Semiconductor quantum dots with spin-orbit interaction

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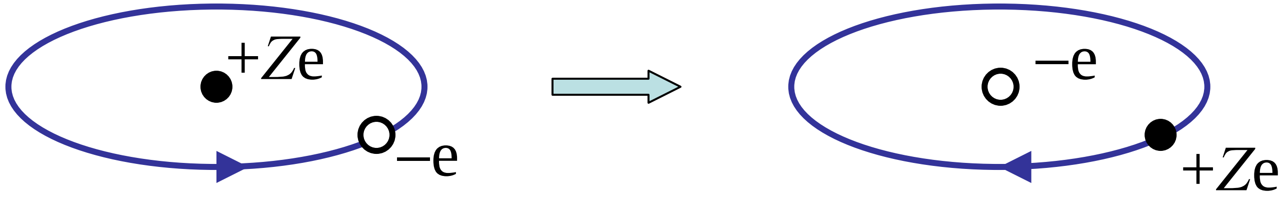
Outline

1. Introduction
    1.1. Spin-orbit interaction
    1.2. Spin Hall effect
2. Spin Hall effect in 2DEG with artificial potential
3. Semiconductor quantum dot
    3.1. Coulomb oscillation
    3.2. Spin Hall effect in quantum dot
4. Search for Majorana fermions
    4.1. Topological quantum computer
    4.2. Majorana fermion

Outline

1.1. Spin-orbit (SO) interaction

Atom in vacuum



- Positive charge rotation makes a magnetic field

Biot-Savart law

$$\boldsymbol{B}_{\text{eff}} = \frac{\mu_0}{4\pi} Ze \frac{(-\boldsymbol{r}) \times (-\boldsymbol{v})}{r^3} = \frac{\mu_0}{4\pi} \frac{Ze}{m} \frac{\hbar \boldsymbol{l}}{r^3}, \qquad \hbar \boldsymbol{l} = m \boldsymbol{r} \times \boldsymbol{v}$$

- Magnetic dipole moment of electron spin

$$\boldsymbol{\mu}_{\text{s}} = -2\mu_{\text{B}} \boldsymbol{s} \quad \left(\mu_{\text{B}} = \frac{e\hbar}{2m} \right)$$

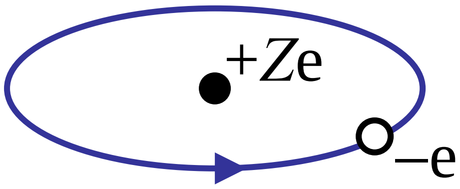
$$H_{\text{SO}} = -\boldsymbol{\mu}_{\text{s}} \cdot B_{\text{eff}} = -\frac{\mu_0}{4\pi} \frac{Ze^2}{2m^2} \frac{\hbar}{r^3} (\hbar \boldsymbol{l} \cdot \boldsymbol{s})$$

Different from Dirac equation by factor 2 (Thomas factor)

$$H_{\mathrm{SO}} = -\frac{\mu_0}{4\pi}\frac{Ze^2}{2m^2}\frac{\hbar}{r^3}(\hbar \boldsymbol{l}\cdot\boldsymbol{s}) \qquad \hbar\boldsymbol{l} = \boldsymbol{r}\times\boldsymbol{p}$$

$$= \frac{e\hbar}{2m^2c^2}\boldsymbol{s}\cdot(\boldsymbol{p}\times\boldsymbol{E}) \qquad \boldsymbol{E} = \frac{1}{4\pi\varepsilon_0}\frac{Ze}{r^2}\frac{\boldsymbol{r}}{r}$$

$$= -\frac{\hbar}{4m^2c^2}\boldsymbol{\sigma}\cdot(\boldsymbol{p}\times\nabla U) \qquad U = \frac{1}{4\pi\varepsilon_0}\frac{-Ze^2}{r}$$



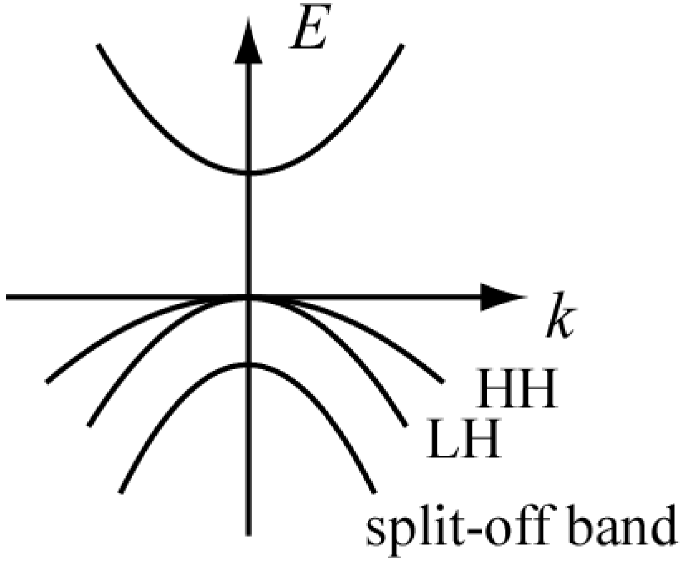
$$\boldsymbol{s} = \frac{1}{2}\boldsymbol{\sigma} \ \text{(Pauli matrices)}$$

$$H_{\mathrm{SO}} = \frac{\lambda}{\hbar}\boldsymbol{\sigma}\cdot(\boldsymbol{p}\times\nabla U), \quad \lambda = -\left(\frac{\hbar}{2mc}\right)^2$$

- Relativistic effect: $2mc^2$=1MeV in the denominator is energy gap between particle and antiparticle

Spin-orbit interaction in semiconductors (I)

- Valence band in compound semiconductors: consists mainly of *p* orbitals (*l*=1) ⟹ SO interaction



$$\boldsymbol{l}\cdot\boldsymbol{s}=\frac{1}{2}\left[(\boldsymbol{l}+\boldsymbol{s})^2-\boldsymbol{l}^2-\boldsymbol{s}^2\right]$$

$$\boldsymbol{j}=\boldsymbol{l}+\boldsymbol{s}:\ j=1\pm\frac{1}{2}=\frac{3}{2},\frac{1}{2}$$

$$\text{HH}:\ \left|j=3/2, j_z=\pm 3/2\right\rangle,$$

$$\text{LH}:\ \left|j=3/2, j_z=\pm 1/2\right\rangle$$

$$\left|j=1/2, j_z=\pm 1/2\right\rangle$$

- Conduction band: consists mainly of *s* orbital (*l*=0)

⟹ No SO interaction?

Spin-orbit interaction in semiconductors (II)

- *k-p* perturbation theory for conduction band
- SO interaction is enhanced, particularly in narrow-gap semiconductors (InAs, InGaAs)

$$H_{\mathrm{SO}} = \frac{\lambda}{\hbar}\boldsymbol{\sigma}\cdot\left(\boldsymbol{p}\times\nabla U\right) \qquad \lambda = \frac{P^2}{3}\left[\frac{1}{E_0{}^2} - \frac{1}{\left(E_0+\Delta_0\right)^2}\right]$$

P: matrix element between conduction and valence bands

E_0: band gap

Δ_0: SO splitting in valence band

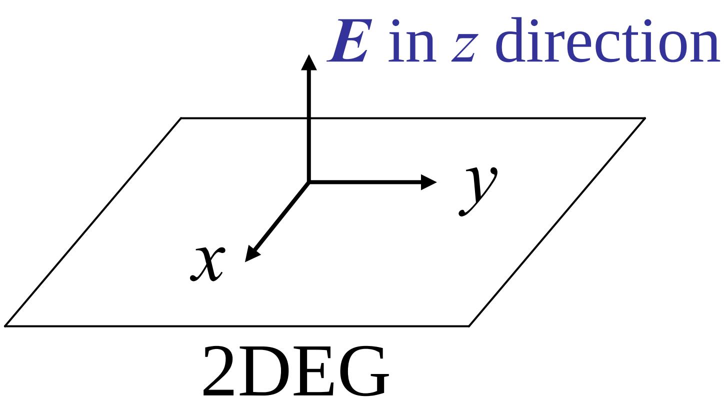
- Roughly speaking, band gap corresponds to particle-antiparticle energy gap in Dirac equation.

(1) Rashba SO interaction

- U: external potential
- Electric field perpendicular to 2D electron gas in InGaAs/GaAs heterostructure

$$H_{\mathrm{RSO}} = \frac{\lambda}{\hbar}\boldsymbol{\sigma}\cdot\left(\boldsymbol{p}\times\nabla U\right) \quad \Leftarrow \quad U = eEz$$

$$= \frac{\alpha}{\hbar}\boldsymbol{\sigma}\cdot\left(\boldsymbol{p}\times\hat{\boldsymbol{z}}\right) = \frac{\alpha}{\hbar}\left(p_y\sigma_x - p_x\sigma_y\right) \quad \left(\alpha = eE\lambda\right)$$



Large α:
Nitta *et al*., PRL (1997);
Grundler, PRL (2000);
Sato *et al*., JAP (2001).

(2) Dresselhaus SO interaction

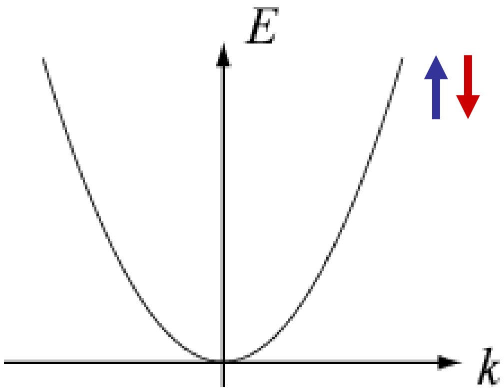
- Inversion symmetry breaking in III-V compound semiconductors
- *U*: crystal field

$$H_{\mathrm{DSO}} = \frac{\beta}{\hbar}\left(-p_x\sigma_x + p_y\sigma_y\right) + \gamma\left(p_x p_y^2\sigma_x - p_y p_x^2\sigma_y\right)$$

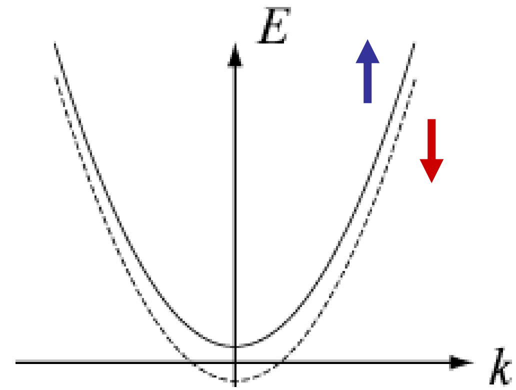
- Same order as Rashba SO (α~β) in GaAs

SO interaction
- Time reversal symmetry, Kramers' degenerate
- One-body problem

SO interaction vs. magnetic field (Zeeman effect)

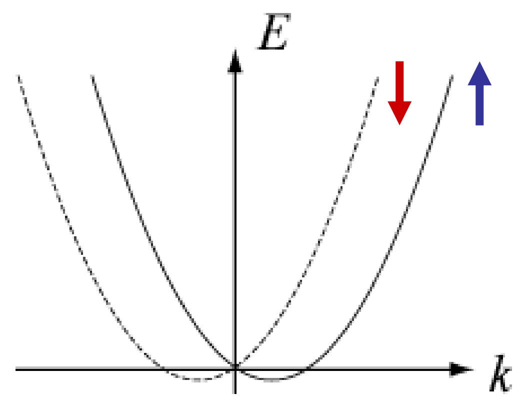


Magnetic field



$$H = \frac{p_x^2}{2m} + \frac{\alpha}{\hbar} p_x \sigma_y$$

SO interaction



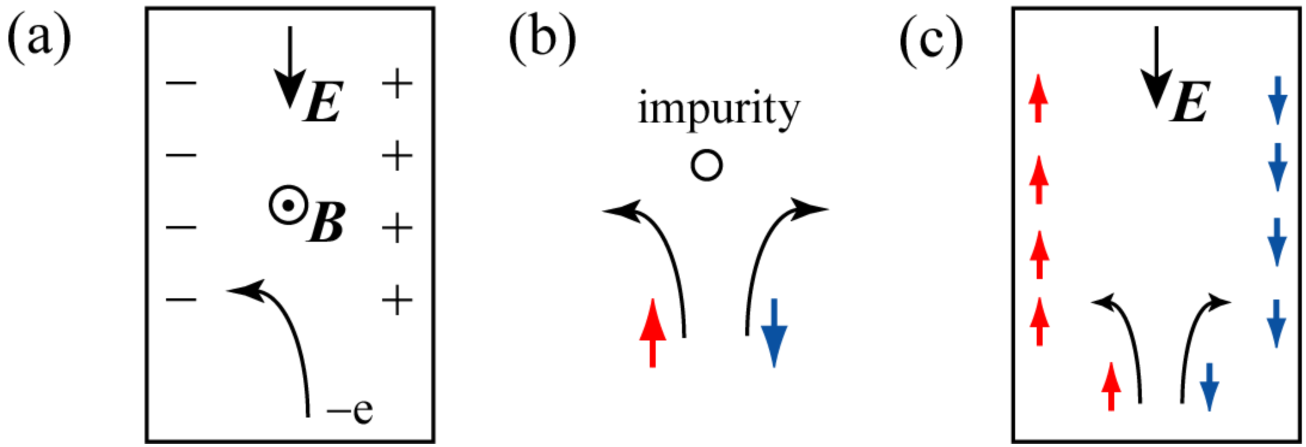
time reversal symmetry:

$E_+(k) = E_-(-k)$

(Kramers degeneracy)

1.2. Spin Hall effect

- "Spin injection" without ferromagnet, magnetic field
- Intrinsic and extrinsic spin Hall effect (SHE)



(a) Hall effect: Lorentz force by magnetic field

(b) Extrinsic SHE: SO interaction + impurity scattering

(c) Intrinsic SHE: topological structure of valence band

S.Murakami, N. Nagaosa and S.-C. Zhang, Science (2003)

Extrinsic SHE

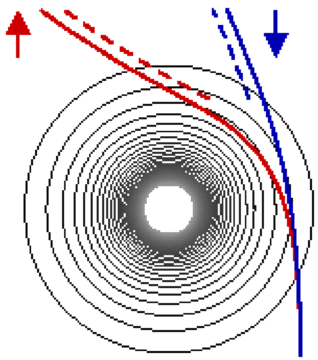
- V: Centrally symmetric potential in 3D (e.g. screened Coulomb potential by charged impurity)

$$\tilde{V} = V(r) + \frac{\lambda}{\hbar}\boldsymbol{\sigma}\cdot[\boldsymbol{p}\times\nabla V(r)] = V(r) + V_1(r)\boldsymbol{l}\cdot\boldsymbol{s}$$

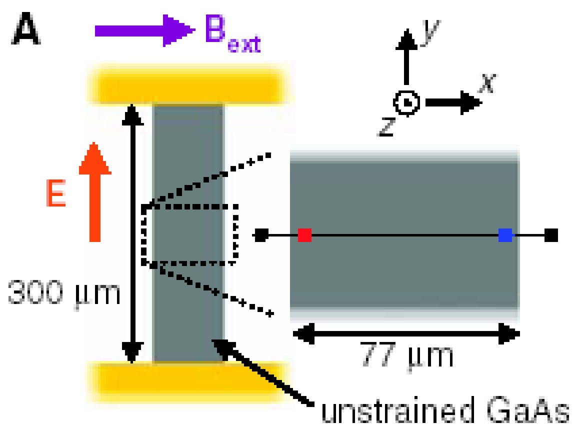
$$V_1(r) = -\lambda\frac{2}{r}\frac{dV}{dr}$$

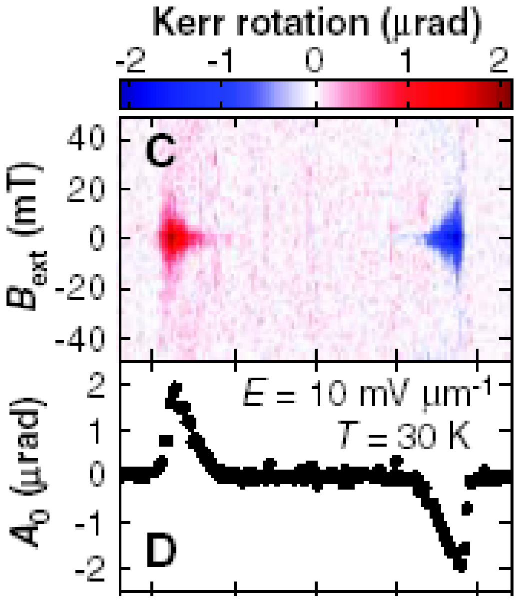
- Semi-classical theory

- "skew scattering"
- "side jump" effect



- Optical experiment (Kerr rotation) by Kato *et al*., Science (2006)



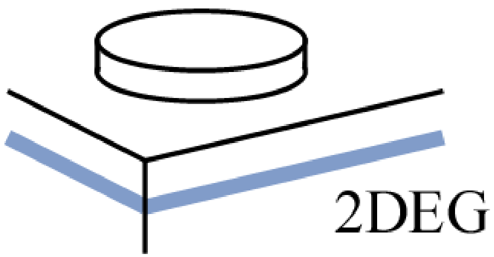


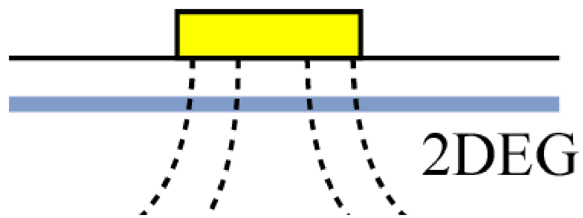
- Ascribable to extrinsic SHE
- Quantitatively explained by semiclassical theory

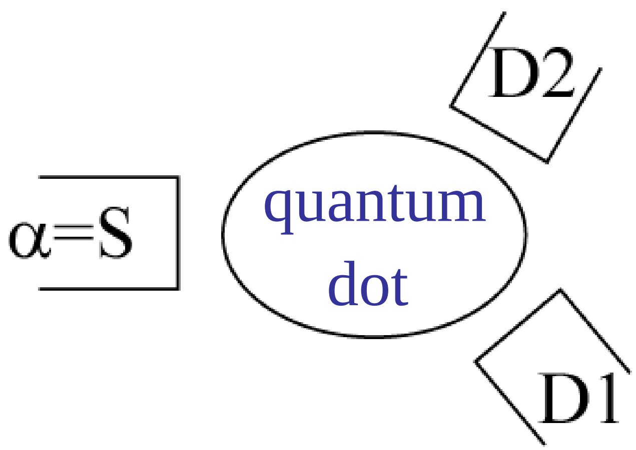
H.-A. Engel, B. I. Halperin, and E. I. Rashba, PRL **95**, 166605 (2005).

In this talk

- Quantum mechanical formulation of SHE for 2DEG in semiconductor heterostructures with "single impurity"
  - Artificial potential by single antidot, STM tip
  - InAs quantum dot with SO interaction





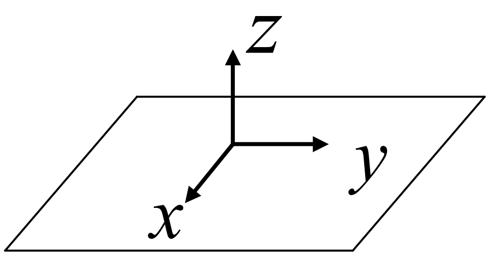


Outline

2.1. Formulation of SHE for 2DEG

- 2D Schrödinger equation (effective mass equation) with axially symmetric potential $V(r)$

$$\left[-\frac{\hbar^2}{2m^*}\Delta+\tilde{V}\right]\psi=E\psi,\quad E=\frac{\hbar^2k^2}{2m}$$



$$\tilde{V}=V(r)+\frac{\lambda}{\hbar}\boldsymbol{\sigma}\cdot\left[\boldsymbol{p}\times\nabla V(r)\right]=V(r)+V_1(r)l_z s_z$$

$$V_1(r)=-\lambda\frac{2}{r}\frac{dV}{dr}$$

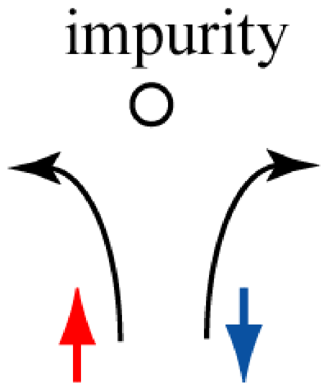
: same sign as $V(r)$ when $|V(r)|$ monotonically decreases with r.

- l_z and s_z are conserved in 2D.

$$\tilde{V} = V(r) + V_1(r) l_z s_z = \begin{cases} V(r) + \frac{1}{2} V_1(r) l_z & (s_z = +1/2) \\ V(r) - \frac{1}{2} V_1(r) l_z & (s_z = -1/2) \end{cases}$$

s_z=1/2:
- Scattering enhanced for l_z >0
- suppressed for l_z <0

s_z=–1/2: (opposite effects)



⇨ extrinsic spin Hall effect

cf. Partial wave expansion:

Eto and Yokoyama, J. Phys. Soc. Jpn. **78**, 073710 (2009).

Appendix

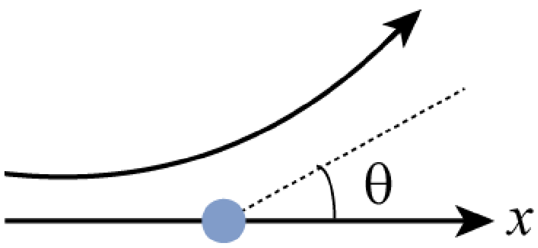
Partial wave expansion*:

$l_z = m(= 0, \pm 1, \pm 2, \pm 3, \cdots)$

- Incident wave

$$e^{ikx} = e^{ikr\cos\theta} = \sum_{m=-\infty}^{\infty} i^m J_m(kr) e^{im\theta}$$

$$J_m(kr) \sim \sqrt{\frac{2}{\pi kr}} \cos\left(kr - \frac{m\pi}{2} - \frac{\pi}{4}\right)$$



(Bessel function; asymptotic form at $r \to \infty$)

- Incident wave + scattered wave

$$J_m(kr) \to R_m^{\pm}(r) \sim \sqrt{\frac{2}{\pi kr}} e^{i\delta_m^{\pm}} \cos\left(kr - \frac{m\pi}{2} - \frac{\pi}{4} + \delta_m^{\pm}\right)$$

phase shift: $\delta_m^{\pm}$ for $l_z = m, \; s_z = \pm 1/2$

(*) 3D: textbooks by Landau-Lifshitz, Mott-Massey

Appendix

$R_m^{\pm}(r)$ is determined from

$$\left[-\frac{\hbar^2}{2m^*}\left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-\frac{m^2}{r^2}\right)+V(r)\pm\frac{m}{2}V_1(r)\right]R_m^{\pm}=ER_m^{\pm}$$

for $s_z=\pm 1/2$

$V_1(r) l_z s_z$

$\Rightarrow \delta_m^{+} \neq \delta_m^{-}$

- $\delta_m^{\pm}=\delta_{-m}^{\mp}$ (time reversal symmetry)
- SO interaction does not work on *S* wave (m=0): $\delta_0^{+}=\delta_0^{-}\equiv\delta_0$

Appendix

- Scattering amplitude (cross section: $\sigma_{\pm}(\theta) = \left| f_{\pm}(\theta) \right|^2$)

$$f_{\pm}(\theta) = A \pm B$$

$$A = \frac{1}{i\sqrt{2\pi k}} \left\{ \left(e^{2i\delta_0} - 1 \right) + \sum_{m=1}^{\infty} \left(e^{2i\delta_m^+} + e^{2i\delta_m^-} - 2 \right) \cos m\theta \right\}$$

$$B = \frac{1}{\sqrt{2\pi k}} \sum_{m=1}^{\infty} \left(e^{2i\delta_m^+} - e^{2i\delta_m^-} \right) \sin m\theta$$

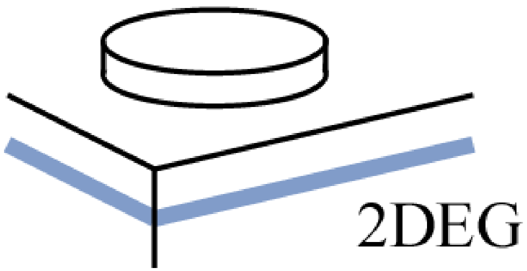
- Spin polarization of scattered wave in θ direction

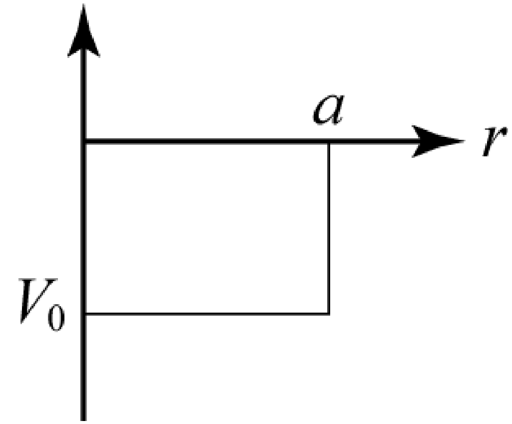
$$P_z = \frac{\left| f_+(\theta) \right|^2 - \left| f_-(\theta) \right|^2}{\left| f_+(\theta) \right|^2 + \left| f_-(\theta) \right|^2} = \frac{2\,\mathrm{Re}\left(AB^* \right)}{\left| A \right|^2 + \left| B \right|^2}$$

⇐ $\delta_m^+ \neq \delta_m^-$

Spin polarization: $-P_z$ for $-\theta$ direction

2.2. SHE by a tunable potential well

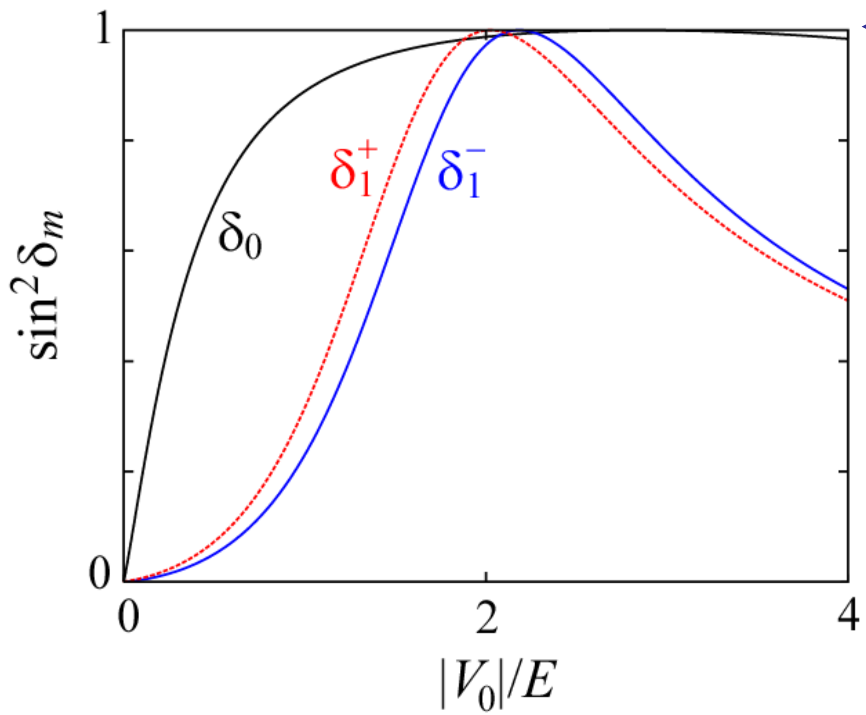




$$V(r) = V_0\theta(a-r) \Rightarrow V_0\left[\theta(a-r) \pm m\frac{\lambda}{a}\delta(a-r)\right]$$

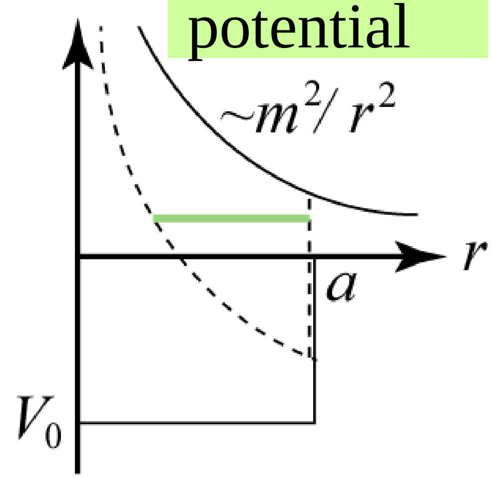
- Fermi wavelength $2\pi/k$: 10~100nm
  Radius of well a: ka=1, 2, 3
- Strength of SO interaction: λ/a^2=0.01 (at ka=1)

- Scattering probability of partial waves (m=0,1)



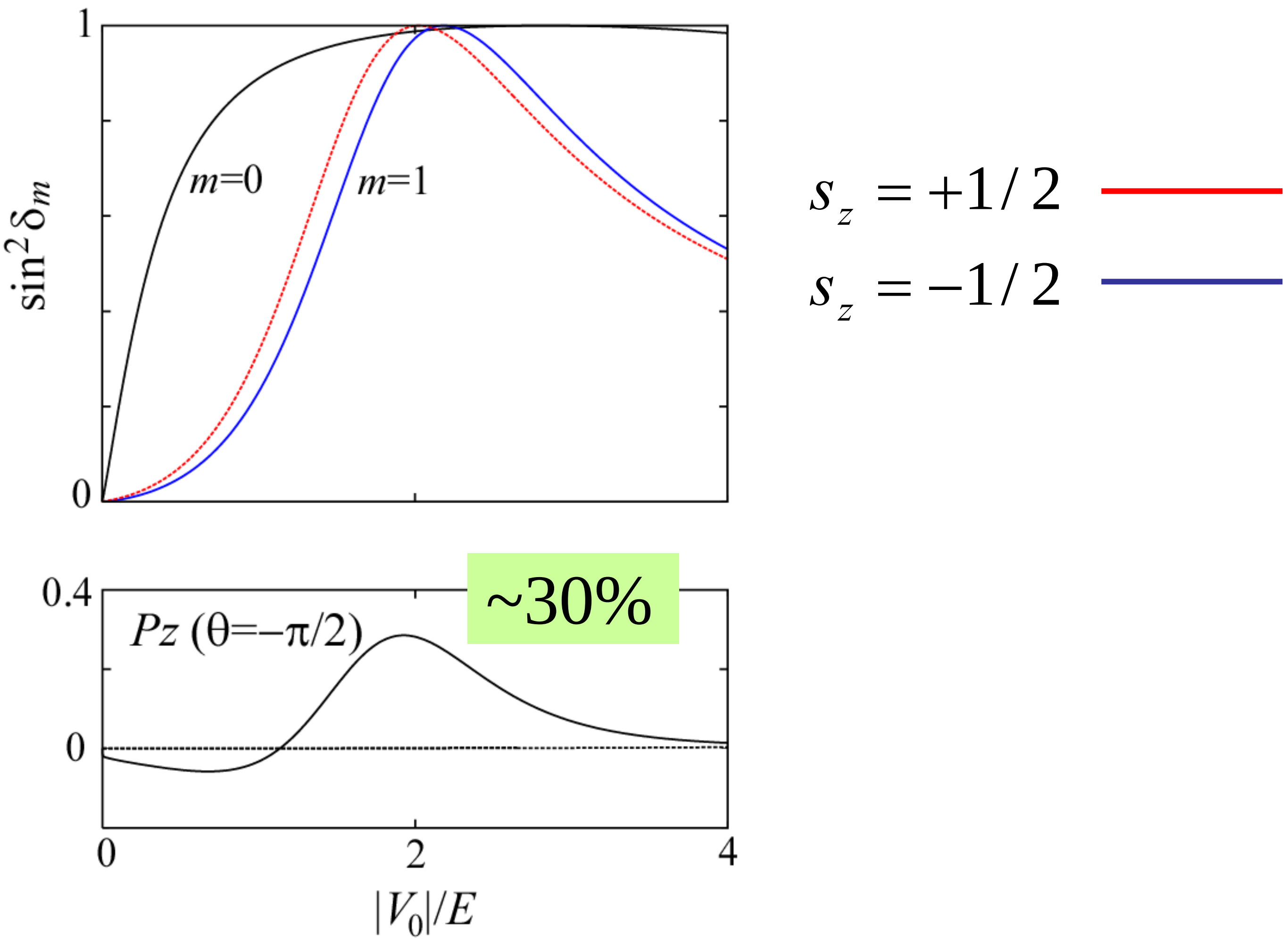
Unitary limit:
$\delta_m^{\pm} = \pi/2$

centrifugal potential



Resonant scattering of S wave (m=0) and P wave (m=0) at some values of $|V_0|$ (unitary limit)

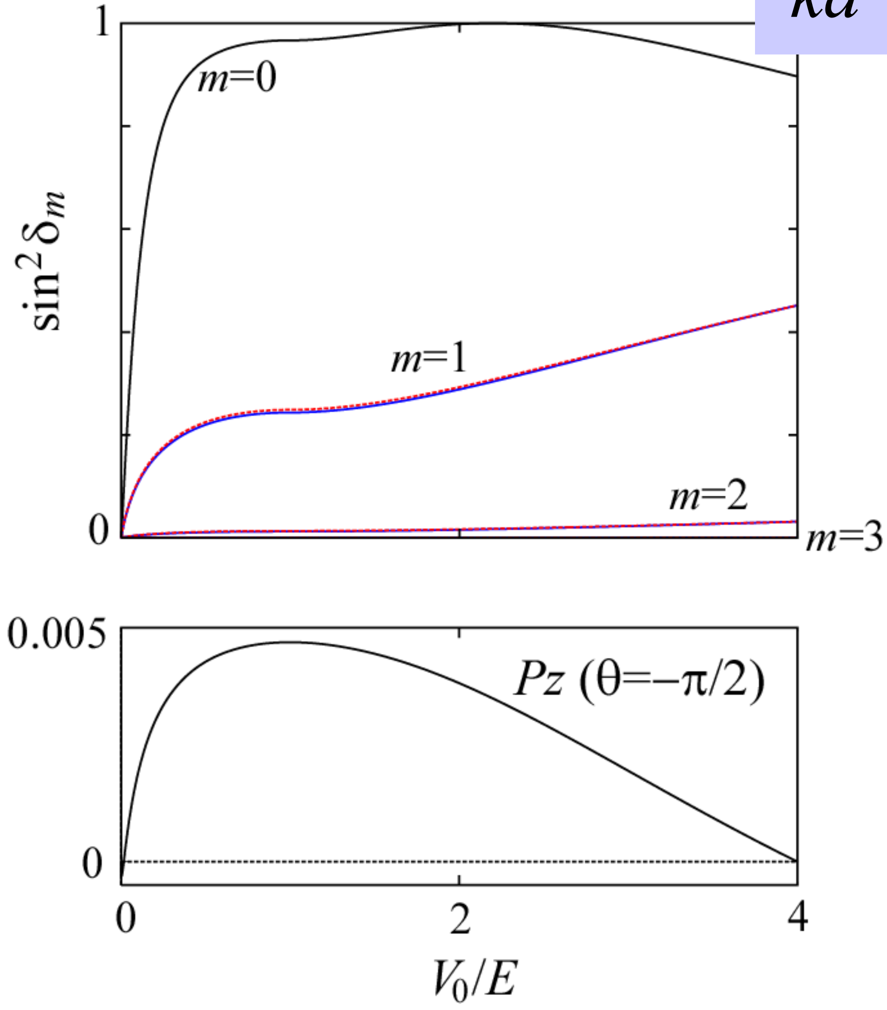
- Spin polatization in $-y$ direction ($\theta=-\pi/2$)



Around resonance, spin polarization is enhanced.

- Repulsive potential

ka=2, $V_0 > 0$

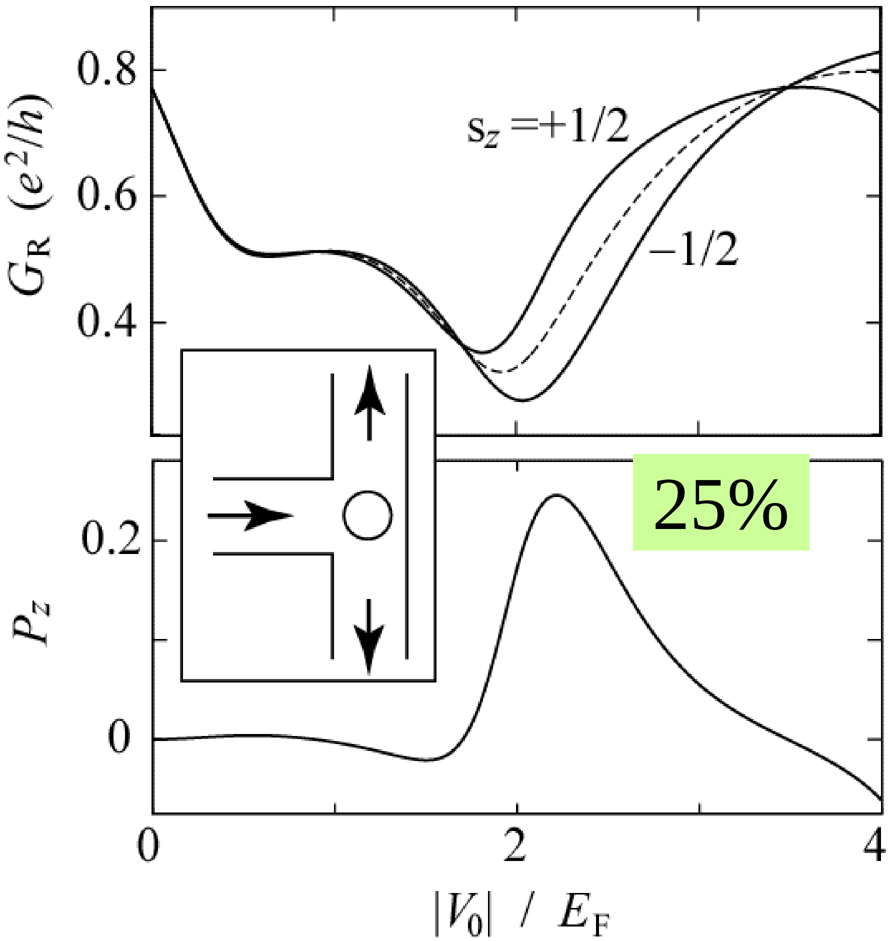


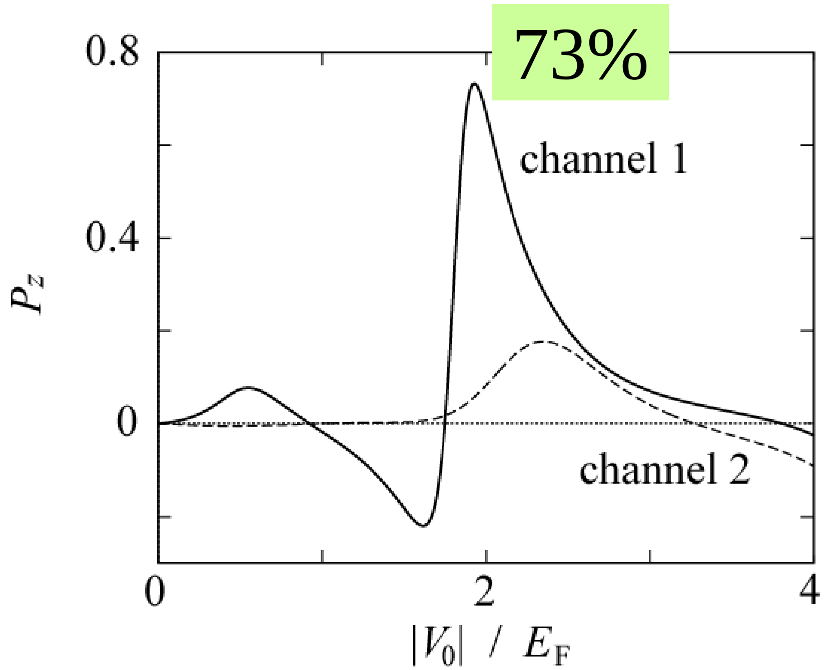
SHE exists, but is very small.

Application to 3-terminal spin filter

Conductance and $P_z = \dfrac{G_+ - G_-}{G_+ + G_-}$

P_z for each channel in incident wave





Channel 1 can be injected selectively using QPC

2.3. Conclusions

- Formulation of extrinsic spin Hall effect (SHE) in 2D using partial wave expansion
- Enhanced SHE by resonant scattering by tuning attractive potential
- Three-terminal device including an antidot for spin injection, showing polarization ~30% for two channels and ~70% for single channel

References

M. Eto and T. Yokoyama, J. Phys. Soc. Jpn. **78**, 073710 (2009); T. Yokoyama and M. Eto, PRB **80**, 125311 (2009).

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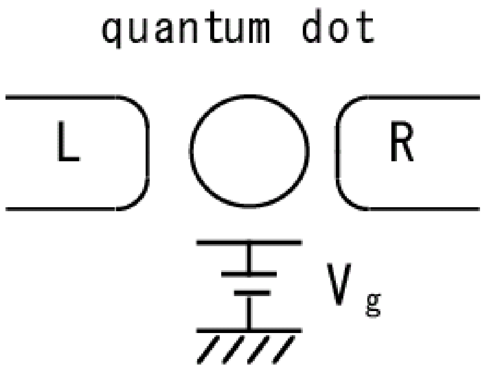
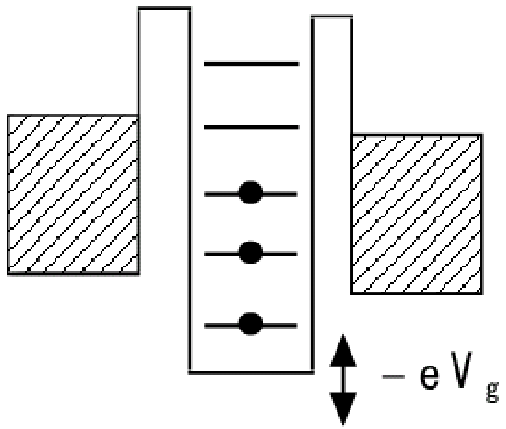
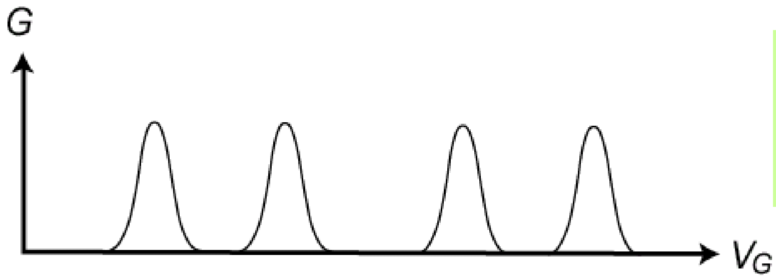
3.1. Coulomb oscillation in quantum dot





- Quantum dots: zero-dimensional systems of nano-meter scale
- Transport through "quantum levels" in quantum dots
- Quantum levels are controlled by gate voltage.

→ <u>peak structure of current</u>

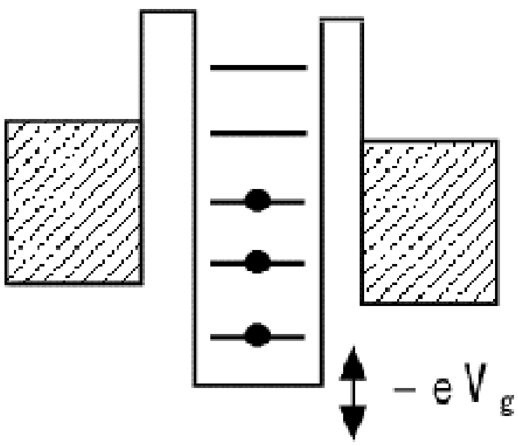


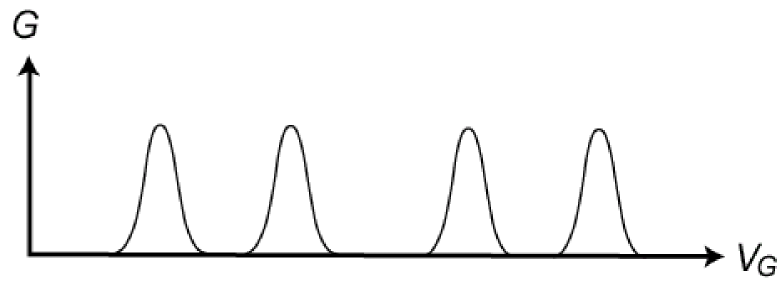
<u>Coulomb oscillation</u>
(<u>Coulomb blockade</u> between peaks)

What are "quantum levels"?

1. In absence of electron-electron interaction, "quantum levels" are single-electron energy levels.
2. In presence of electron-electron interaction (charging energy), increase in energy to put an electron on the dot (electro-chemical potential):

$$\mu_N = E_N - E_{N-1}$$

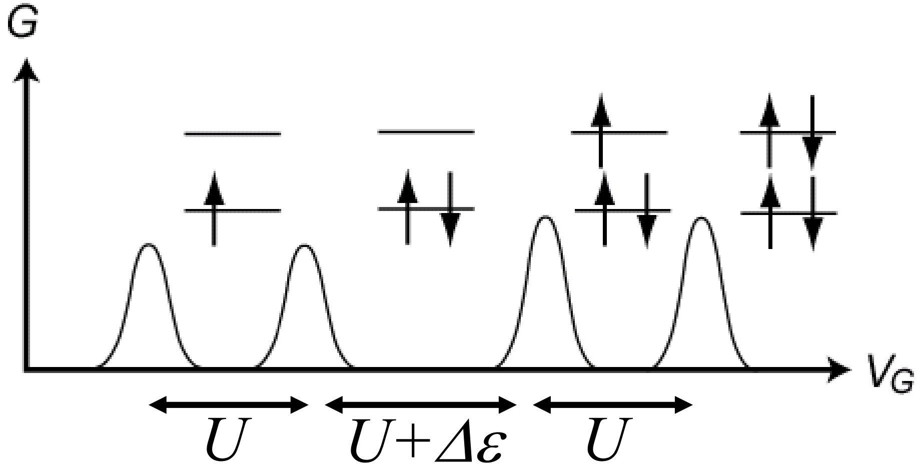




Constant interaction model with spin-degenerate levels

$$E_N = \sum_{i=1}^{N} \varepsilon_i + \frac{N(N-1)}{2} U,$$

$$\mu_N = E_N - E_{N-1} = \varepsilon_N + (N-1)U$$

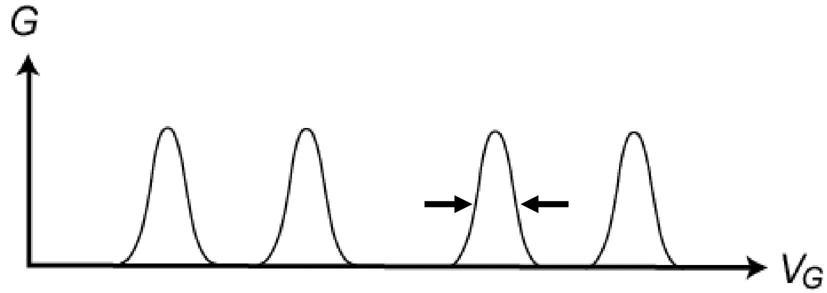


$$\mu_1 = \varepsilon_1$$
$$\mu_2 = \varepsilon_1 + U$$
$$\mu_3 = \varepsilon_2 + 2U$$
$$\mu_4 = \varepsilon_2 + 3U$$

- Coulomb peak: resonant tunneling at $k_B T < \Gamma$
- Coulomb blockade regime: Kondo resonance at $T < T_K$

Condition to observe Coulomb oscillation, blockade

(level spacing), (Charging energy) >> $k_B T$, Γ



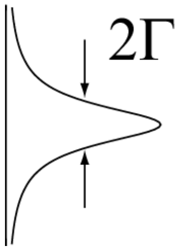
- Quantum fluctuation: "level broadening" Γ

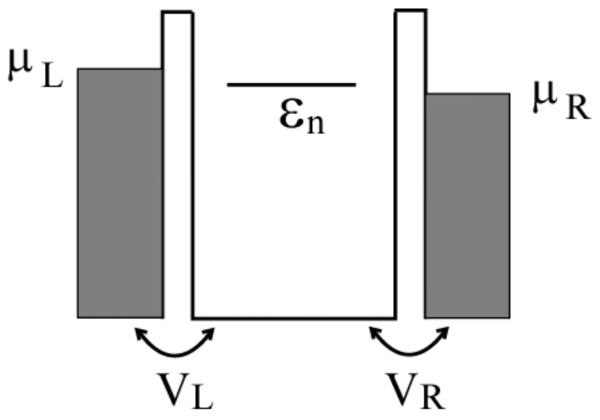
(due to finite lifetime by tunnel coupling to the leads)

$$\frac{1}{\tau} = \sum_{\alpha=L,R;k} \frac{2\pi}{\hbar} \left| \langle \alpha, k | H_T | d_n \rangle \right|^2 \delta(\varepsilon_k - \varepsilon_n)$$

$$= \frac{2\pi}{\hbar} \nu \left(|V_L|^2 + |V_R|^2 \right)$$

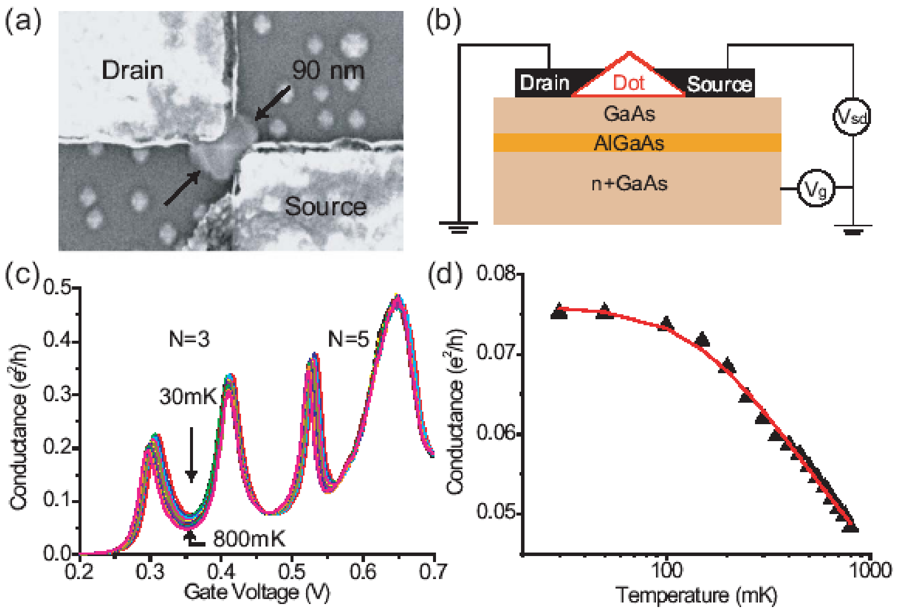
$$\Gamma = \frac{1}{2}\frac{\hbar}{\tau} = \pi \nu \left(|V_L|^2 + |V_R|^2 \right)$$





3.2. Spin Hall effect in quantum dot

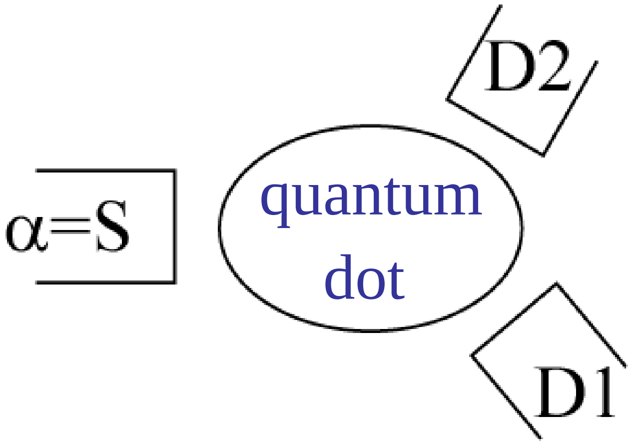
- InAs quantum dot: strong SO interaction
  - Y. Igarashi *et al*., PRB **76**, 081303(R) (2007).
  - S. Takahashi *et al*., PRL **104**, 246801(2010).



- Energy level splitting by SO interaction: 0.23meV
- Kondo effect

- C. Fasth *et al*., PRL **98**, 266801 (2007).
- A. Pfund *et al*., PRB **76**, 161308(R) (2007).

“Spin Hall effect” at quantum dot



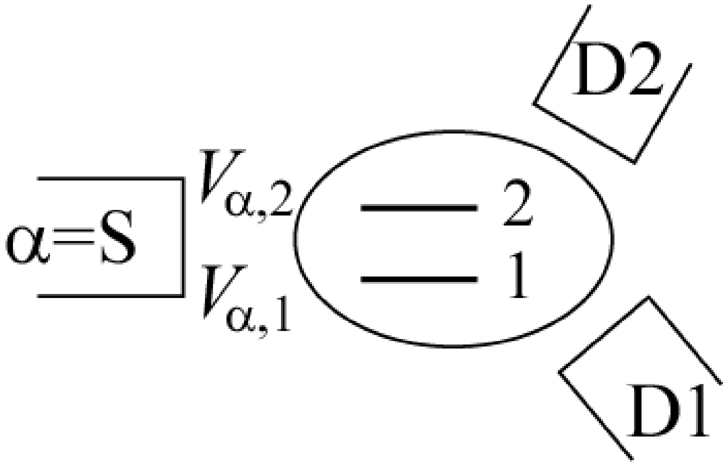
- Strong SO interaction is present only in quantum dot
- Multi-terminal system
  - unpolarized current is injected from lead S
  - spin-polarized current to leads D1, D2,…

“Spin filter” effect

cf. Previous work for “open quantum dot” without tunnel barriers: Krich and Halperin, PRB 78, 035338 (2008).

Model

- Two energy levels in a quantum dot (minimal model)
- Single channel in leads
- No magnetic field: wavefunctions are real



Number of leads

$N \geq 2$

$$\varepsilon_d = \frac{\varepsilon_1 + \varepsilon_2}{2}, \ \Delta = \varepsilon_2 - \varepsilon_1$$

tunable by V_G

- SO interaction in the quantum dot

$$H_{\mathrm{SO}} = \frac{\lambda}{\hbar}\boldsymbol{\sigma}\cdot\left(\boldsymbol{p}\times\nabla U\right)$$

$$\left\langle 1\middle|H_{\mathrm{SO}}\middle|1\right\rangle = \left\langle 2\middle|H_{\mathrm{SO}}\middle|2\right\rangle = 0$$

$$\left\langle 2\middle|H_{\mathrm{SO}}\middle|1\right\rangle = i\,\boldsymbol{h}_{\mathrm{SO}}\cdot\boldsymbol{\sigma}/2$$

$$i\boldsymbol{h}_{\mathrm{SO}}/2 = \left(\lambda/\hbar\right)\left\langle 2\middle|\left(\boldsymbol{p}\times\nabla U\right)\middle|1\right\rangle$$

Quantization axis // $\boldsymbol{h}_{\mathrm{SO}}$

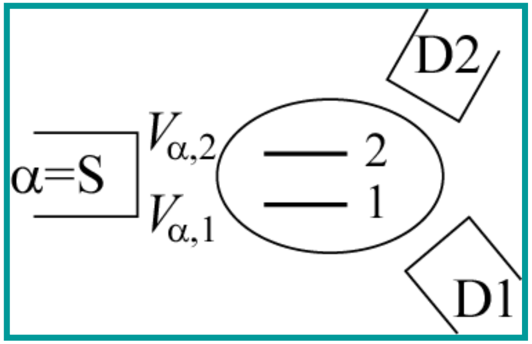
$$H_{\mathrm{dot},\sigma=\pm 1} = \varepsilon_d + \frac{1}{2}\begin{pmatrix} -\Delta & \mp i\Delta_{\mathrm{SO}} \\ \pm i\Delta_{\mathrm{SO}} & \Delta \end{pmatrix}$$

$$\Delta_{\mathrm{SO}} = \left|\boldsymbol{h}_{\mathrm{SO}}\right|$$

For details, see
Eto and Yokoyama,
Poster WP-62

Three-terminal system

$\Delta = \varepsilon_2 - \varepsilon_1 = 0.2\Gamma$ $\quad \Delta = \Gamma$



$\Gamma_S = \Gamma_{D1} \equiv \Gamma$

$e_{S,1}/e_{S,2} = 1/2$

$e_{D1,1}/e_{D1,2} = -3, e_{D2,1}/e_{D2,2} = 1$

(a) $\Gamma_{D2} = 0.2\Gamma$

(b) $\Gamma_{D2} = 0.5\Gamma$

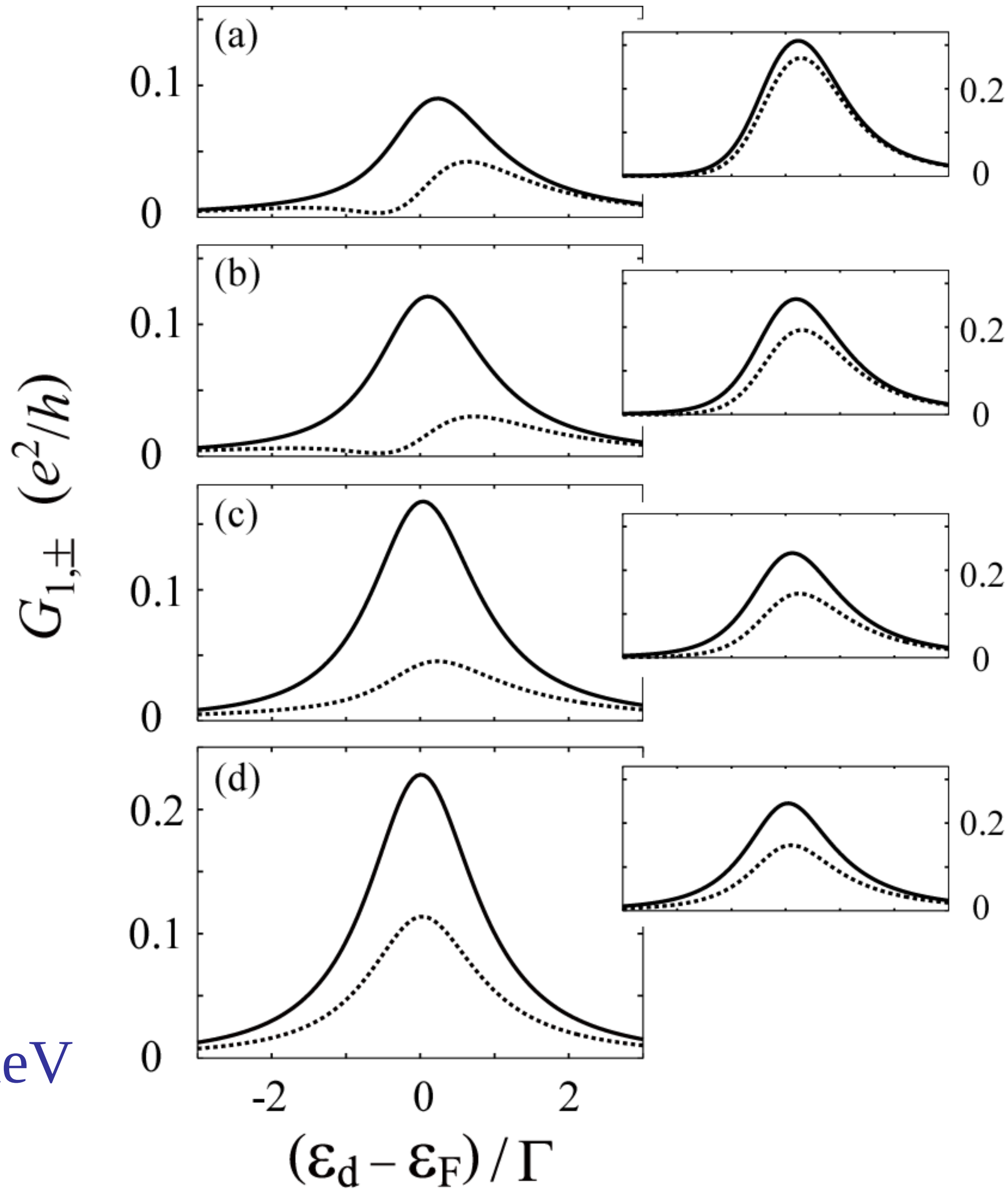
(c) $\Gamma_{D2} = \Gamma$

(d) $\Gamma_{D2} = 2\Gamma$

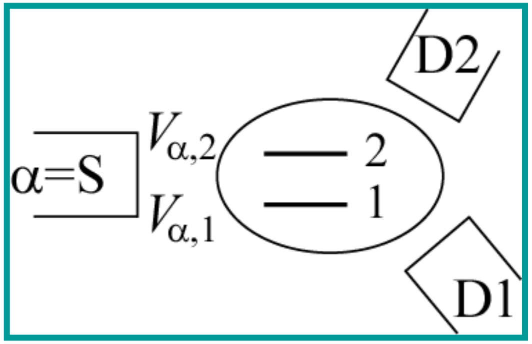
$\Delta_{SO} = 0.2\Gamma$

Δ_{SO}=0.23 meV

Γ ~ 1 meV



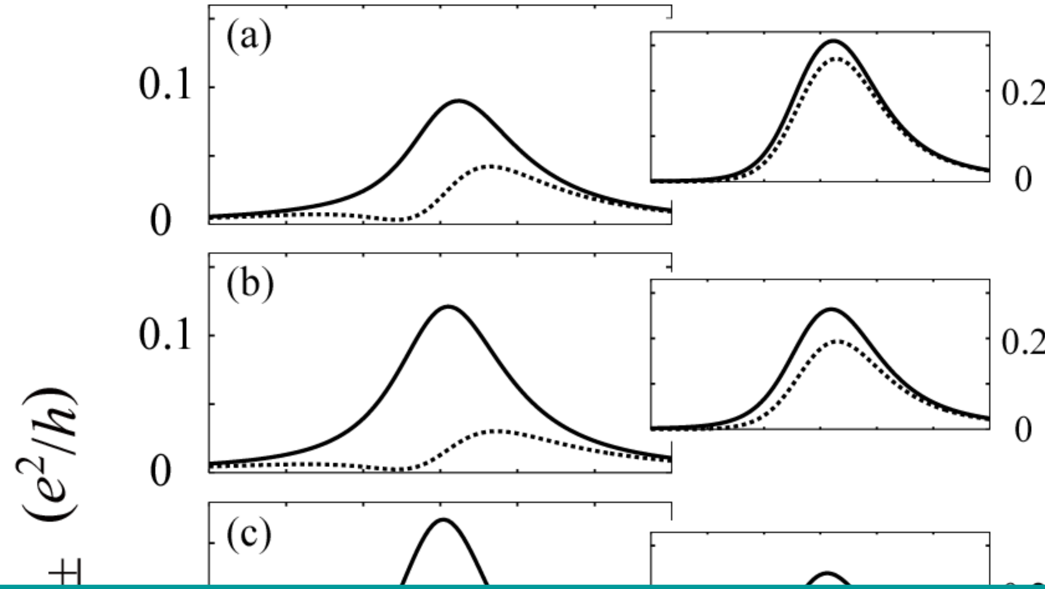
Three-terminal system



$\Gamma_S = \Gamma_{D1} \equiv \Gamma$

$e_{S,1}/e_{S,2} = 1/2$

$\Delta = \varepsilon_2 - \varepsilon_1 = 0.2\Gamma$ $\Delta = \Gamma$



(1) Large spin polarization around current peak

Enhancement of SHE by resonant tunneling

(2) Level spacing Δ ~ broadening Γ

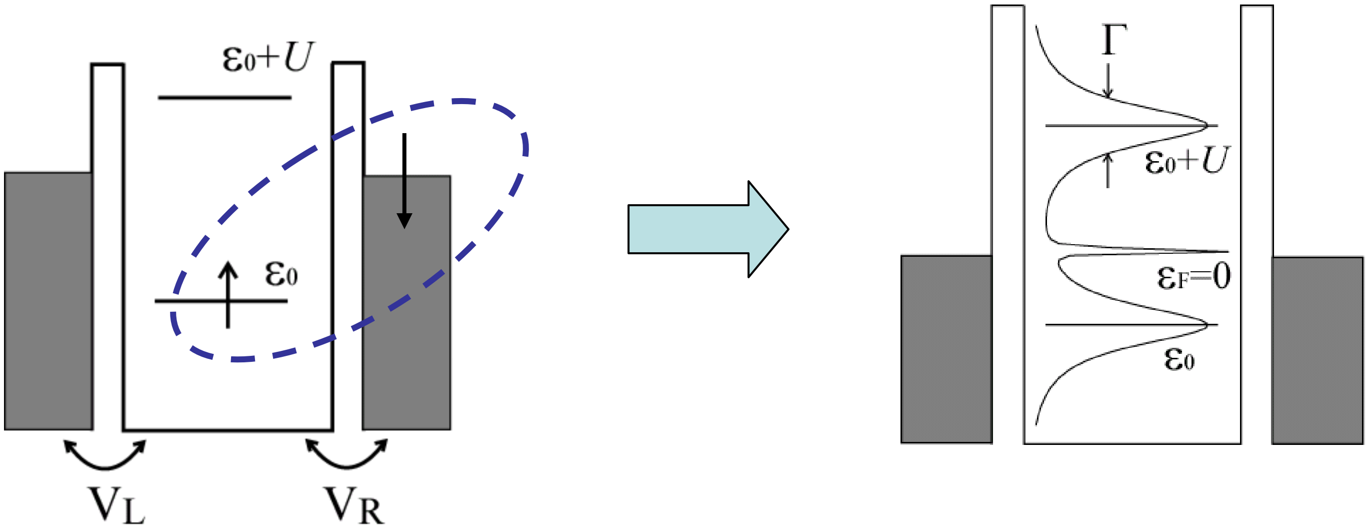
Two levels should contribute to transport

(3) Control of SHE by tuning Γ_{D2} (tunnel coupling to D2)

3.3. Enhanced SHE by Kondo resonance

<u>Kondo effect in quantum dot</u>

- Spin S=1/2 in quantum dot + Fermi sea in leads
- Spin-singlet state (S=0)



Many-body ground state
(Binding energy:
Kondo temperature T_K)

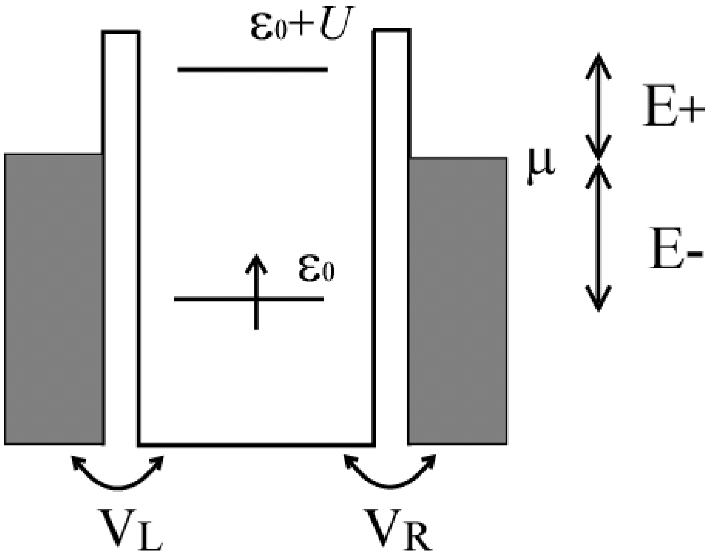
Resonant level at E_F
with width of T_K

Coulomb blockade with single electron

$$E^{+}, E^{-} >> k_{\mathrm{B}}T, \Gamma$$

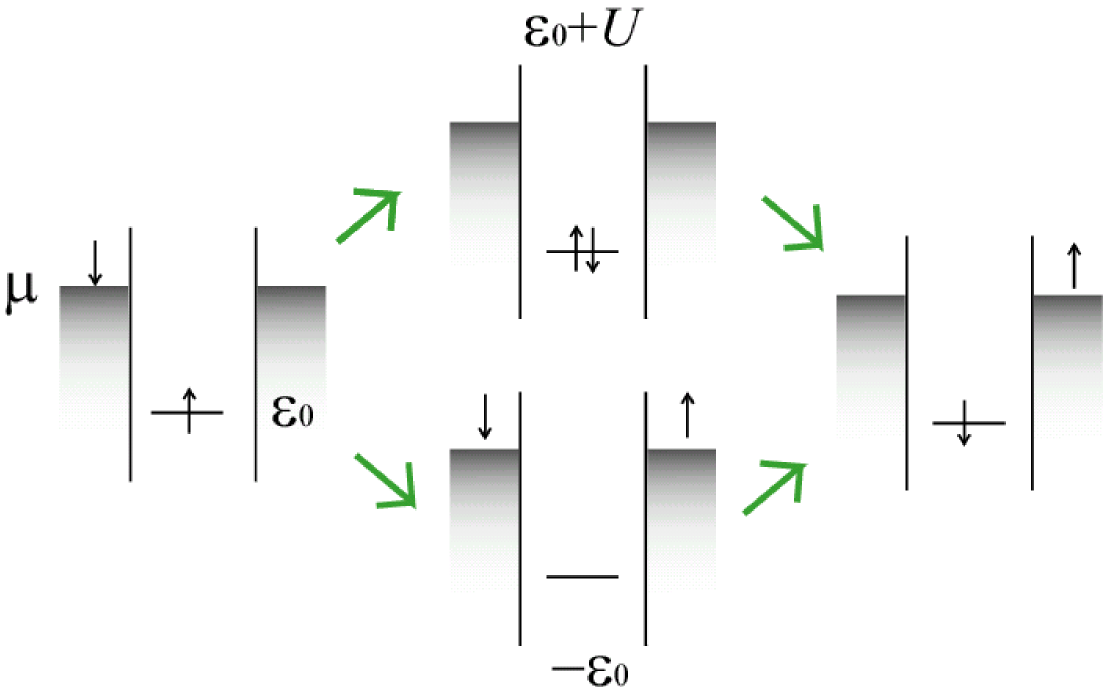
- Addition and extraction energies

$$\begin{cases} E^{+} = \mu_2 - \mu = \varepsilon_0 + U - \mu \\ E^{-} = \mu - \mu_1 = \mu - \varepsilon_0 \end{cases}$$



- Sequential tunnel process is forbidden.
- Higher-order tunnel process “cotunneling”

- Spin-flip by cotunneling

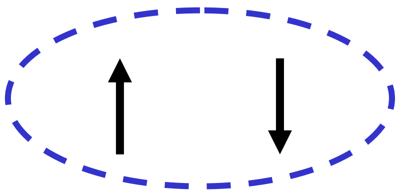


- Anti-ferromagnetic coupling between localized spin and conduction electrons

$$H = 2J\sum_{kk'}\left[S_+ c^+_{k'\uparrow} c_{k\downarrow} + \cdots\right] = 2J\boldsymbol{S}\cdot(\boldsymbol{s})_{k'k},\ \ J = V^2\left(\frac{1}{E^+} + \frac{1}{E^-}\right)$$

Ground state with antiferromagnetic coupling

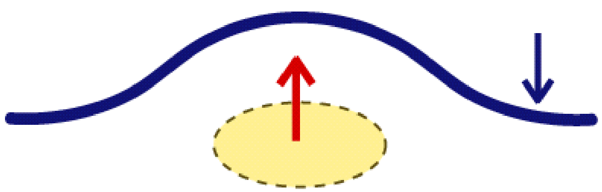
- Two interacting spins:



$$\left|\mathrm{Grd}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle_1\left|\downarrow\right\rangle_2 - \left|\downarrow\right\rangle_1\left|\uparrow\right\rangle_2\right)$$

Spin-singlet state

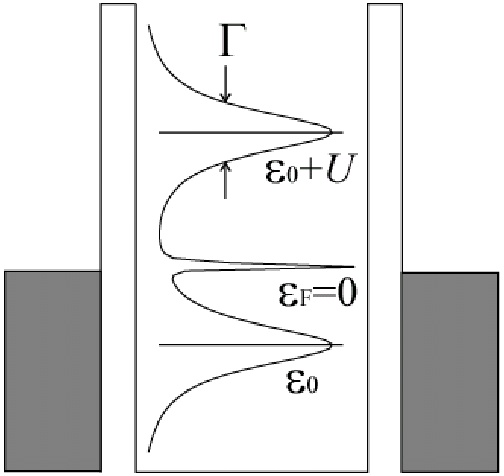
- One spin and Fermi sea:



Kondo singlet state
(Many-body state)

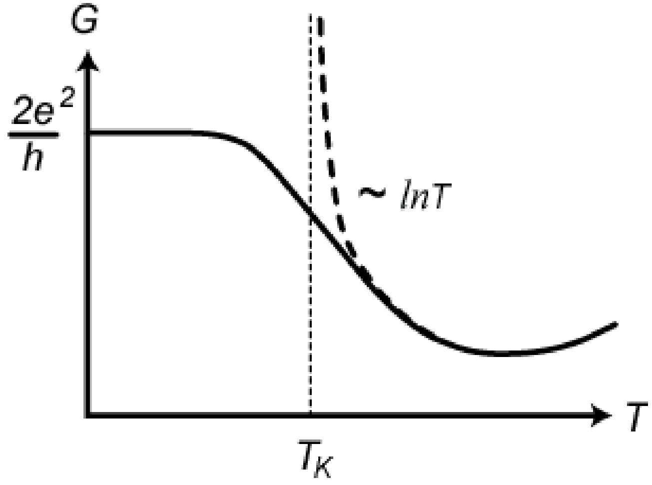
Conduction electrons coherently couple with a localized spin (spin is completely screened).

- $T >> T_K$: Spin S=1/2 is localized in the quantum dot (small G by Coulomb blockade)
- $T << T_K$: Kondo singlet state is formed. Resonant tunneling through the singlet state.



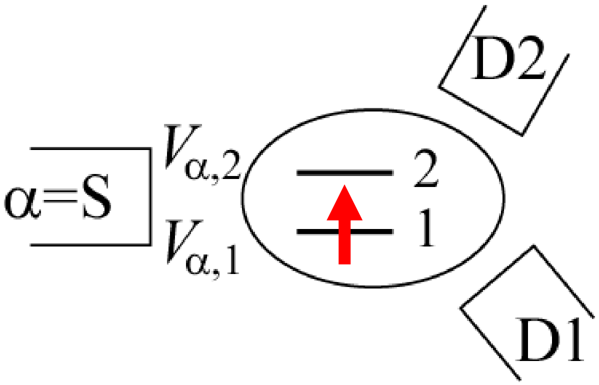
$$G = \frac{2e^2}{h}$$

Conductance



Two levels in quantum dot

$$\Delta = \varepsilon_2 - \varepsilon_1$$



(1) $T_K < \Delta$: level 2 is irrelevant

Spin S=1/2 in level 1 is screened out
(conventional "spin SU(2)" Kondo effect)

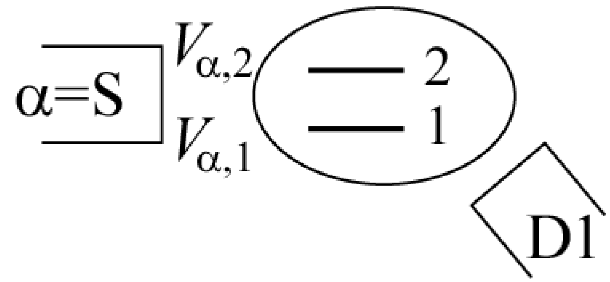
(2) $T_K > \Delta$:

Both pseudo-spin (levels 1 and 2) and spin S=1/2 are screened out (SU(4) Kondo effect)

Crossover of (1) and (2): slave-boson mean-field theory
cf. J. S. Lim *et al*., PRB **74**, 205119 (2006).

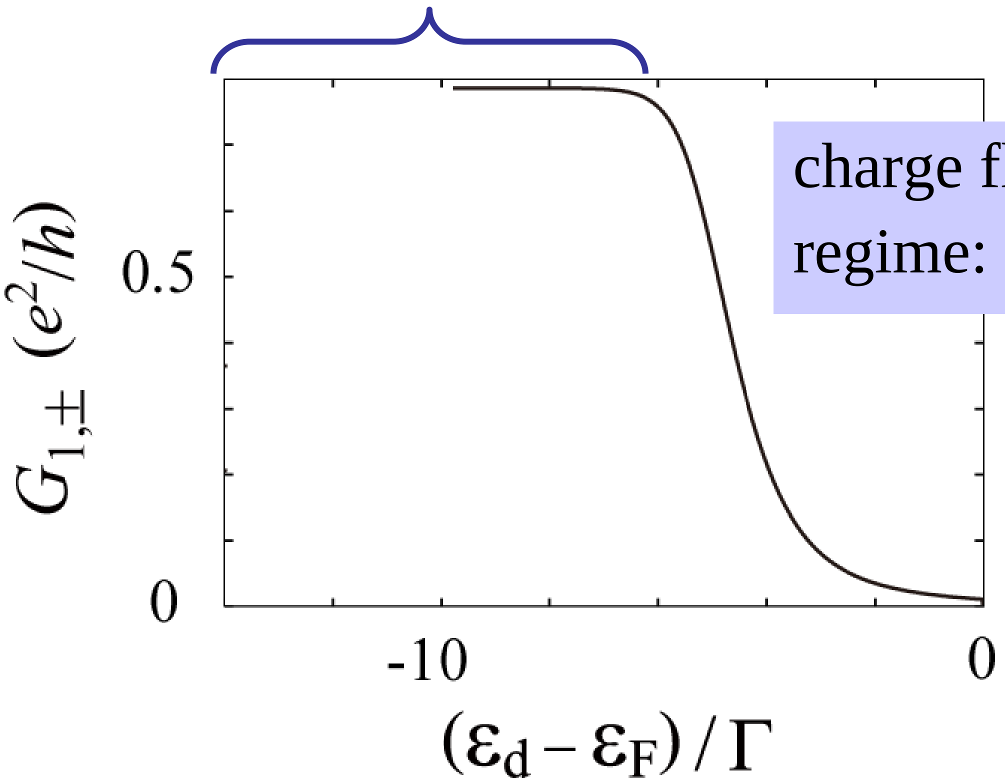
Two-terminal system $U = \infty$

$G_{1,+} = G_{1,-}$



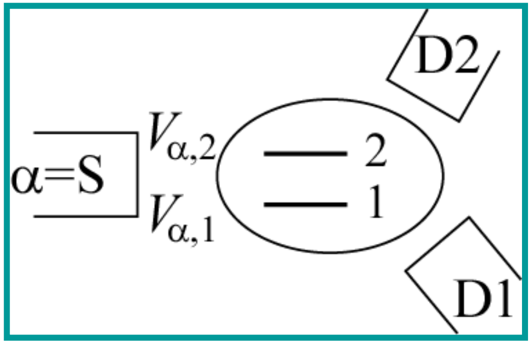
Kondo regime : $-\varepsilon_d \ll \Gamma$

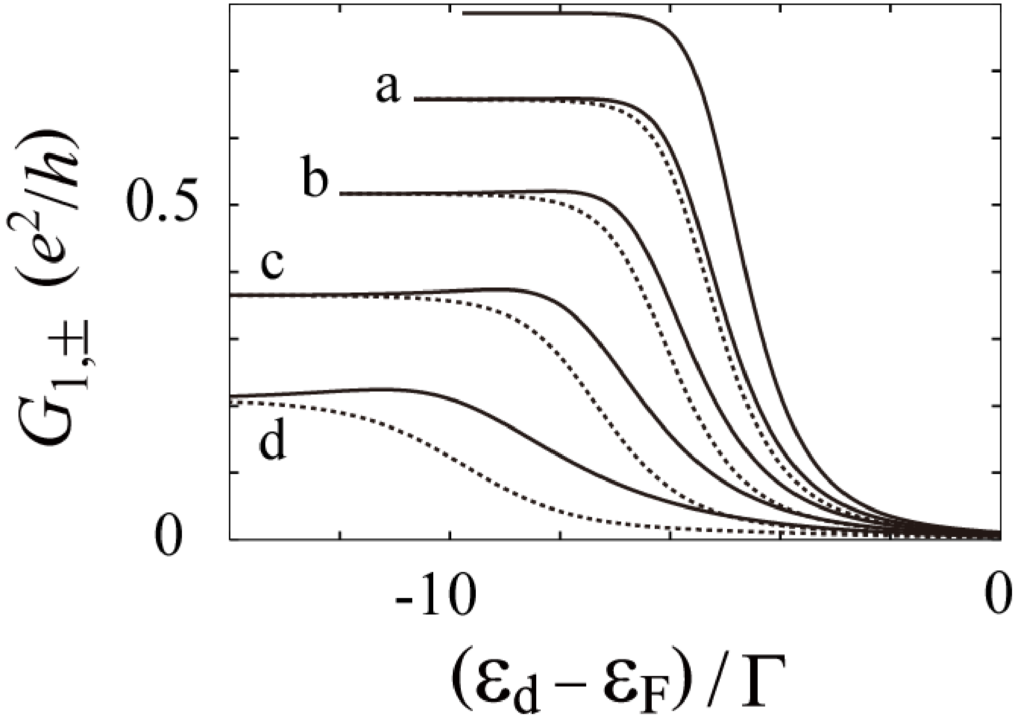
charge fluctuation regime: $-\varepsilon_d \sim \Gamma$



Three-terminal system $U = \infty$

$\Delta = \varepsilon_2 - \varepsilon_1 = 0.2\Gamma$





$\Gamma_S = \Gamma_{D1} \equiv \Gamma$

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$e_{D1,1}/e_{D1,2} = -3, e_{D2,1}/e_{D2,2} = 1$

(a) $\Gamma_{D2} = 0.2\Gamma$

(b) $\Gamma_{D2} = 0.5\Gamma$

(c) $\Gamma_{D2} = \Gamma$

(d) $\Gamma_{D2} = 2\Gamma$

$\Delta_{SO} = 0.2\Gamma$

T_K decreases with decreasing ε_d:

- SU(4) Kondo: $T_K > \Delta$
- SU(2) Kondo: $T_K < \Delta$

3.4. Conclusions

- Formulation of spin Hall effect at quantum dot
  - needs two levels in the dot, more than two leads
- Enhancement of SHE by resonant tunneling and Kondo resonance (SU(4) Kondo effect)

[Reference]

Eto and Yokoyama, J. Phys. Soc. Jpn. **79**, 123711 (2010).

[Observation by "inverse SHE"]

- Ferromagnet (magnetization p) + InAs quantum dot: Hamaya *et al*., APL (2007)

$$G_1 = \frac{1 + p\cos\theta}{2} G_{1,+} + \frac{1 - p\cos\theta}{2} G_{1,-}$$

θ: angle between magnetization and $\boldsymbol{h}_{\mathrm{SO}}$

Outline

1. Introduction

    1.1. Spin-orbit interaction

    1.2. Spin Hall effect

2. Spin Hall effect with artificial potential

3. Semiconductor quantum dot

    3.1. Coulomb oscillation

    3.2. Spin Hall effect in quantum dot

4. Search for Majorana fermions

    4.1. Topological quantum computer

    4.2. Majorana fermion

4.1. Topological quantum computer

- Conventional quantum computer

  (1) qubit: single electron spin

$$|\psi\rangle = C_0|\uparrow\rangle + C_1|\downarrow\rangle$$

  (2) Easy to manipule using ESR, etc.

  (3) Decoherence problem: continuous phase error

$$C_0|\uparrow\rangle + C_1|\downarrow\rangle \Rightarrow C_0|\uparrow\rangle + C_1 e^{i\theta}|\downarrow\rangle$$

overcome by "quantum error correction"

- Topological quantum computer

(1) Many-body states of interacting electrons

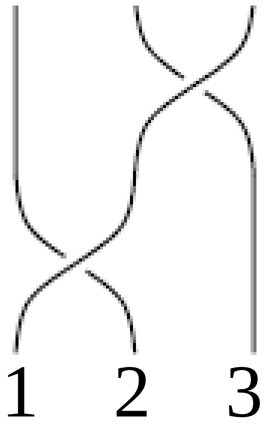
$$\left|\psi\right\rangle = \sum_j C_j \left|\Psi_j\right\rangle \quad \text{(degenerate ground states)}$$

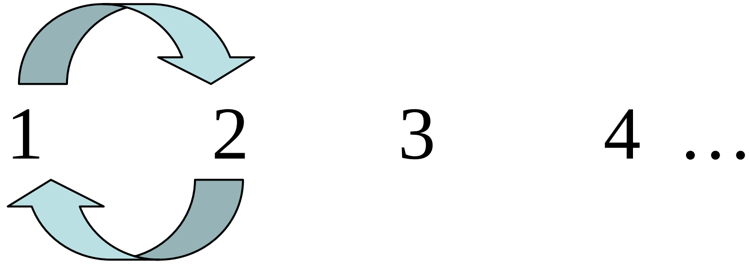
(2) Topologically protected:

robust against local perturbation

(3) Manipulation by exchanging quasi-particles

Non-Abelian braiding statistics





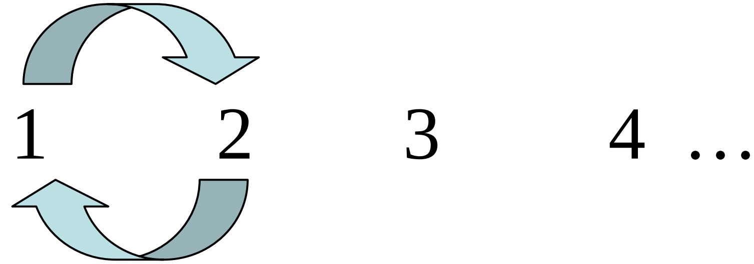
$$\Psi(2,1,3,\ldots) = \Psi(1,2,3,\ldots)$$ Boson

$$\Psi(2,1,3,\ldots) = -\Psi(1,2,3,\ldots)$$ Fermion

$$\Psi(2,1,3,\ldots) = e^{i\alpha}\Psi(1,2,3,\ldots)$$ “Anyon”

2D, $U = \infty$

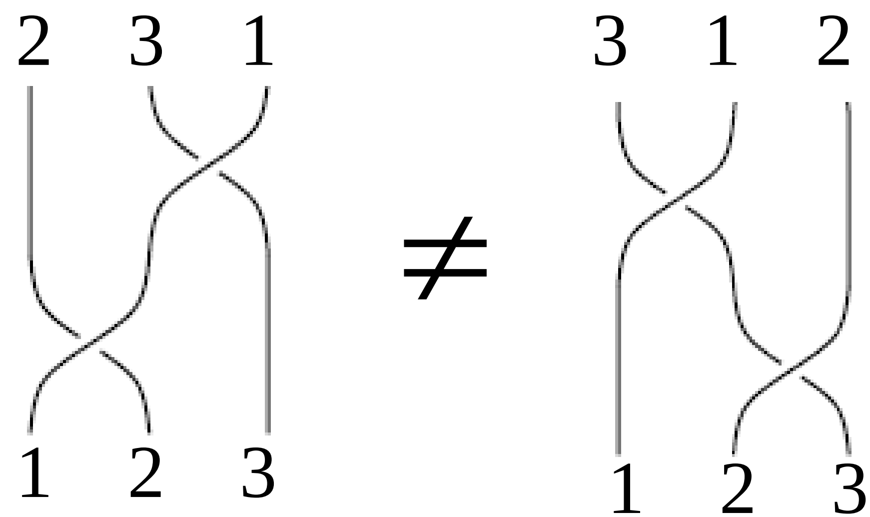
Suggested for high Tc cuprates



$$\Psi_i(2,1,3,\ldots) = \sum_j C_{ij}\Psi_j(1,2,3,\ldots)$$

Non-Abelian

$$T_2T_1 \neq T_1T_2$$



Candidates

- Fractional Quantum Hall effect with ν=5/2
- Half-quantum vortex in *p*-wave superconductor

Strontium ruthenate $Sr_2Ru_2O_4$: Science **331**, 186 (2011).

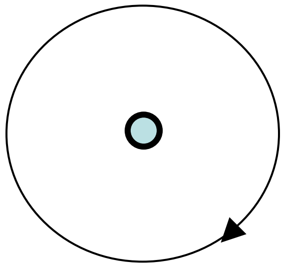
$$\Delta = \Delta_0 \underbrace{\left(k_x \pm i k_y\right)}_{L=1,\, L_z = \pm 1} \underbrace{\frac{1}{\sqrt{2}}\left(\left|\uparrow\uparrow\right\rangle - \left|\downarrow\downarrow\right\rangle\right)}_{S=1}$$

“chiral states”
clockwise vs. anticlockwise

Spin triplet
“equal spin pairing”

Vortex core

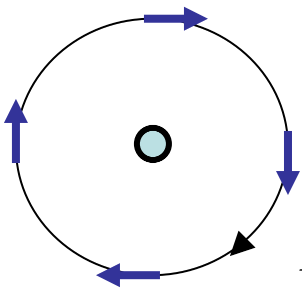
Usual spin-singlet superconductor



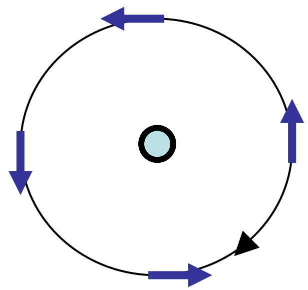
$$\Delta(\theta+2\pi)=\Delta(\theta) \Rightarrow \text{flux } \Phi = n\frac{h}{2e}$$

p-wave (chiral) superconductor

$$\text{flux } \Phi = \frac{n}{2}\frac{h}{2e}$$



$L_z = 1$



$L_z = -1$

Bogoliubov-de Gennes eq. yields quasi-particle operators

$$\gamma^+ = u\Psi^+ + v\Psi$$

$$\gamma^+(E) = \gamma(-E)$$

$$\therefore \gamma^+(E=0) = \gamma(E=0)$$

4.2. Majorana fermions

- Creation and annihilation operators are self conjugate

$$\gamma^+ = \gamma$$

- possibly neutrino
- Half-quantum vortex at E=0

- $2n$ half-quantum vortices: $\gamma_i^+ = \gamma_i \left(i = 1,2,\cdots,2n\right)$

$$\begin{cases} d_i = \left(\gamma_{2i-1} + i\gamma_{2i}\right)/2 \\ d_i^+ = \left(\gamma_{2i-1} - i\gamma_{2i}\right)/2 \end{cases}$$

n "usual" fermions

$$n_i = d_i^+ d_i = 0,1$$

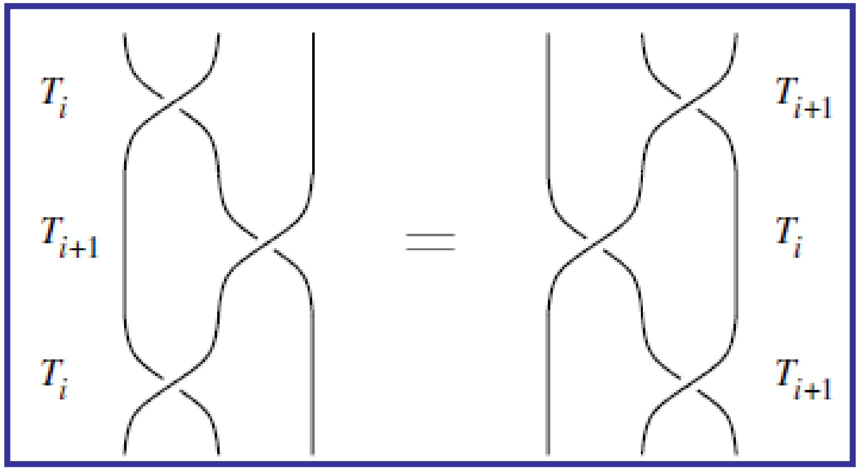
⇒ 2^n degenerate states

* Majorana fermion is a "half" of usual fermion

- 2^n degenerate states
- Non-Abelian braiding statistices

$$T_i T_j = T_j T_i \qquad |i-j|>1$$

$$T_i T_j T_i = T_j T_i T_j \quad |i-j|=1$$



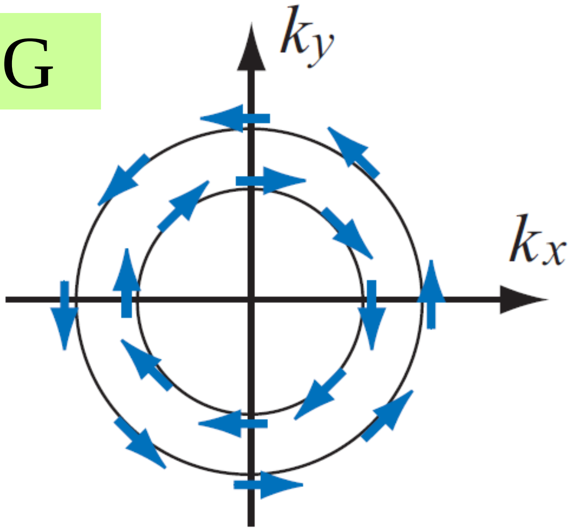
D. A. Ivanov, PRL **86**, 268 (2001).

* Majorana fermions enable topological quantum computation

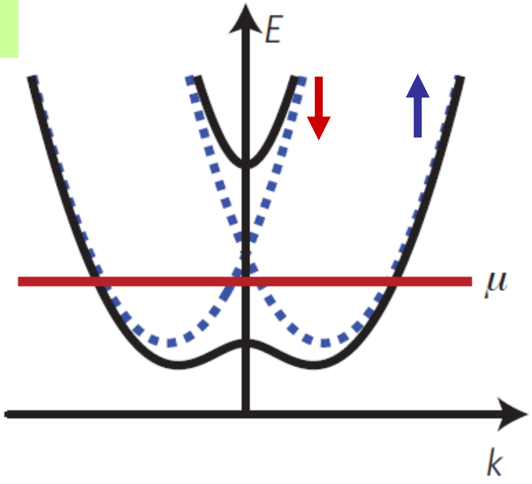
4.3. Majorana fermions in semiconductor

- Rashba spin-orbit interaction

2DEG



1D



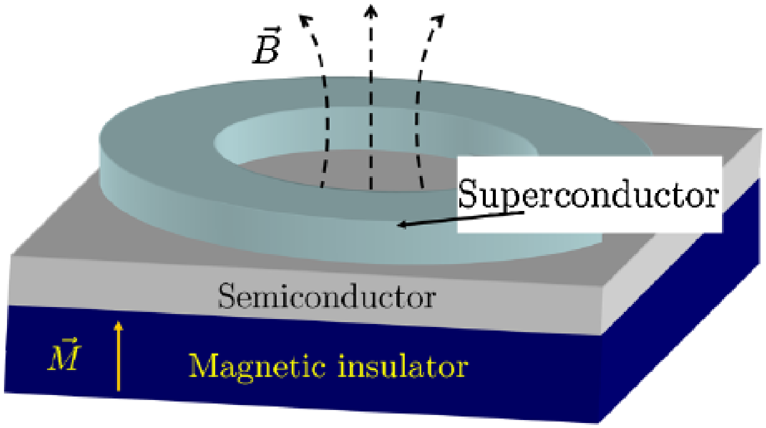
- *S*-wave superconductivity (proximity effect) + magnetic field

⇒ Similar to chiral superconductivity

cf. Wray's talk: topological insulator + superconductivity

(1) Semiconductor thin film

J.D.Sau *et al*., PRL **104**, 040502 (2010)



(2) InAs nanowire

J. Alicea *et al*., Nat. Phys. **7**, 412 (2011)