

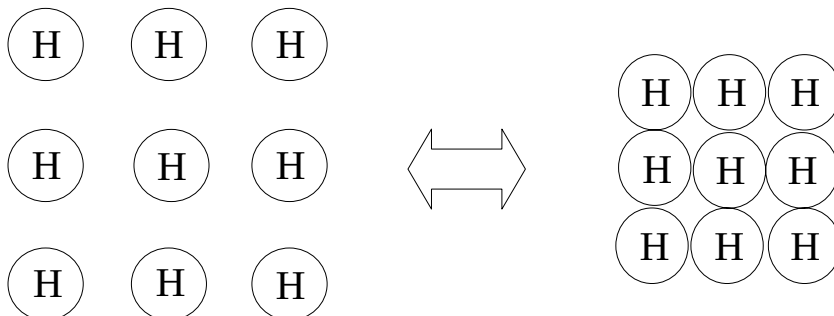
Critical Exponents for Mott-Anderson Transition

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Mott Transition

Neutral hydrogen in a simple squared lattice at $T=0\text{K}$



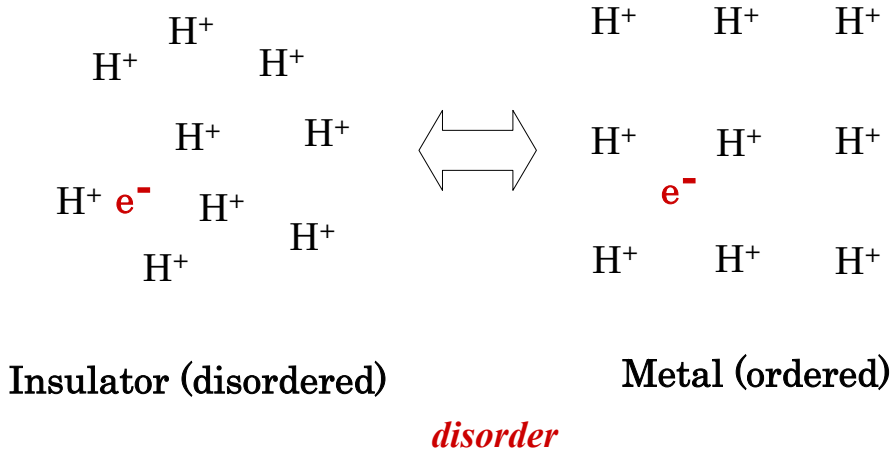
Insulator $N < N_C$

Metal $N > N_C$

electron-electron interaction

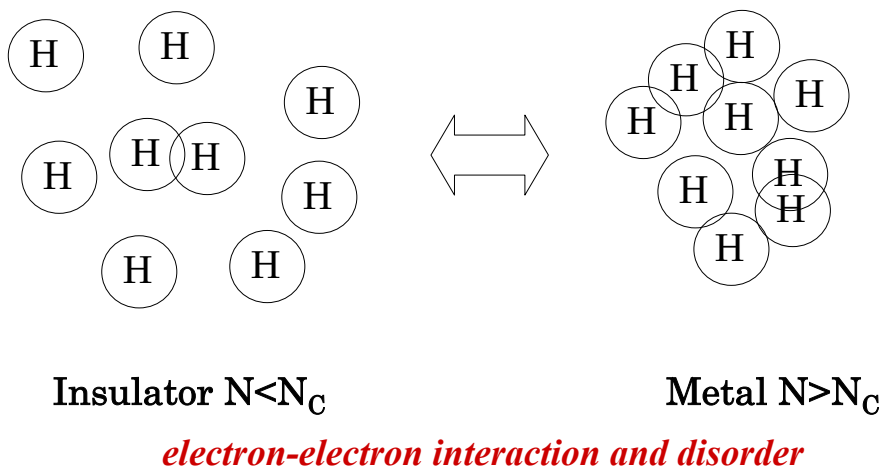
Anderson Transition

One electron in ordered and disordered potential at $T=0K$



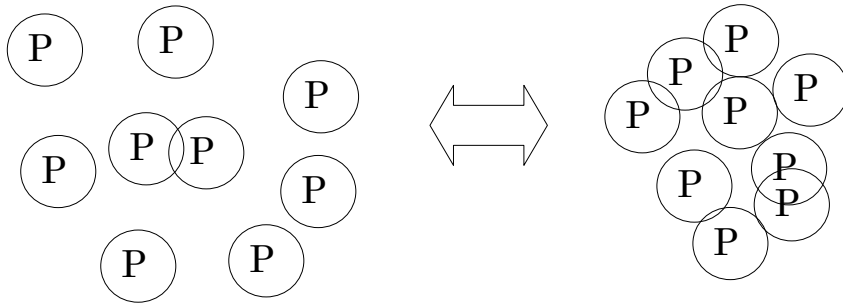
Mott-Anderson Transition

Neutral hydrogen in the random state at $T=0K$



Mott-Anderson Transition in uncompensated Si:P

Neutral donors in semiconductors in
the random state at $T=0\text{K}$



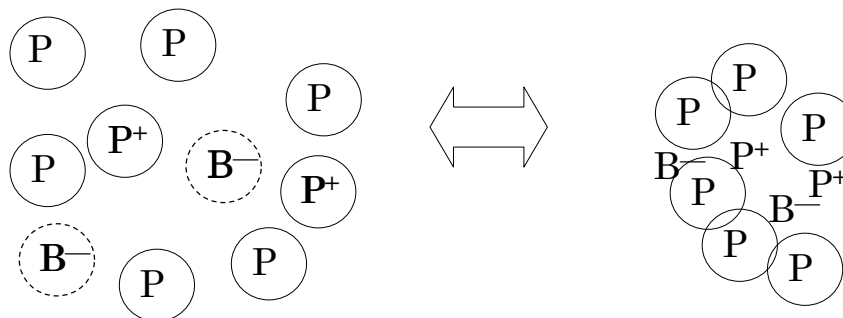
Insulator $N < N_C$

Metal $N > N_C$

electron-electron interaction and disorder

Mott-Anderson Transition in compensated Si:P

Neutral and ionized donors in semiconductors in
the random state at $T=0\text{K}$



Insulator $N < N_C$

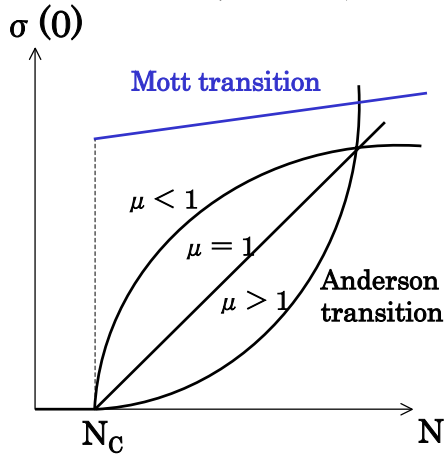
Metal $N > N_C$

electron-electron interaction and stronger disorder for larger $K \equiv \frac{[B]}{[P]}$

Critical exponents

Conductivity (experiment)

$$\sigma(0) \propto \left(\frac{N}{N_C} - 1 \right)^\mu$$



correlation length (theory)

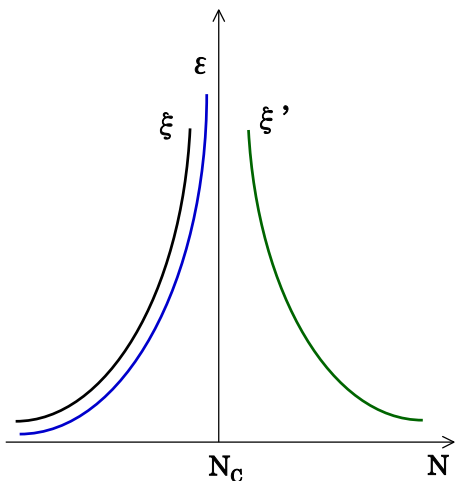
$$\xi' \propto \left(\frac{N}{N_C} - 1 \right)^{-\nu}$$

$$\left\{ \begin{array}{ll} \nu \approx 1.5 & \text{for pure-Anderson} \\ \nu > \frac{2}{3} & \text{Chayes's relation} \\ & \text{for Mott-Anderson} \end{array} \right.$$

Compare theory and experiment assuming Wegner's relation for 3-dimension ($d=3$)

$$\mu = (d - 2)\nu = \nu$$

Critical exponents



Metallic side ($N > N_C$)

Correlation length (theory)

$$\xi' \propto \left(\frac{N}{N_C} - 1 \right)^{-\nu}$$

Insulating side ($N < N_C$)

Dielectric constant

$$\epsilon(N) = \epsilon_h + \chi_{imp}(N)$$

$$\chi_{imp}(N) = \chi_0 \left(\frac{N_C}{N} - 1 \right)^{-\zeta}$$

Localization length

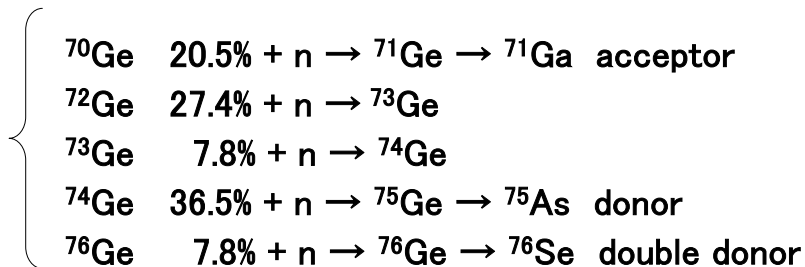
$$\xi(N) = \xi_0 \left(1 - \frac{N}{N_C} \right)^{-\nu}$$

**Experimentally reported μ for
nominally uncompensated doped semiconductors**

System	μ
Si:P	0.5, 1.0, 1.2
Si:As	0.5, 1.0
Si:B	0.65, 1.6
Ge:As	0.5, 1.2
Ge:Sb	0.9
Ge:Ga	0.5, 1.2

For compensated semiconductors $\mu \sim 1$

**Fabrication of homogeneously doped sample
by neutron transmutation doping (NTD)**



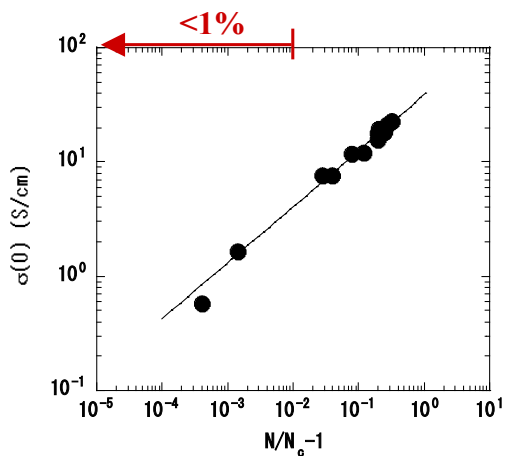
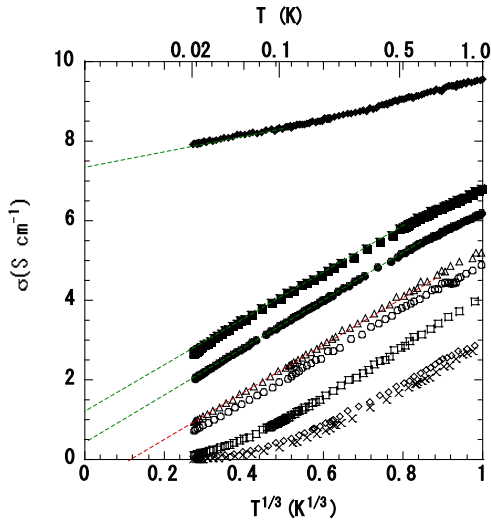
1. Nominally uncompensated series $K \sim 0$

NTD of ${}^{70}\text{Ge}$ (96.3%) + ${}^{72}\text{Ge}$ (3.7%) single crystal

2. Compensated series $K \sim 0.32$

NTD of ${}^{\text{nat}}\text{Ge}$

Nominally uncompensated series (K~0)



$$\sigma(0) \propto \left(\frac{N}{N_C} - 1 \right)^{\mu=0.55}$$

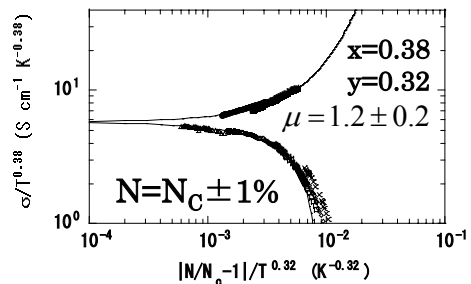
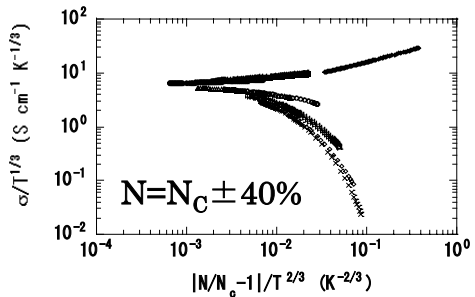
Nominally uncompensated series (K~0)

Finite temperature scaling

$$\sigma(N, T) \propto T^x f\left(\frac{N/N_C - 1}{T^y}\right)$$

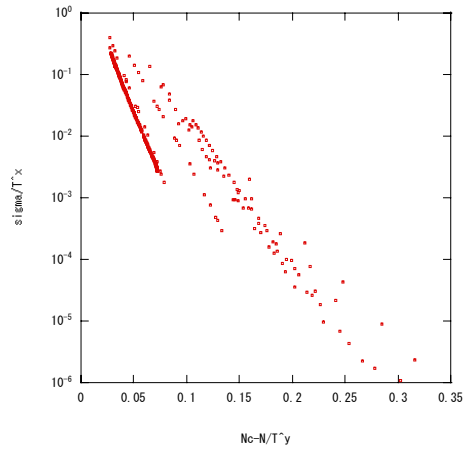
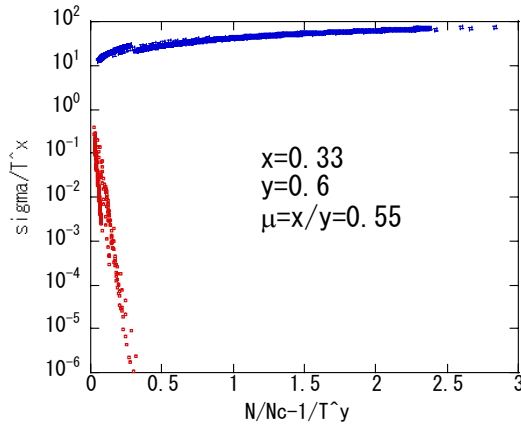
$$\frac{\sigma(N, T)}{T^x} \propto f\left(\frac{N/N_C - 1}{T^y}\right)$$

$$\mu = \frac{x}{y}$$



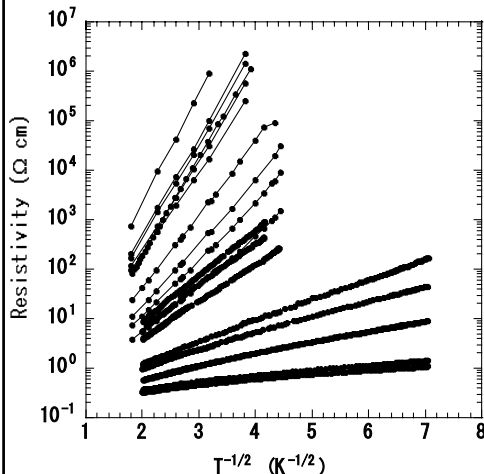
Nominally uncompensated series (K~0)

Finite temperature scaling **without** $N=N_C \pm 1\%$



Nominally uncompensated series (K~0)

Variable range hopping conduction in insulating samples



$$\rho(N, T) \propto \rho_0 T^{-1/3} \exp\left(\frac{T_0}{T}\right)^{1/2}$$

Dielectric constant

$$\varepsilon(N) = \varepsilon_h + \chi_{imp}(N)$$

$$\chi_{imp}(N) = \chi_0 \left(\frac{N_C}{N} - 1 \right)^{-\zeta}$$

Localization length

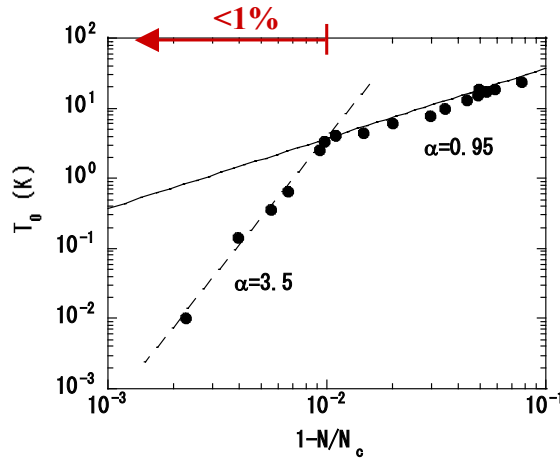
$$\xi(N) = \xi_0 \left(1 - \frac{N}{N_C} \right)^{-\nu}$$

$$k_B T_0 \propto \frac{1}{4\pi\varepsilon_0} \frac{2.8e^2}{\varepsilon(N)\xi(N)} = \frac{2.8e^2}{4\pi\varepsilon_0 \chi_0 \xi_0} \left(1 - \frac{N}{N_C} \right)^{\alpha = \nu + \zeta}$$

Nominally uncompensated series (K~0)

Variable range hopping conduction in insulating samples

$$k_B T_0 \propto \frac{1}{4\pi\epsilon_0} \frac{2.8e^2}{\epsilon(N)\xi(N)} = \frac{2.8e^2}{4\pi\epsilon_0\chi_0\xi_0} \left(1 - N/N_C\right)^{\alpha=\nu+\zeta}$$



Nominally uncompensated series (K~0)

Scaling of the dielectric constant and localization length in insulators

Wegner's law

$$\mu = (d - 2)\nu = \nu$$

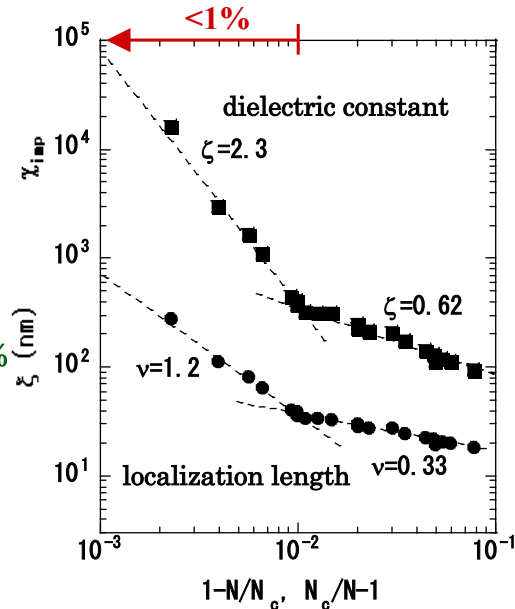
$\mu = \nu = 1.2$ within 1%

$\mu = 0.55 \neq \nu = 0.33$ for $>1\%$

Dynamical scaling exponents

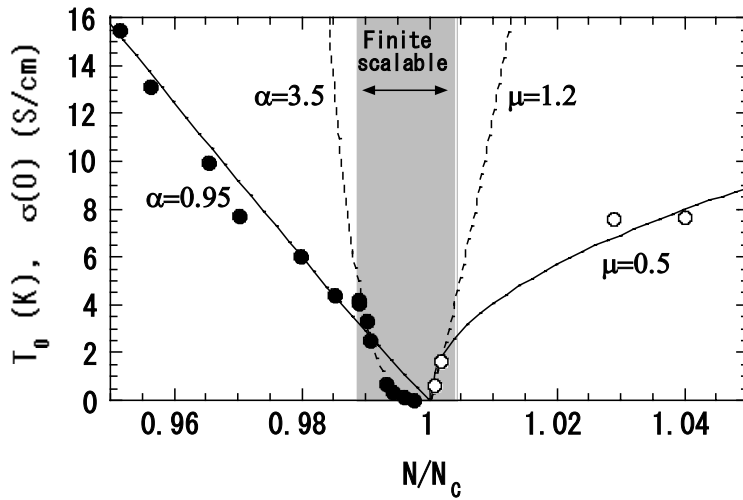
$$z = \frac{1}{y\nu} \approx 3 \text{ within } 1\%$$

≈ 5 for $> 1\%$



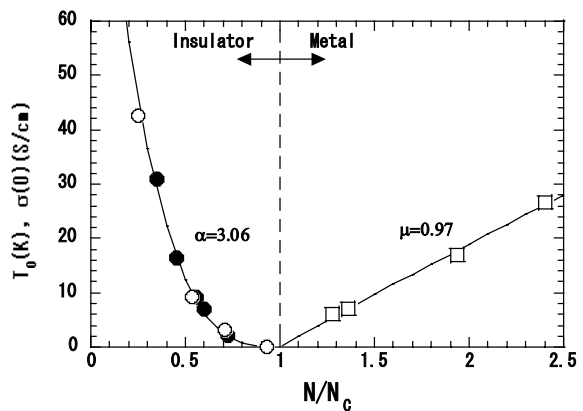
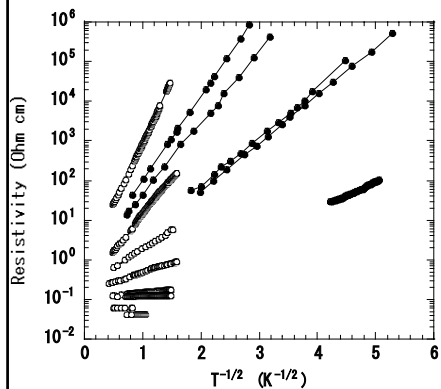
Nominally uncompensated series (K~0)

Variable range hopping conduction in insulating samples



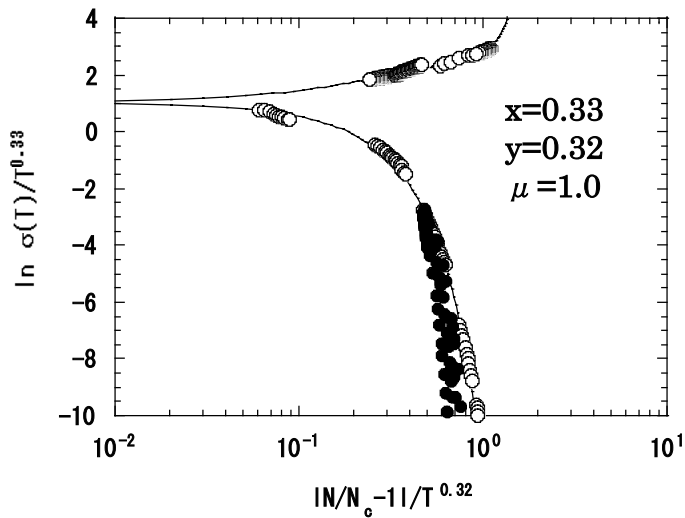
Intentionally compensated series (K~0.32)

Variable range hopping conduction in insulating samples



Intentionally compensated series ($K \sim 0.32$)

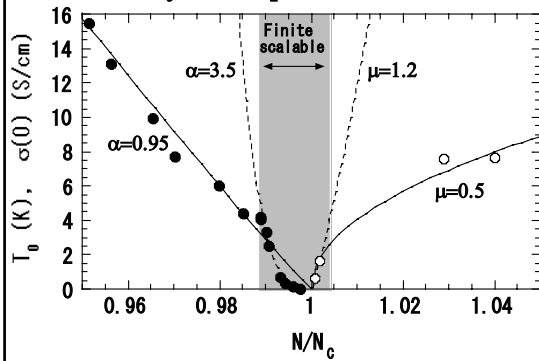
Finite temperature analysis for $N = N_c \pm 80\%$



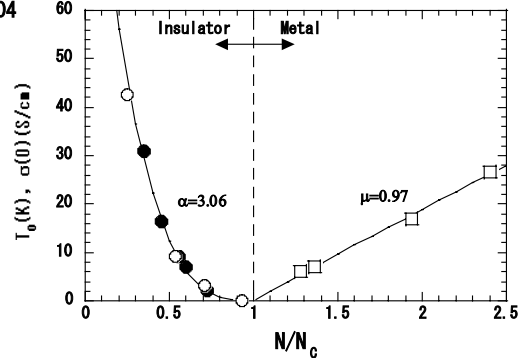
$$z = \frac{1}{y\nu} \approx 3$$

Nominally uncompensated

Comparison

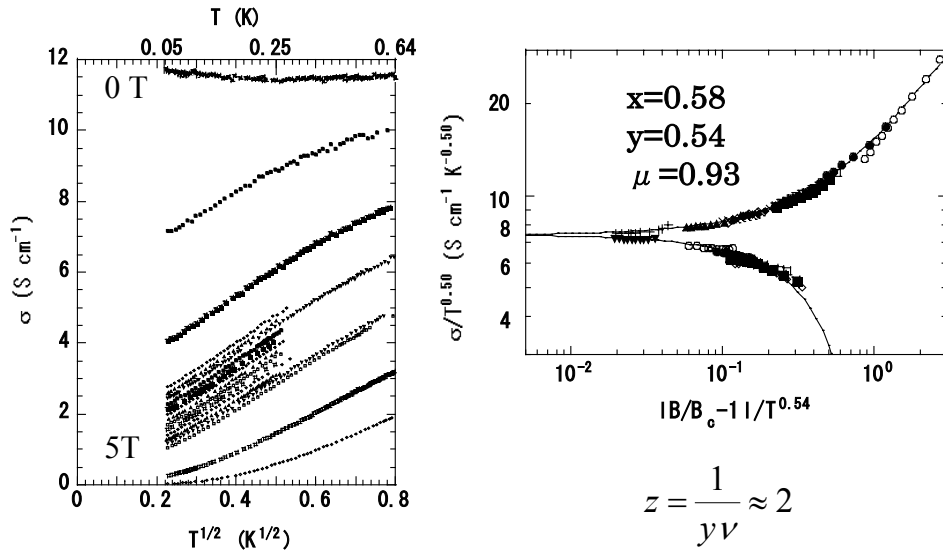


intentionally compensated



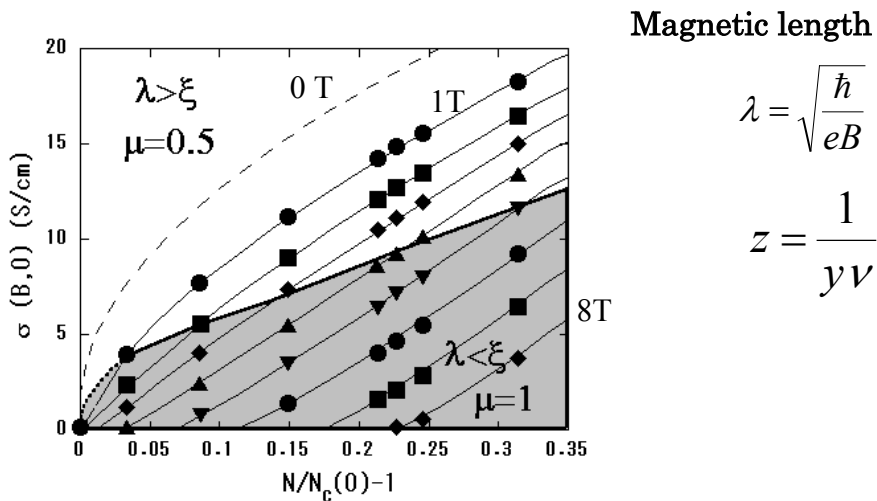
Nominally uncompensated series (K~0)

Magnetic field induced transition



Nominally uncompensated series (K~0)

Effect of externally apply constant magnetic field



Summary

exponents	Compensation $K \sim 0, B=0$	$K=0.32, B=0$	$K \sim 0, B > 0$	
	up to 50%	within 1%	up to 80%	
Conductivity μ from $\sigma(N, T=0)$	0.55	----	0.97	1
Conductivity μ from $\sigma(N, T)$	0.55 ?	1.2 ± 0.2	1.0	0.93
To of hopping α	0.95	3.5 ± 0.5	3.1	---
Localization and correlation ν	0.33	1.2	~ 1 (Katsumoto)	---
Dielectric const. ζ	0.62	2.3	~ 2 (Katsumoto)	---
Dynamical Z	5 ?	3	3	2
Wegner's law	$\mu \neq \nu$	$\mu = \nu$	$\mu = \nu$	---