of total operation \(\equiv\) \(\frac{\text{Phase decoherence time } T_2}{\text{Switching time } t_s}\)

<table>
<thead>
<tr>
<th>qubit</th>
<th>(T_2) (sec)</th>
<th>(t_s) (sec)</th>
<th># of operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electronic state</td>
<td>(10^{-9})</td>
<td>(10^{-13})</td>
<td>(10^4)</td>
</tr>
<tr>
<td>Electronic spin</td>
<td>(10^{-6})</td>
<td>(10^{-10})</td>
<td>(10^4)</td>
</tr>
<tr>
<td>Ion state</td>
<td>(10^{-1})</td>
<td>(10^{-14})</td>
<td>(10^{13})</td>
</tr>
<tr>
<td>Nuclear spin</td>
<td>(10^3)</td>
<td>(10^{-4})</td>
<td>(10^7)</td>
</tr>
</tbody>
</table>

 photon

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The World of Experimental Quantum Computation

- Neutral Atoms in Optical Lattices
- Atomic Cavity QED
- Trapped Ions
- Solution NMR
- Solid-State Systems
- Optically Driven Electronic States in Quantum Dots
- Flux States in Superconductors
- Charge States in Superconductors
- Electrons Floating on Liquid Helium
- Others: Nonlinear Optics, STM, . . .

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Experimental situation

- A small number of stable qubits provided by distinct spin 1/2 nuclei
- Perfect initialization is not possible. Instead, “pseudo-pure” states must be used
- Dipolar interactions controllable by RF
- Measurement is facilitated by huge ensemble of independent copies
- Decoherence times are long since nuclei interact only weakly with environment

Solution NMR Quantum Computation

1. A small number of stable qubits provided by distinct spin 1/2 nuclei
2. Perfect initialization is not possible. Instead, “pseudo-pure” states must be used
3. Dipolar interactions controllable by RF
4. Measurement is facilitated by huge ensemble of independent copies
5. Decoherence times are long since nuclei interact only weakly with environment

N. A. Gershenfeld and I. Chuang, Science 275, 350 (1997)
### Solld-State Impurity NMR QC

1. Isolated impurity nuclei provide qubits
2. Low temperature electrons allow initialization
3. Electron-mediated interactions controlled by gates
4. Single-spin measurement via nuclear-electron coupling is proposed
5. Well-separated impurities have long decoherence times

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### Motivation

**Solution NMR QC**
- Advantages:
  - Ensemble measurement
  - Natural (chemical) fabrication
- Disadvantages:
  - Challenging to scale to many qubits and/or gates
  - Initialization difficult

**Solid-State Impurity QC**
- Advantages:
  - Scalable!
  - Can cool to low temperatures for initialization
- Disadvantages:
  - Need single-spin measurement
  - Challenging fabrication

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**All Silicon QC uses advantages of both!**

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Semiconductor Isotope Engineering

Silicon: Si
Germanium: Ge
Silicon-Germanium: SiGe

<table>
<thead>
<tr>
<th>Element</th>
<th>Isotopes</th>
<th>Abundance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>$^{28}$Si, $^{29}$Si, $^{30}$Si</td>
<td>92.2%, 4.7%, 3.1%</td>
</tr>
<tr>
<td>Ge</td>
<td>$^{70}$Ge, $^{72}$Ge, $^{73}$Ge, $^{74}$Ge, $^{76}$Ge</td>
<td>20.5%, 27.4%, 7.8%, 36.5%, 7.8%</td>
</tr>
</tbody>
</table>

List of stable isotopes

- $^{28}$Si: 92.2% → 1/2 (nuclear spin)
- $^{29}$Si: 4.7% → 1/2 (nuclear spin)
- $^{30}$Si: 3.1% (nuclear spin)
- $^{69}$Ga: 60.1% → 3/2
- $^{71}$Ga: 39.9% → 3/2 (nuclear spin)
- $^{75}$As: 100% → 3/2

Nuclear spin control through manipulation of stable isotopes

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Relaxation time of $^{29}$Si (Sasaki)

$^{29}$Si NMR in natural silicon at $T=300K$

- $T_1 \approx 3.0 \times 10^3$ sec
- $T_2 \approx 6.5$ msec
- $T_2 \gg 100$ msec

Decoupling

29Si nuclear spin quantum computer

Copies

Position

$\omega_{n+1} - \omega_n \sim 20$ kHz

large field gradient 1.5 Tesla/µm
Elimination of background spins

qubits: phosphorus donors in Si and SiGe
Kane: nuclear spin of $^{31}$P
Yablonovitch: electron spin

Elimination of $^{29}$Si (s=1/2) and $^{73}$Ge(s=9/2) in the background is important!

Kane’s nuclear spin quantum computer

Ref. B.Kane, Nature 393, 133 (1998)

Mono isotopic Si without $^{29}$Si with I=1/2

Qubit: nuclear spins of $^{31}$P(I=1/2)
Operation: A- and J-gates

T=100mK

Energy of $^{31}$P(I=1/2)

Energy level diagram:

\[ |1\rangle \quad \Delta E = g\beta H \quad |0\rangle \]
Yablonovitch’s ESR quantum computer


SiGe hetero structures  (ESR transistor)

• Band structures
• g-values
• Bohr radius

Qubit: Spins of bound electrons of $^{31}$P (I=1/2)
Operation: A-gate

$^{29}$Si nuclear spin quantum computer

Position

$|\omega_{n+1} - \omega_n| \sim 20$kHz

large field gradient
1.5 Tesla/µm

$^{29}$Si nuclear spin quantum computer

Si and Ge without $^{29}$Si (I=1/2) and $^{73}$Ge (I=9/2)
Qubits are spin-1/2 $^{29}\text{Si}$ nuclei in a $^{28}\text{Si}$ crystal. They are distinguished by a one-dimensional field gradient.

Initialization is accomplished by cooling, optical pumping, "boosting," and "pseudo-pure state" techniques.

Qubit interactions (decoupling and recoupling) are accomplished with RF pulse sequences.

An ensemble of copies, orthogonal to the gradient direction, allow measurement by MRFM.

Decoherence times are limited by pulse sequence design, crystal purity, and cantilever stability.

An all silicon quantum computer

$^{29}\text{Si}$ wires embedded in the $^{28}\text{Si}$ matrix

$^{28}\text{Si}$ Bridge

$^{28}\text{Si (111)}$

Alternative configuration

$^{28}\text{Si}$ cantilever

Dy magnet

Optical fiber

Active Region: 100$\mu$m x 0.2$\mu$m

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Initialization

Polarization by optical pumping

Long $T_1$ means nuclei may be cooled much lower than lattice temperature (Optical Pumping)

Then, excess qubits may be sacrificed to cool a subset (Boosting)

Finally, logical labeling may be used to establish an effective pure state

Higher polarization $\Leftrightarrow$ Lower $T_1$ $\Leftrightarrow$ More logically labeled qubits


Polarization by optical pumping

The spin-polarized conduction electrons are cleared away after polarization, removing them as decoherence source

Optical initialization of electron (and nuclear) spins

Various excitation of electrons in Si

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Operation (decoupling)

RF

\[ \omega_1 \]

\[ \omega_2 \]

\[ \omega_3 \]

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Operation (recoupling)

- Qubits may be selectively recoupled
- Add single spin rotations to make controlled-NOT

Read-out by the MRFM cantilever

Qubit #1 $\omega_1$
Qubit #2 $\omega_2$
Qubit #3 $\omega_3$
Qubit #4 $\omega_4$

Read out of the qubit #2

Resonance $\omega_c$ (10-100kHz) of the cantilever

$\omega_c$ (10-100kHz)
SNR and number of qubits

Force resolution for a cantilever in the thermal limit:
\[ F_{\text{min}} = \sqrt{4k_B T B / \omega_0 Q} \]

Force generated from a single atomic plane:
\[ F(t) = M_z(r,t) \frac{\partial B_z}{\partial z} \]

Magnetization for nuclear spins in plane:
\[ M_z = \gamma h IN \left[ \left( \frac{1+P}{2} \right)^n - \left( \frac{1-P}{2} \right)^n \right] \]

Number of qubits \( n \) for SNR = 1 vs. nuclear polarization

\[ N = \text{number of qubit copies} \]
\[ n = \text{number of qubits in QC} \]

Decoherence and the maximum operation step

\[ \frac{1}{T_2} = \frac{1}{2T_1} + \frac{\gamma^2}{2} \int_{-\infty}^{\infty} \langle \partial B_Z(t) \partial B_Z(0) \rangle dt + \left( \langle [H_{\text{Dip}}^{\text{PES}}, M_{\text{Dip}}^{\text{PES}}] \rangle \right)^{1/2} \]

DC spectral density of local fluctuating field

Clock period \( t_c = \frac{\ln 2}{\Delta \omega} \) set by pulse sequence.
\( \Rightarrow \) Number of logic gates \( T_2 / t_c \) is limited

Primary Decoherence Sources:

- Residual Dipolar Couplings
  - Reversible in principle
  - Present sequence: \( T_2 \approx 10 \text{ ms} \)

- Cantilever Drift
  - Thermal equilibrium: \( T_2 \approx 200 \text{ ms} \)
  - Feedback control \( \Rightarrow T_2 \approx 1 \text{ hour} \)

- Paramagnetic Impurities
  - Assuming very dilute impurities, \( T_2 = (\omega_0 T_1 \tau)^{1/2} T_1 \approx 1 \text{ minute} \), but much shorter for nuclei near impurity

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**29Si wire fabrication**

- Form regular step arrays on slightly miscut 28Si(111)7×7 surface (~ 1° from normal)
- Steps are straight, with about 1 kink in 20000 sites.
- 29Si chains formed by “Step Decoration” from 28Si steps
- Angle of miscut controls chain spacing

**Row-by-row growth**

The step-flow growth was observed as the appearance of new adatoms at the edge.

Short rows are thermally diffused to form a longer row which is energetically stable.

\[
T_{\text{sub}} = 350°C \\
\text{Growth rate } 0.8 \times 10^{-2} \text{BL/min}
\]

MBE fabrication of $^{29}\text{Si}$ wire copies

Progress at Keio
Progress on Isotope Engineering at Keio

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J. Mater. Res. 8, 1341 (1993)

Isotopically controlled Si fabrication

99.92% $^{28}$Si single crystal

<table>
<thead>
<tr>
<th>Isotopically controlled Si fabrication</th>
<th>99.92% $^{28}$Si single crystal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural abundance</td>
<td>96% $^{28}$Si single crystal</td>
</tr>
<tr>
<td>$^{28}$Si</td>
<td>92.2%</td>
</tr>
<tr>
<td>$^{28}$Si</td>
<td>4.7%</td>
</tr>
<tr>
<td>$^{30}$Si</td>
<td>3.1%</td>
</tr>
</tbody>
</table>

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28Si/ 30Si Isotope Superlattices

Depth profile of 28Si and 30Si of the [(28Si)16/(30Si)16]50 superlattice

Phonons in Si SLs

Phonons of (28Si)n/(30Si)n SLs

Calculation of Phonon Frequency

Planar-Bond Charge (PBC) Model

Raman Spectrogram of SL

Raman spectroscopy

\[ ([\text{natur. Si}_{16}/(\text{Si}_{30})_{16}]_{50}) \]

Conditions

✓ Laser: 514.5 nm
✓ Temp.: \(~ 4 \text{ K}\)

Phonon folded mode can be confirmed.

Fig. Raman spectrum of \([\text{natur. Si}_{16}/(\text{Si}_{30})_{16}]_{50}\) superlattice

Expected Raman Peaks

Ex) \((\text{Si}_{28})_{16}/(\text{Si}_{30})_{16}\)

530.721 cm\(^{-1}\)
523.298 cm\(^{-1}\)
513.942 cm\(^{-1}\)

Expected Raman peaks of \([\text{natur. Si}] / (\text{Si})_n\) superlattices calculated by PBC model (\(n\): even numbers)
**Micro-magnet fabrication**

- Si substrate
- Micro-magnet

**Homogeneity and Strong gradient**

- Qubit#1 $\omega_1$
- Qubit#2 $\omega_2$

**Magnetic field simulation**

- Width of magnet (µm)
- Distance (µm)
- Gradient (T/µm)

- Shaded area:
  - Si
  - Micro-magnet

- Distance and gradient

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Field simulation and micro-fabrication

**NH$_3$/CO/Xe reactive etching of NiFe**

- Ti buffer 80 Å
- Si substrate
- Optical pumping and NMR

*Sputter growth and reactive ion etching of NiFe*

- Ti buffer 80 Å
- Si substrate
- NiFe
- After reactive ion etching

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Next-step: $^{29}\text{Si}/^{28}\text{Si}$ isotope superlattice

Proof of concepts

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Qubit #1 $\omega_1$</th>
<th>Qubit #2 $\omega_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{28}\text{Si}$ ($I=0$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{29}\text{Si}$ ($I=1/2$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{29}\text{Si}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>substrate</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In-plane coupling of $^{29}\text{Si}$ may severely limit the scaling

Summary

- Semiconductor Isotope Engineering
- New quantum computation scheme exclusively with silicon
- Fabrication of the all silicon quantum computer at Keio
Collaborators
Eisuke Abe (Keio University)
Takeharu Sekiguchi (Keio University)
Ryusuke Nebashi (Keio University)
Yoshinori Matsumoto (Keio University)
Hideo Ohno (Tohoku University)
Yuzo Ohno (Tohoku University)
Susumu Sasaki (Nigata University)
Yoshhisa Yamamoto (Stanford University)
Thaddeus Ladd (Stanford University)
Jonathan Goldman (Stanford University)