Quantum Information Processing with Spins in Semiconductors

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2011 August 2

CIFAR CANADIAN INSTITUTE for ADVANCED RESEARCH
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Quantum Information Processing with spins

0. Historical overview

I. Physical requirements

II. Errors and spin coherence

III. Scaling up

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Timeline

1959: There's plenty of room at the bottom

"What I want to talk about is the problem of **manipulating and controlling things on a small scale**."
-R. P. Feynman

1982: Simulating physics with computers

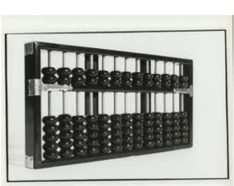
"Nature isn't classical, dammit, and **if you want to make a simulation of nature, you'd better make it quantum mechanical**..."
-R. P. Feynman

1984: Quantum communication (Bennett and Brassard)

1994: Shor's algorithm - *factoring*

1995: Quantum error correction (Shor/Steane)

1996: Grover's algorithm - *database search*

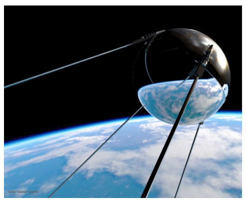


1990's: **Age of proposals:** Cavity QED, ions (1995), NMR, electron spins, superconductors (1997), ...

1996: DiVincenzo Criteria

Timeline: Spins in quantum dots

2000: Single electrons in (lateral) quantum dots (NRC, Ottawa) **(DiV. I)**



2002: Spin lifetime 200 μs (NTT, Tokyo)

2003: Spin initialization and readout (lateral dots, Delft) **(DiV. II,V)**

2005: Spin coherence (Harvard), T_2^{FID}=10 ns **(DiV. III)**

2006: Single-spin echo (Delft): $T_2^{echo} > 1$ μs

2007: Electron spin relaxation (energy dissipation): $T_1 > 1$ s (Munich, Delft, MIT)

2008: Electrically controlled selective spin rotation (Tokyo) **(DiV. IV)**

2008/2009: Hole spin T_1> 1 ms, T_2^{FID}> 1 μs (lower bound) (Heriot-Watt)

2010: Extension to T_2^{FID}~100 ns via nuclear spin state narrowing (Harvard, Tokyo)

2010: Dynamical decoupling: T_2^{echo} ~ 270 μs (Harvard)

Why Spintronics?

Low power, faster devices, additional control

$$\text{“0”} = \uparrow \qquad \text{“1”} = \downarrow$$ (new bits)

Why Quantum Information Processing?

Exponential speedup in algorithms, physical simulation, secure communication, metrology

$$|0\rangle = |\uparrow\rangle \qquad |1\rangle = |\downarrow\rangle$$ (qubits)

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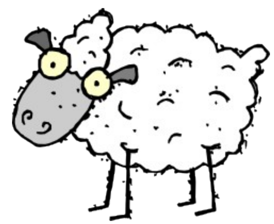
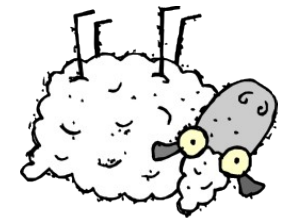
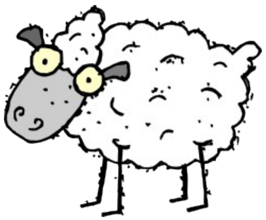
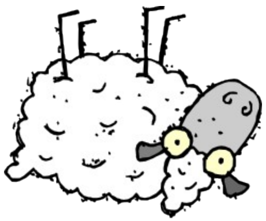
Requirements for Physical Quantum Computing

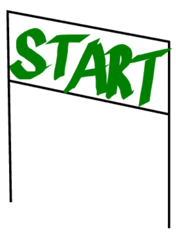
- Encoding

 $= |0\rangle$  $= |1\rangle$

Requirements for Physical Quantum Computing

- Encoding  $= |0\rangle$  $= |1\rangle$

- Initialization/readout



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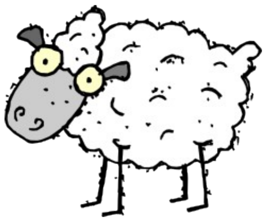
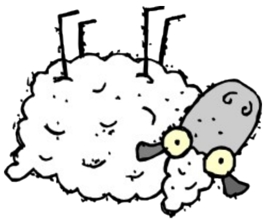
- Encoding  $= |0\rangle$  $= |1\rangle$

- Initialization/readout

- Universal control

(one- and two-qubit control sufficient)

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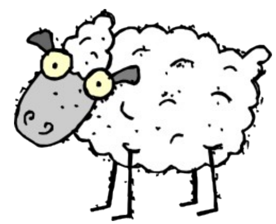
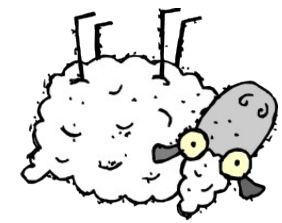
- Encoding  $= |0\rangle$  $= |1\rangle$
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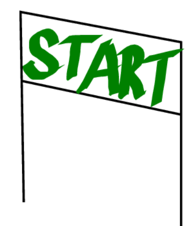
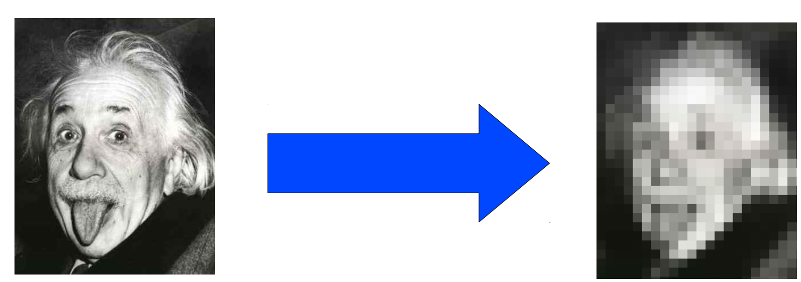


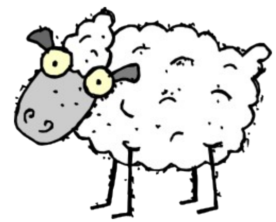
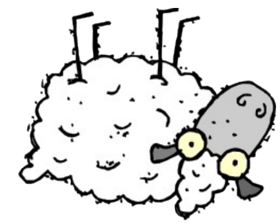
- Universal control

  (one- and two-qubit control sufficient)

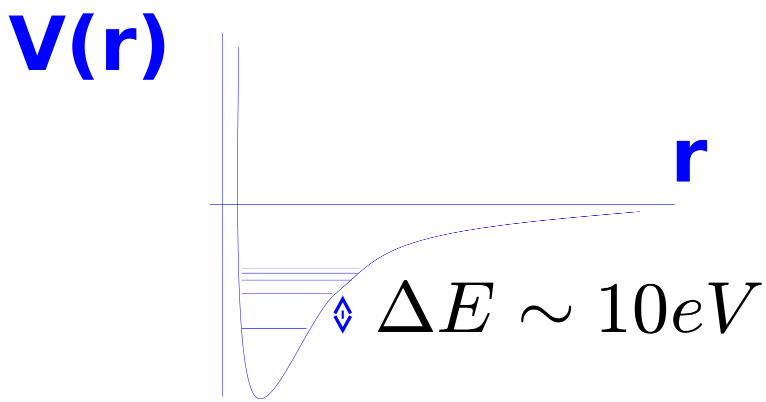
- Coherence

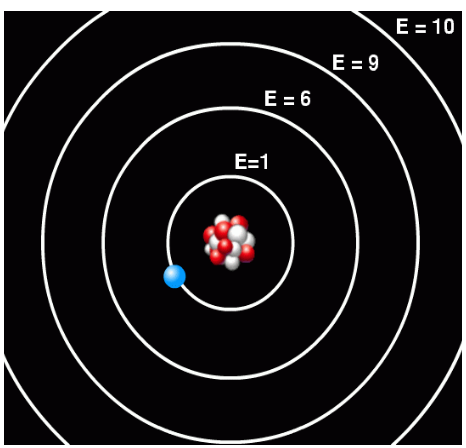


- Encoding

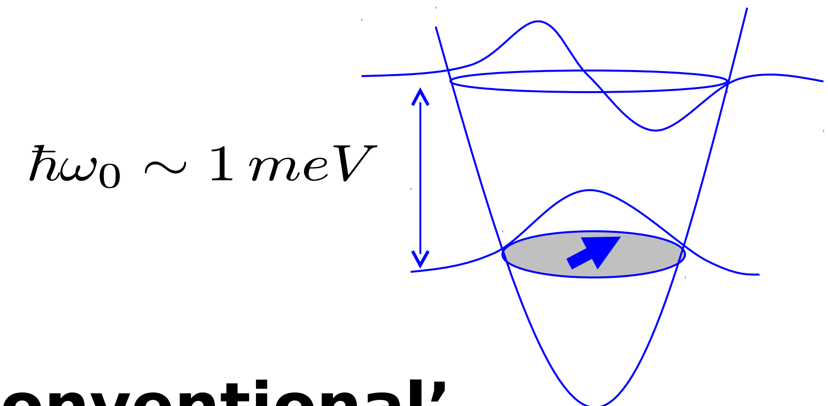
 $= |0\rangle$  $= |1\rangle$

Encoding: Single spins?

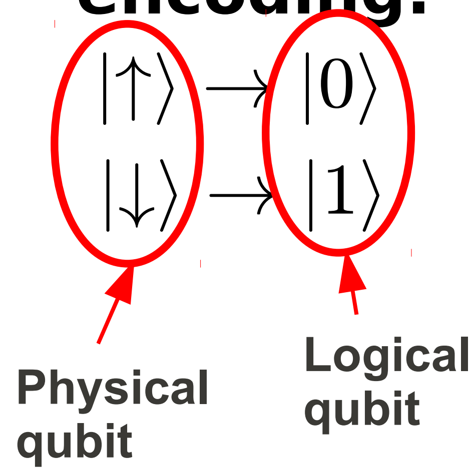


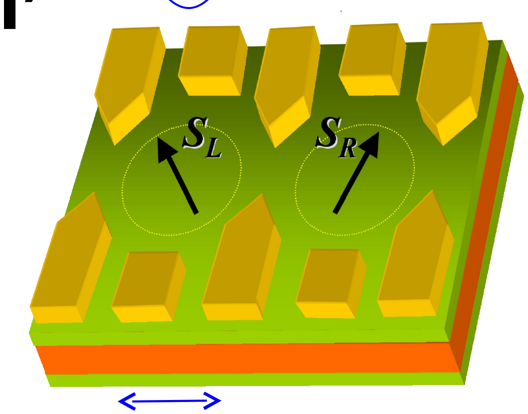


$\sim 10^{-10}\,\mathrm{m}$



'conventional' encoding:





$\sim 10^{-7}\,\mathrm{m}$

Encoding: Alternatives?

Conventional Encoding (single spin)

$$|\uparrow\rangle \rightarrow |0\rangle \qquad\qquad |\downarrow\rangle \rightarrow |1\rangle$$

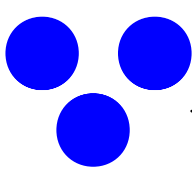
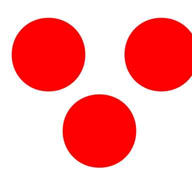
Pro: Well-defined two-level system.
Con: Difficult to control electrically?

Two-spin Encoding (singlet-triplet)

$$|T_0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) \rightarrow |0\rangle \qquad\qquad |S\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) \rightarrow |1\rangle$$

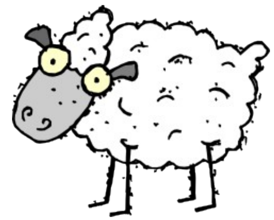
Pro: Easier to control electrically.
Con: One logical qubit for two spins; Susceptible to charge fluctuations.

Three-spin Encoding

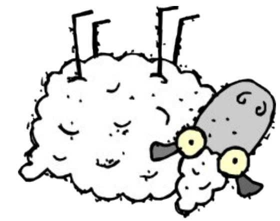
 $\rightarrow |0\rangle$  $\rightarrow |1\rangle$

Pro: All-electrical control, qubits immune to global noise.
Con: One logical qubit for three spins; predominant noise source (nuclei) is local.

- Encoding

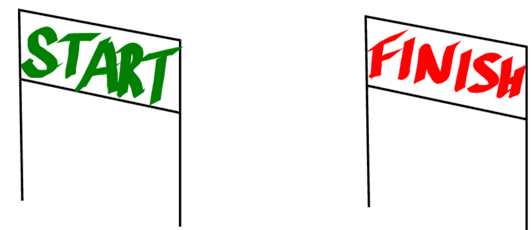


$= |0\rangle$

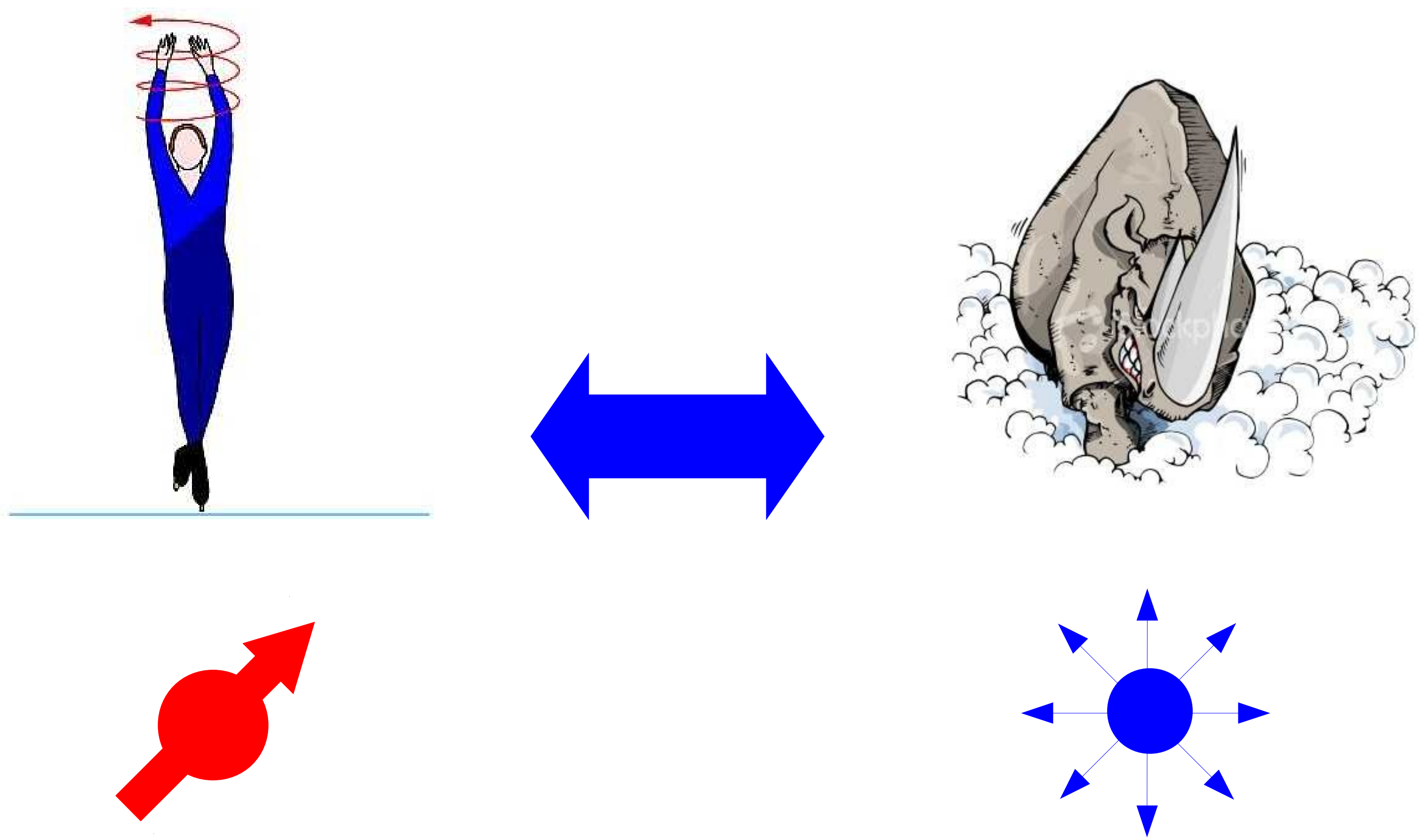


$= |1\rangle$

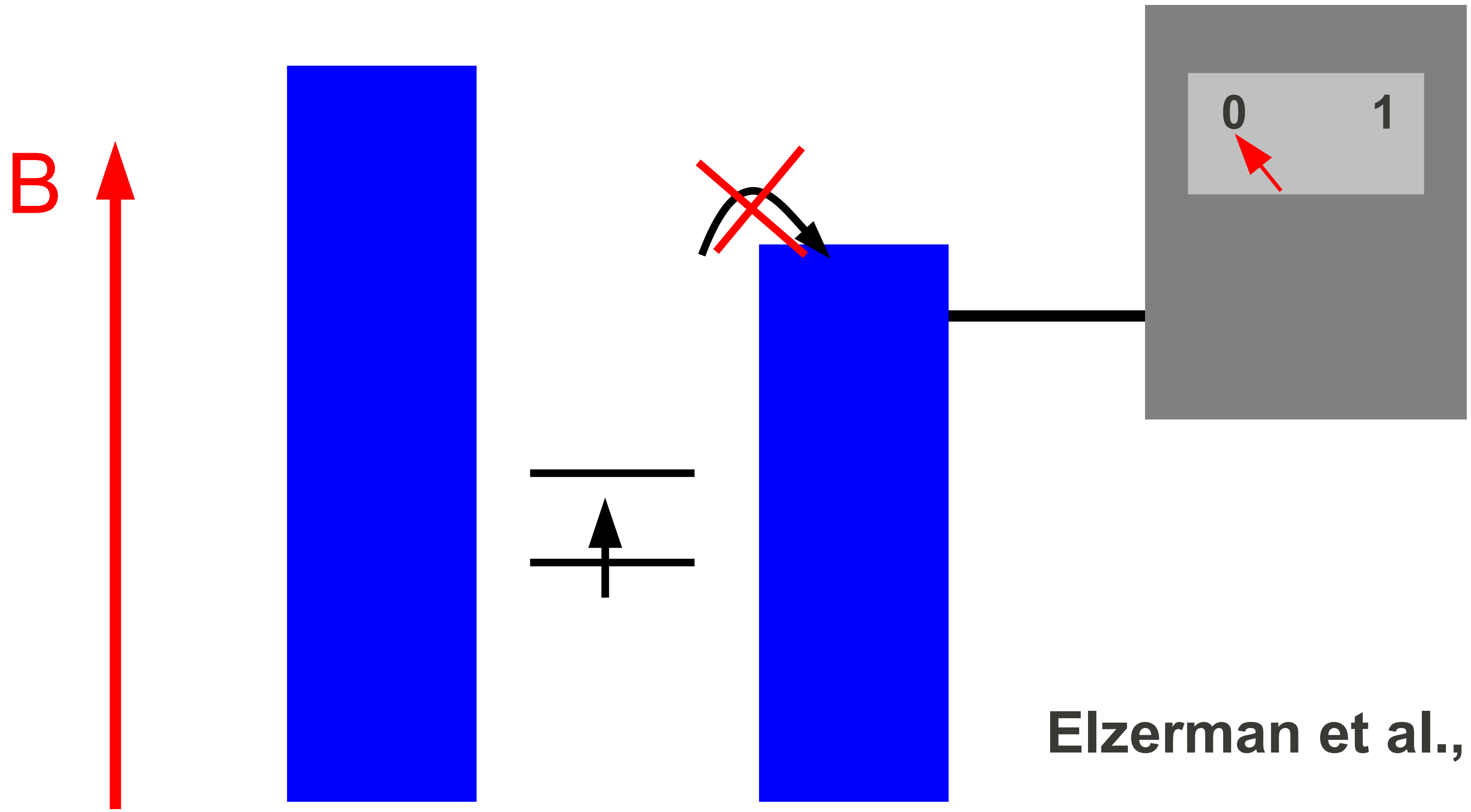
- Initialization/readout



Readout: Spin-to-charge conversion

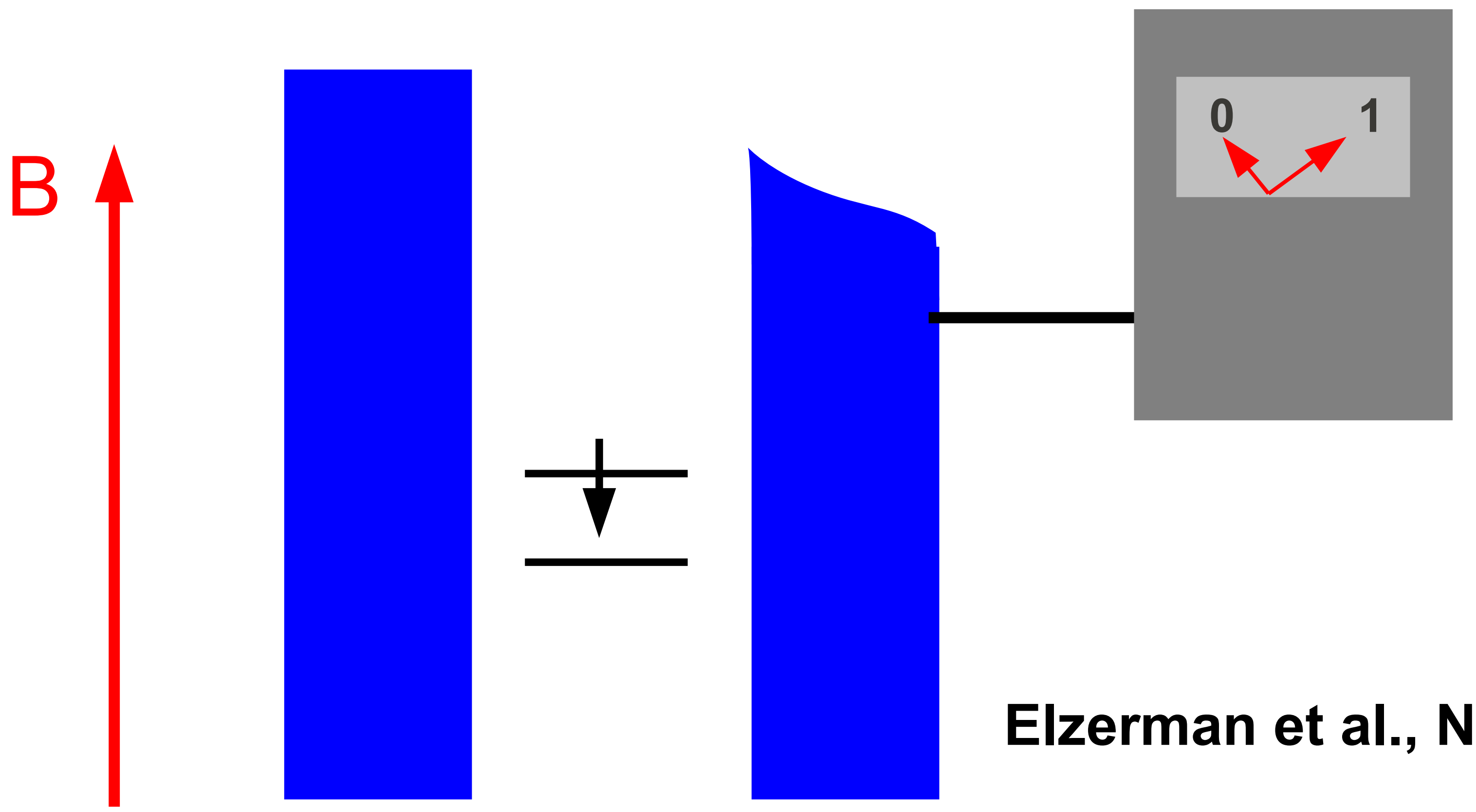


Spin-to-charge conversion: Energy-dependent tunneling



Elzerman et al., Nature (2004)

Spin-to-charge conversion: Energy-dependent tunneling

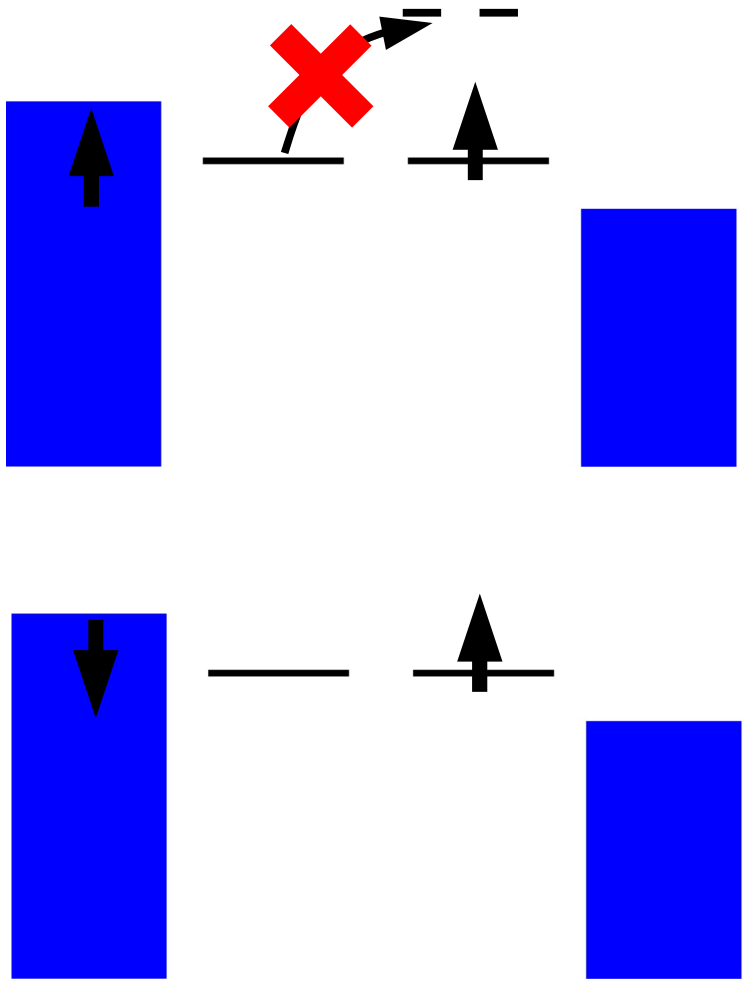


Elzerman et al., Nature (2004)

Finite T: Readout efficient only for sufficiently large B

$$g^* \mu_B B > k_B T$$ (B>1T, GaAs)

Spin-to-charge conversion: Pauli spin blockade (high T)



Current blocked for triplets due to Pauli exclusion:

$$|T_+\rangle = |\uparrow\uparrow\rangle$$

$$|T_-\rangle = |\downarrow\downarrow\rangle$$

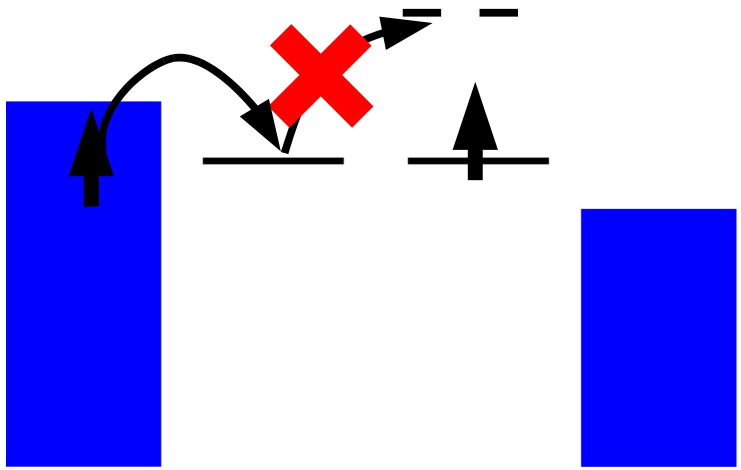
$$|T_0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$$

Current allowed for singlets:

$$|S\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Ono, Austing, Tokura, Tarucha, Science (2002)

Spin-to-charge conversion: Pauli spin blockade



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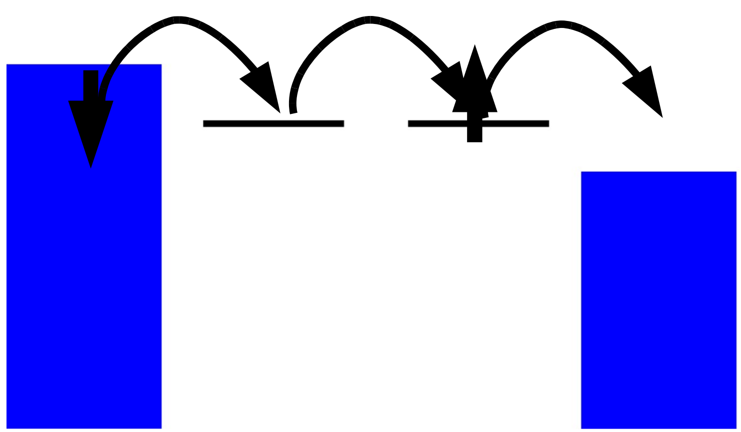
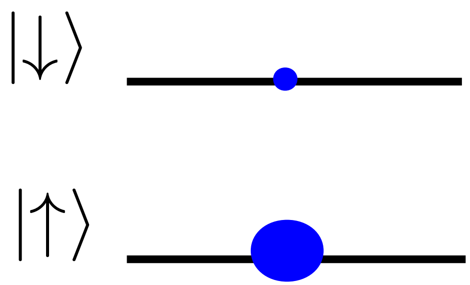


Current allowed for singlets:

$$|S\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Initialization: Thermalize

$$H = g^* \mu_B B S_z$$



$$g^* = -0.4 \quad \text{(GaAs)}$$

$$B = 1\,\mathrm{T}$$

$$T = 100\,\mathrm{mK}$$

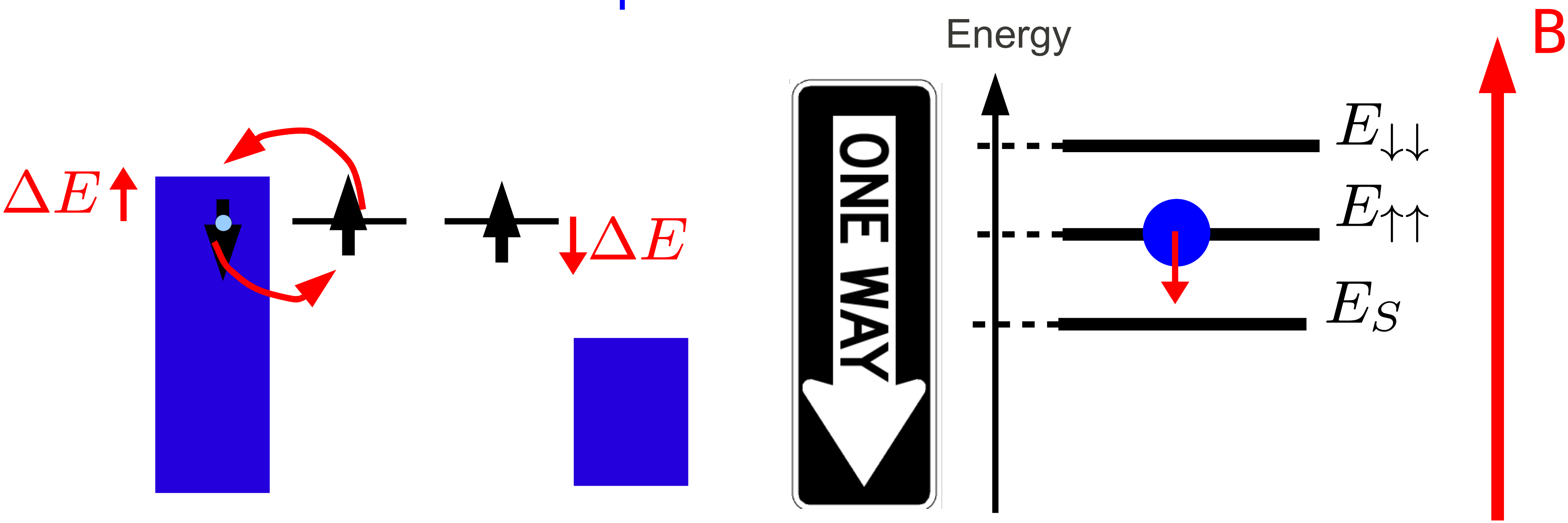
$$\frac{p_\downarrow}{p_\uparrow} = e^{-|g^* \mu_B B|/k_B T} = 0.06$$

But: Need to wait a time T_1 (long) for relaxation.

Alternatives:

(1) Pump the spin to non-equilibrium initial state (e.g., optically).
(2) Briefly decrease T_1 to equilibrate rapidly.

Initialization:
Decrease T_1: Inelastic cotunneling

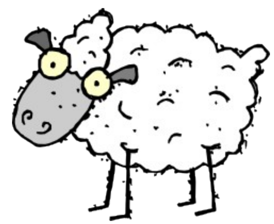
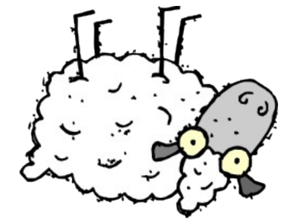


$$|S\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

F. Qassemi, WAC, F. K. Wilhelm, PRL (2009)

N. S. Lai et al., arXiv (2010)

Only $|\uparrow\uparrow\rangle$ prepared!

- Encoding  $= |0\rangle$  $= |1\rangle$

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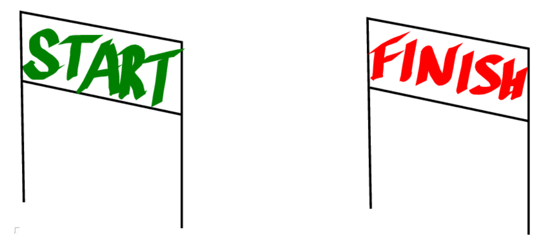
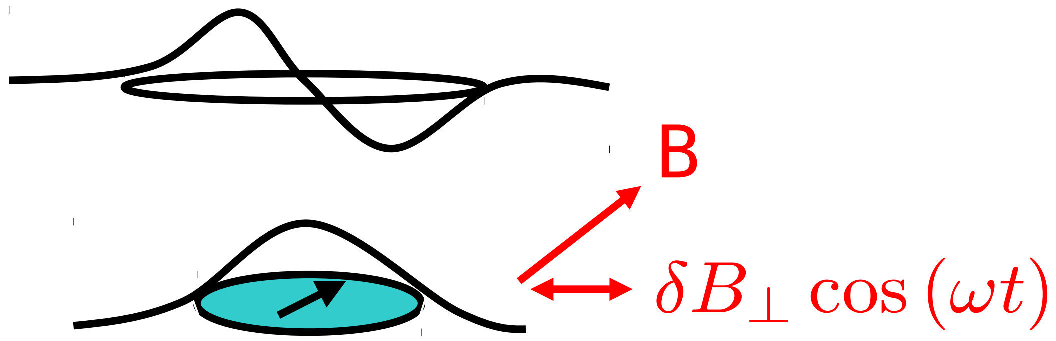


- Universal control

(one- and two-qubit control sufficient)

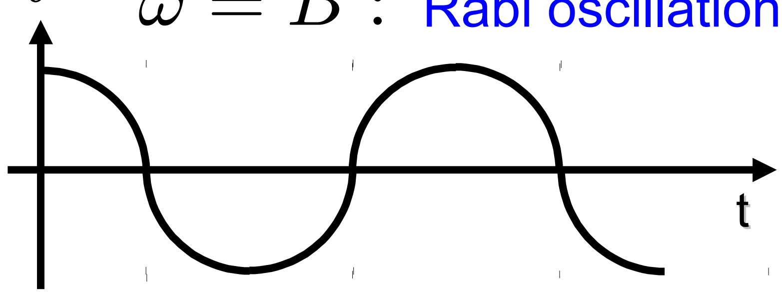
Universal control

Single-qubit control:
Electron spin resonance



$\langle S_z \rangle_t$

$\omega = B$: Rabi oscillations



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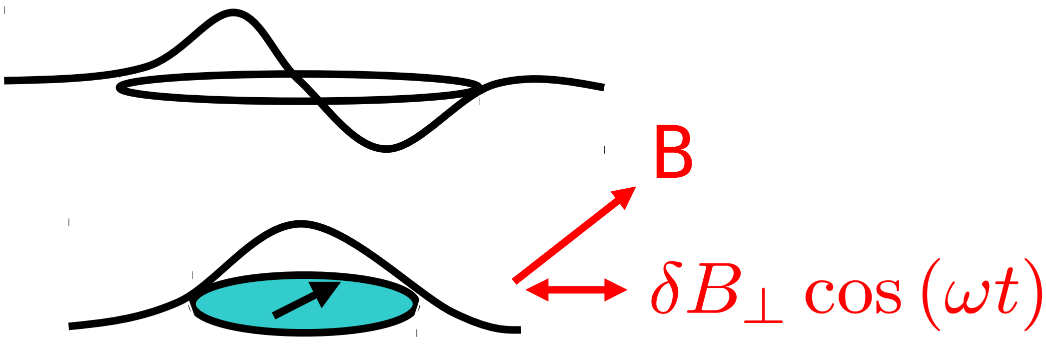
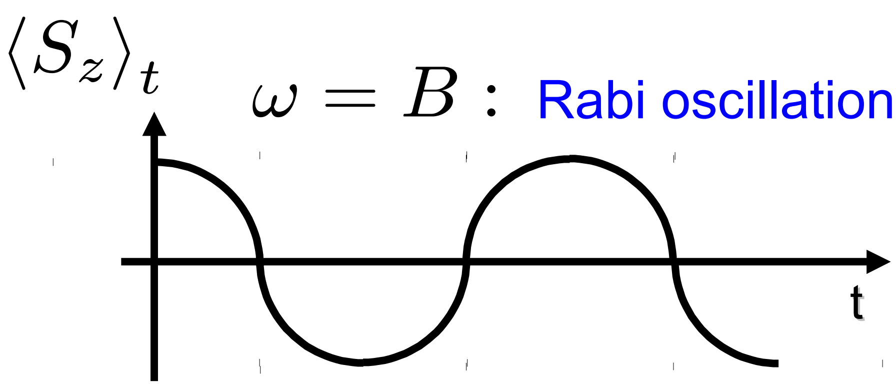
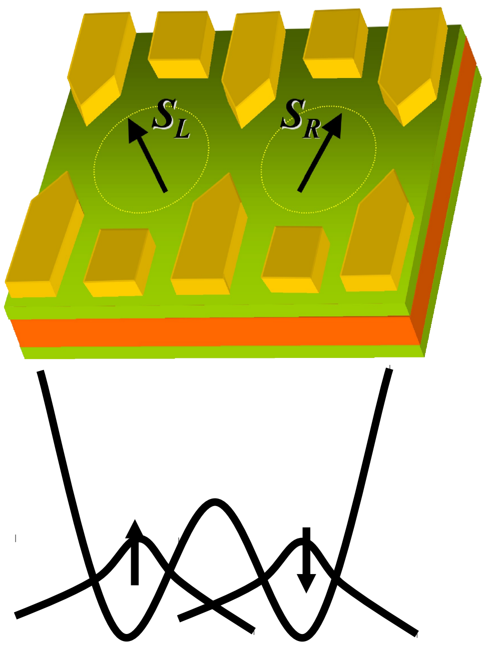
Electron spin resonance





$\omega = B$: Rabi oscillations

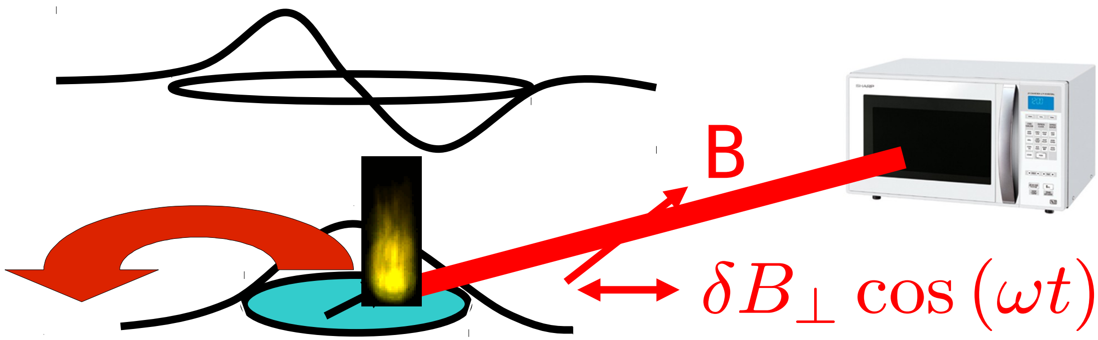
Two-qubit control:
pulsed exchange



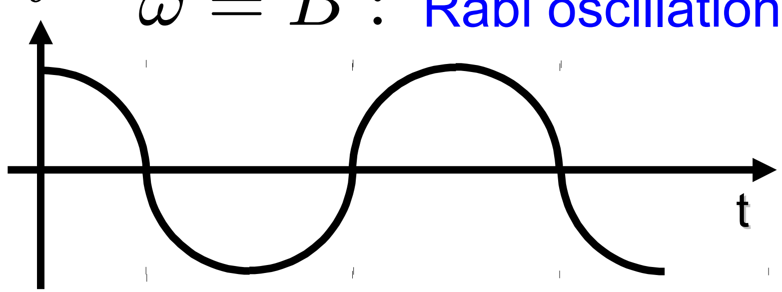
$$H(t) = J(t)\mathbf{S}_L \cdot \mathbf{S}_R$$

Universal control

Single-qubit control:
Electron spin resonance

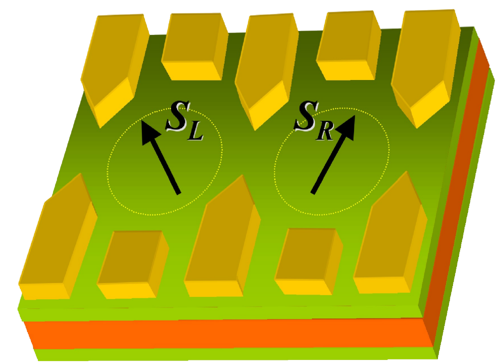


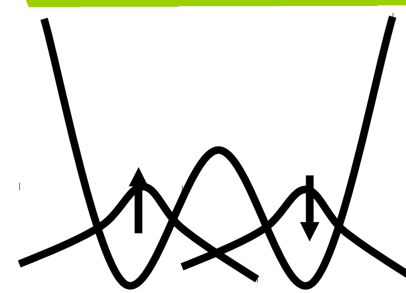
$\langle S_z \rangle_t$ $\quad \omega = B:$ Rabi oscillations



Slow!

Two-qubit control:
pulsed exchange

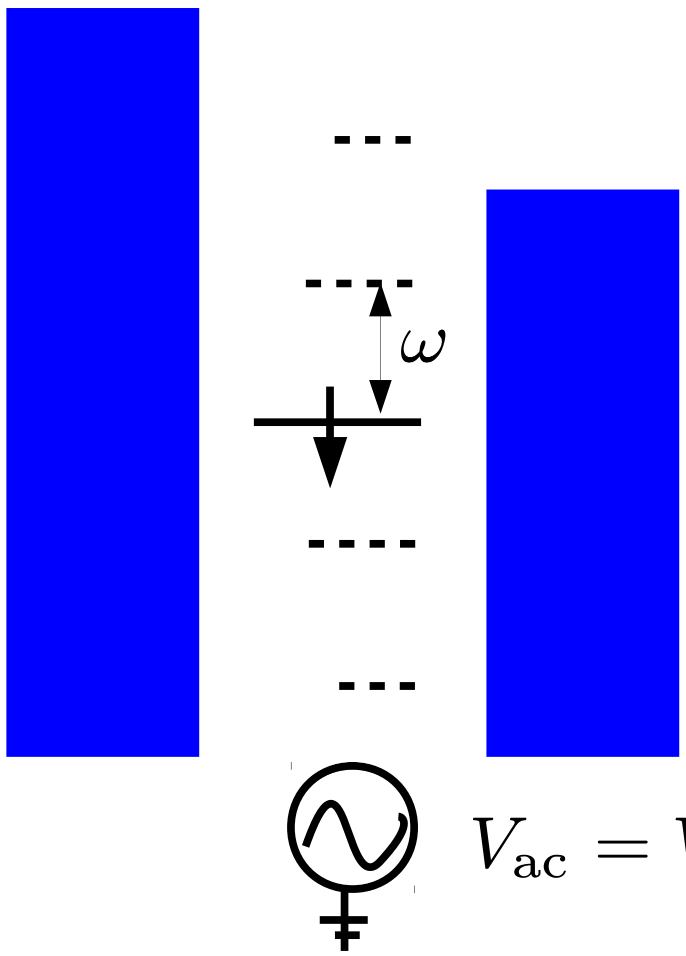




$$H(t) = J(t)\mathbf{S}_L \cdot \mathbf{S}_R$$

Fast!

Problem: Photon-assisted tunneling



$$V_{ac} = V_0 \cos(\omega t)$$

$$\Gamma = 0$$

$$\Gamma(\epsilon) = \sum_n J_n^2\left(\frac{V_0}{\omega}\right)\delta(\epsilon - n\omega)$$

To reduce escape rate:

Small amplitude V_0

Electric-field modulation.

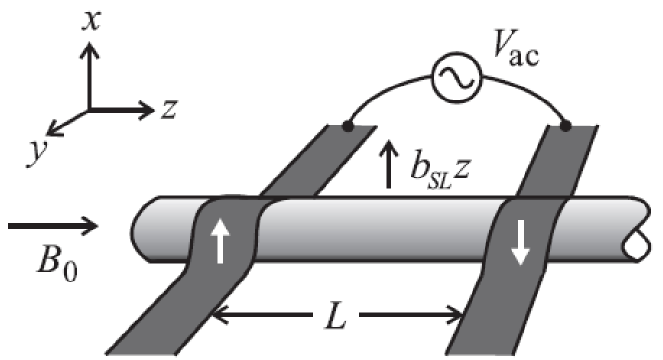
Low frequency ω

Decrease B-field or use pulsed scheme (no ac modulation).

Kouwenhoven et al., PRB (1994)

Universal control:
Electrically controlled Spin Resonance

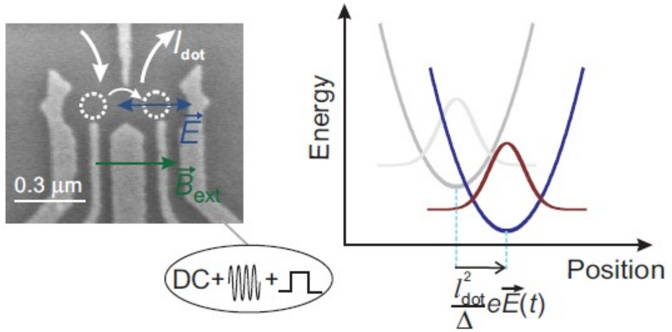
Idea 1)
Move spin periodically using an electric field in presence of slanting Zeeman field; get an ac magnetic field in the rest frame of the electron.



Tokura et al., PRL (2006)

Pioro-Ladrière et al., Nat. Phys. (2008)

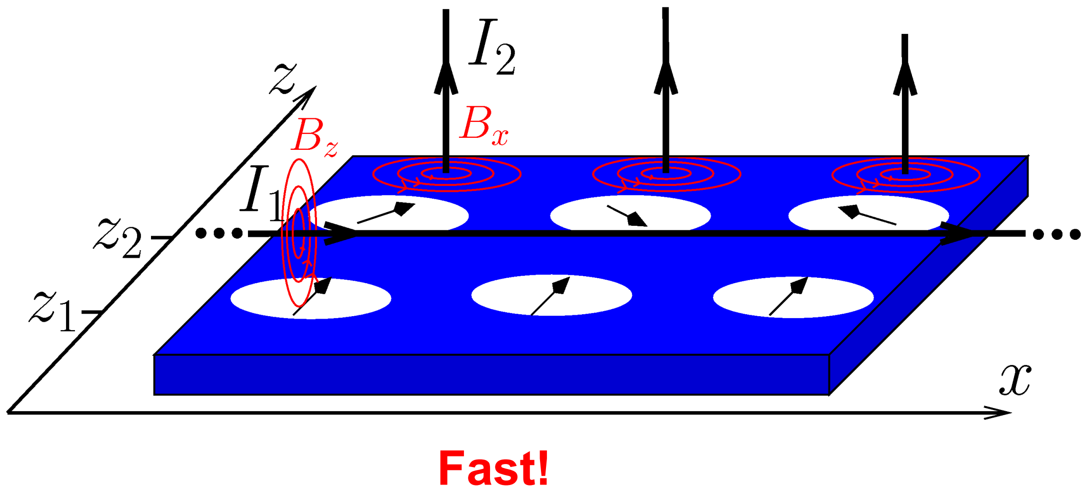
Idea 2)
Move spin periodically in spin-orbit field to generate an effective ac magnetic field:
EDSR = "Electric dipole spin resonance".

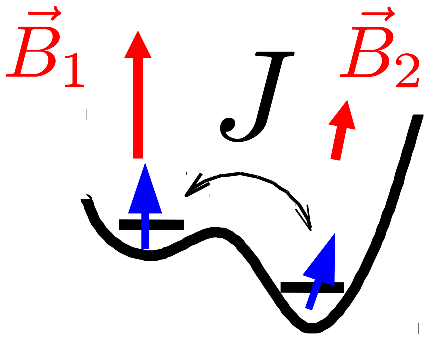


Golovach, Borhani, Loss, PRB (2006)

Nowack et al., Science (2007)

Universal control:
Pulsed exchange (single spins)





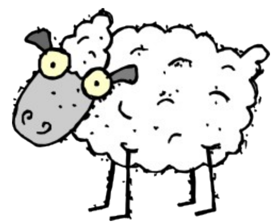
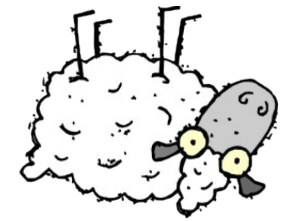
Achievable error rate:

$$\eta \sim 10^{-3}$$

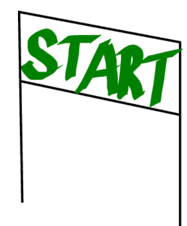
WAC and Loss, PRB (2007)

Requirements for Physical Quantum Computing

- Encoding

 $= |0\rangle$  $= |1\rangle$

- Initialization/readout



- Universal control

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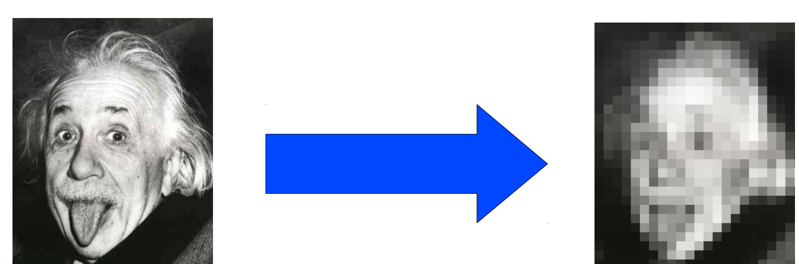
- Coherence



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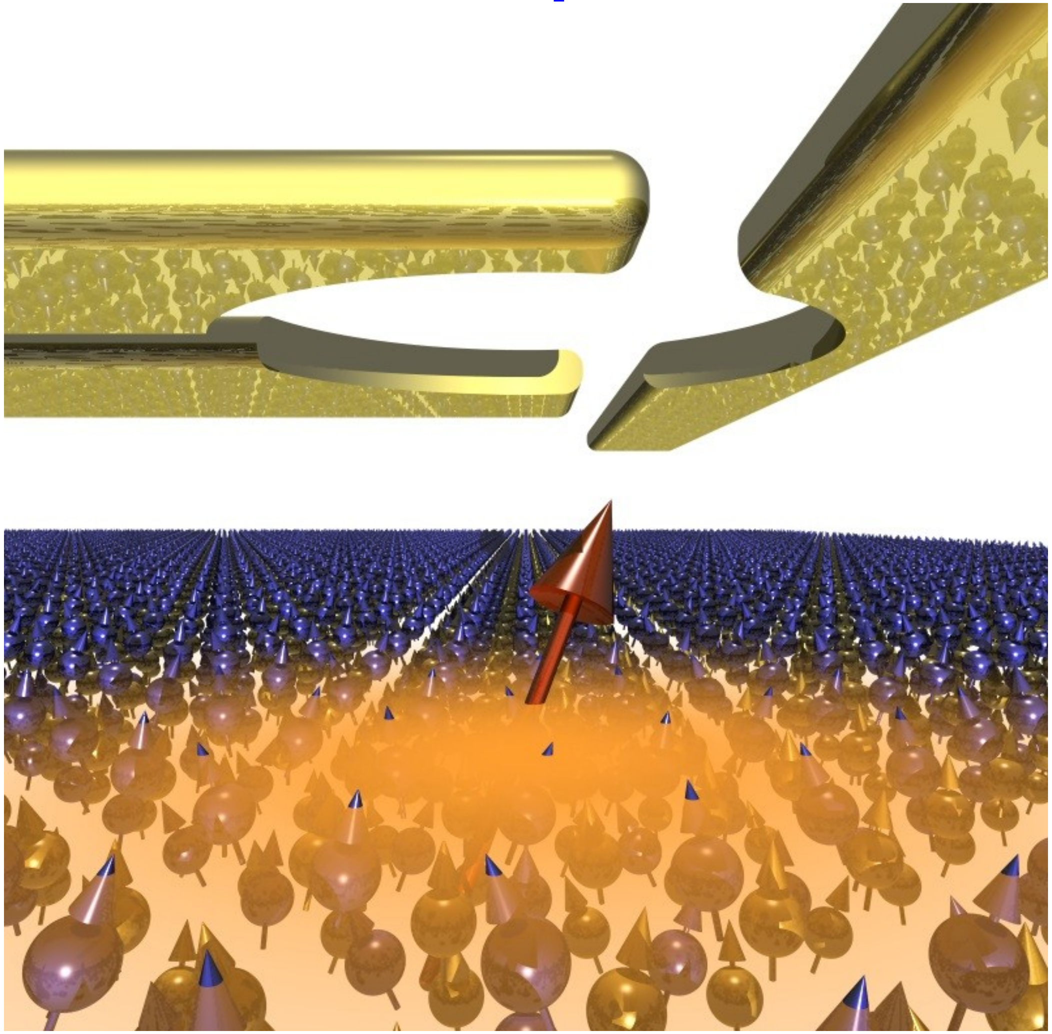
Coherence

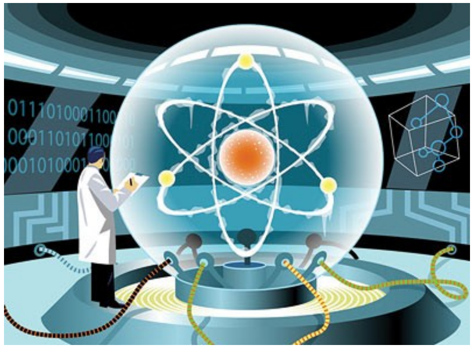
Problem: One spin sees many



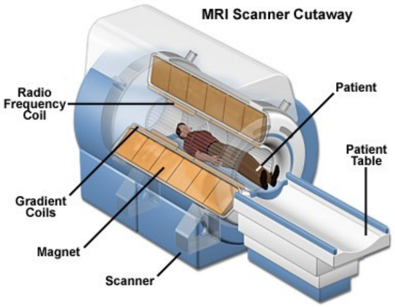
WAC and Baugh, “Nuclear spins in nanostructures” Phys. Stat. Solidi B (2009)

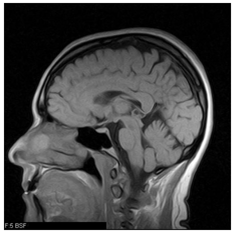
Quantum Coherence: Why do we care?

Computing and information technology

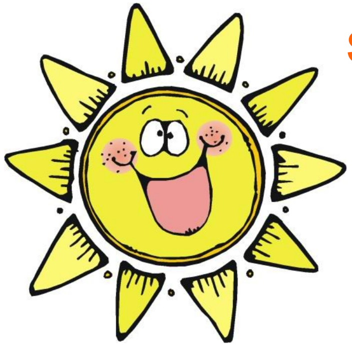


MRI Imaging





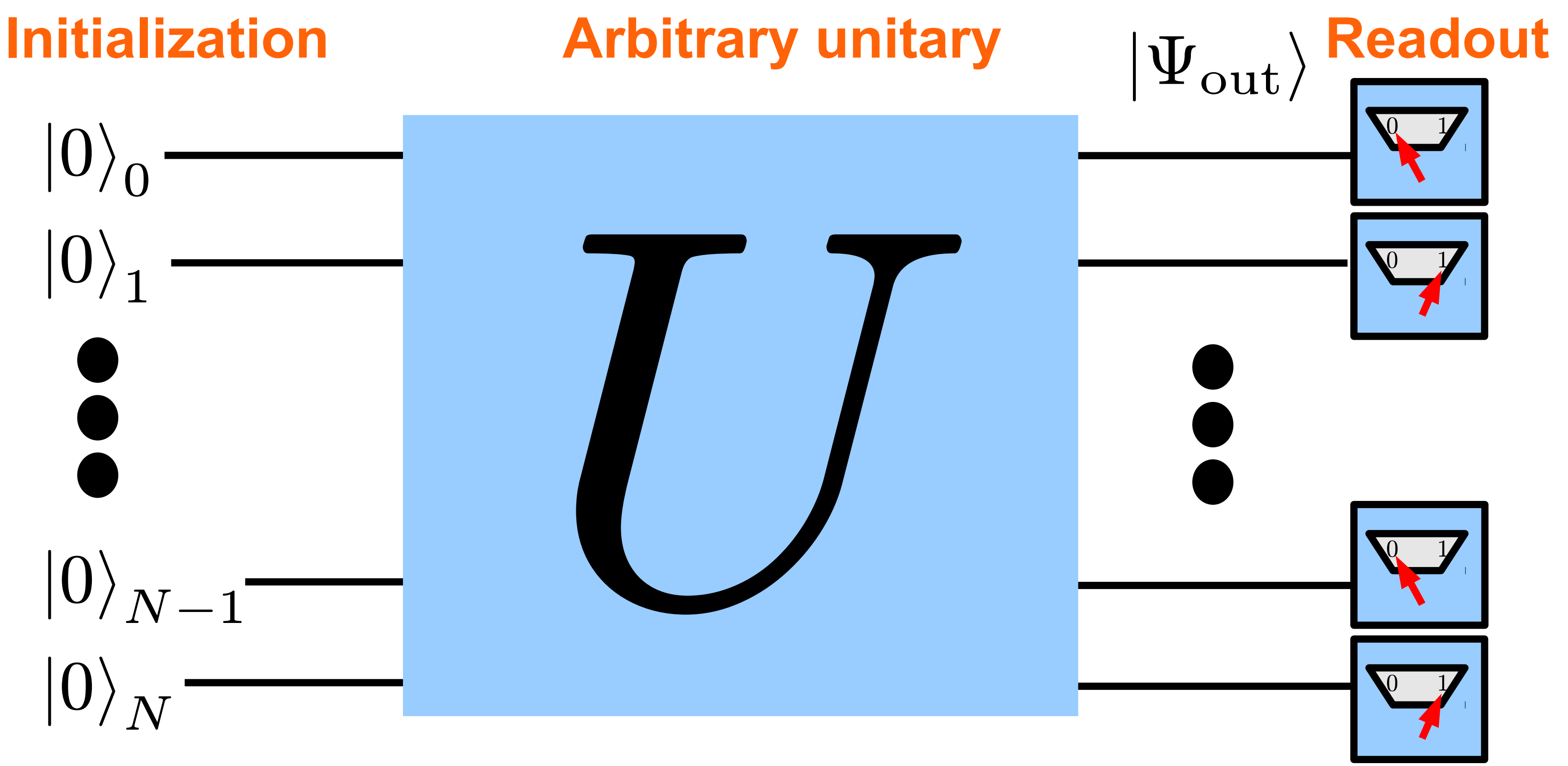
Photosynthesis



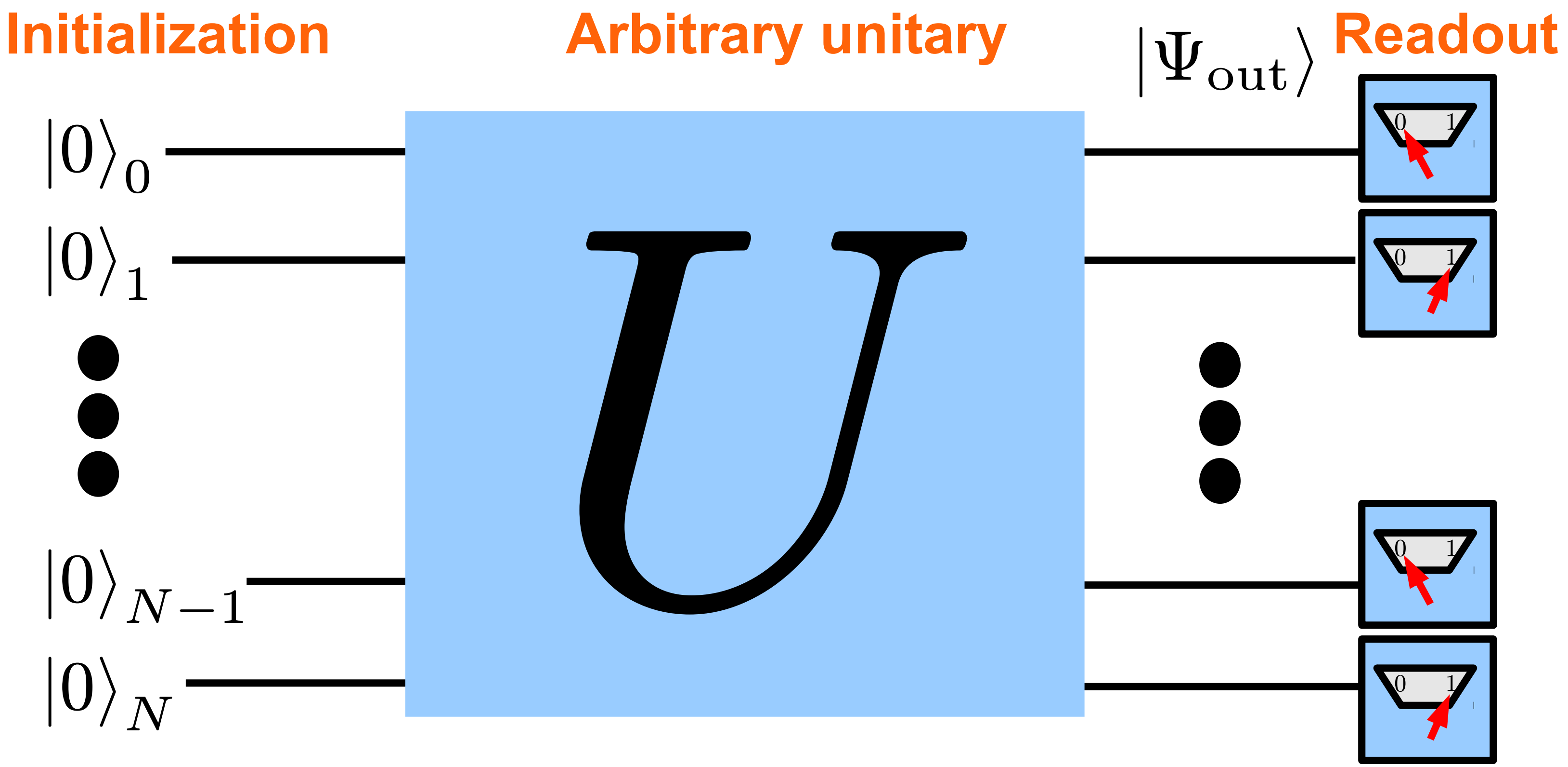
Solar energy

The avian compass

Goal: Quantum Information Processing



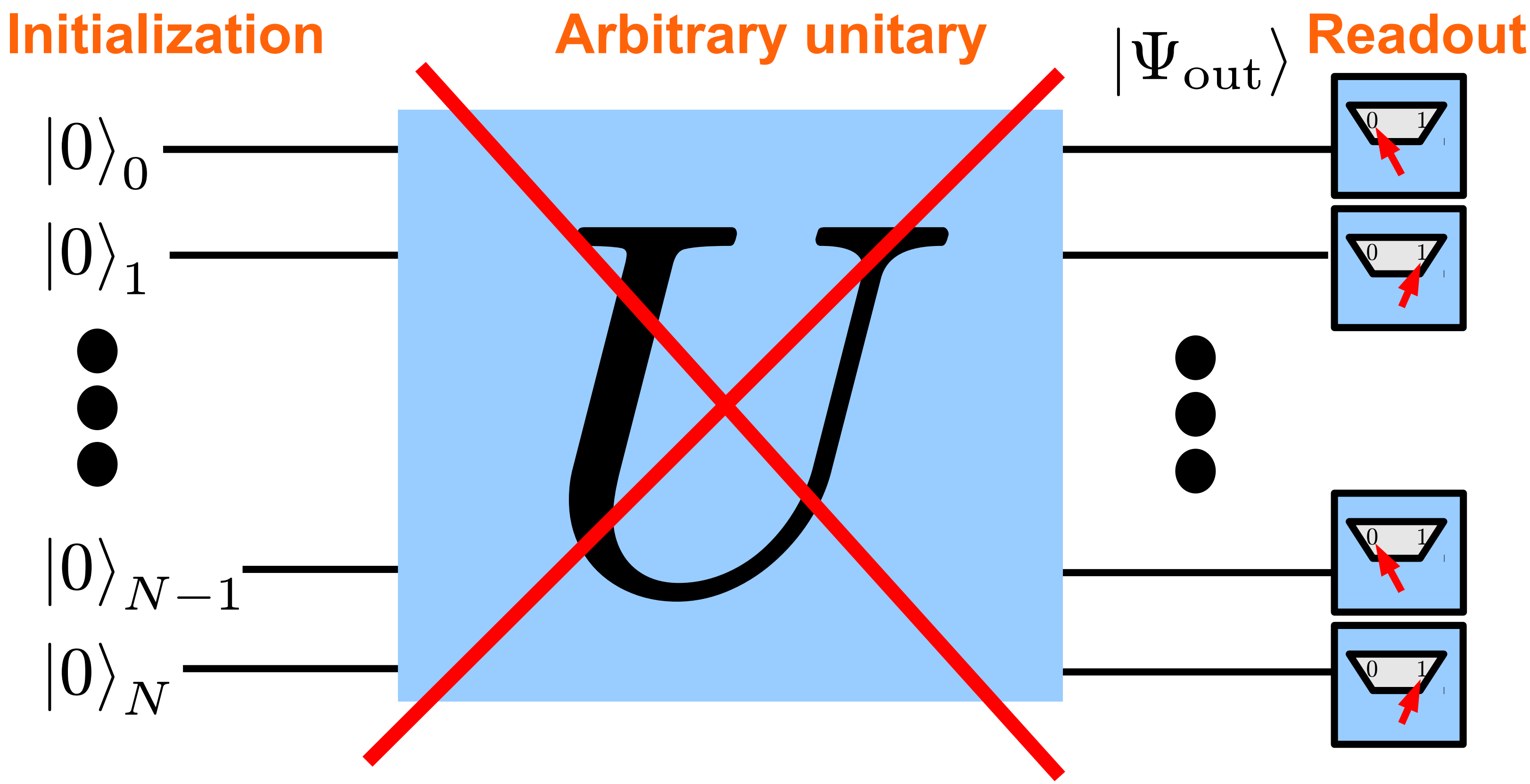
Goal: Quantum Information Processing



Physical Implementation:

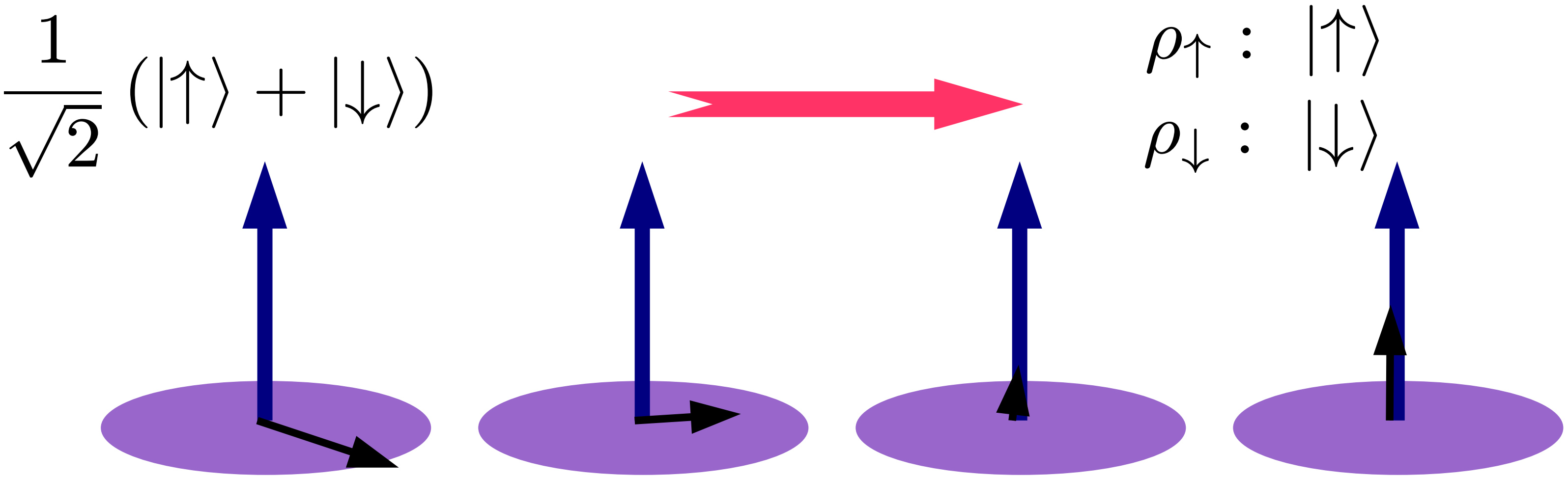
$$U = \mathcal{T} \exp\left\{ -i \int_0^t dt' H(t') \right\} \qquad H \in \mathcal{H}_S$$

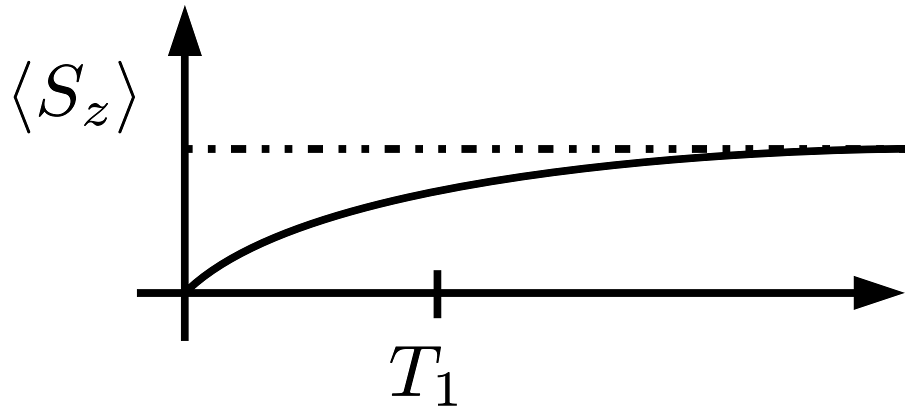
Reality: Imperfections



$$\tilde{U} = \mathcal{T} \exp \left\{ -i \int_0^t dt' \, (H(t') + \delta H(t')) \right\} \qquad \delta H \in \mathcal{H}_S \otimes \mathcal{H}_E$$

Energy Relaxation: “T_1”



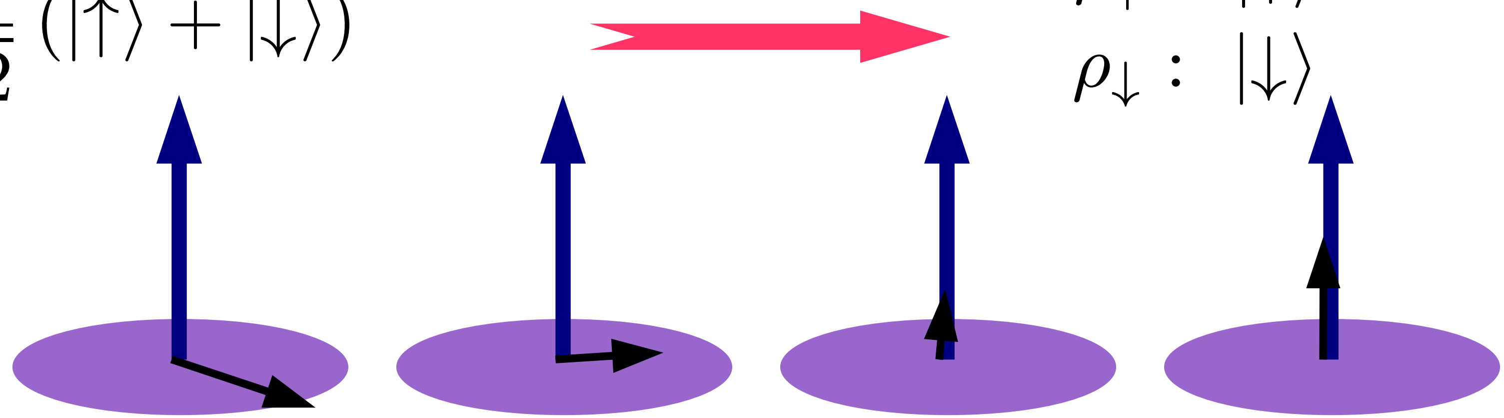


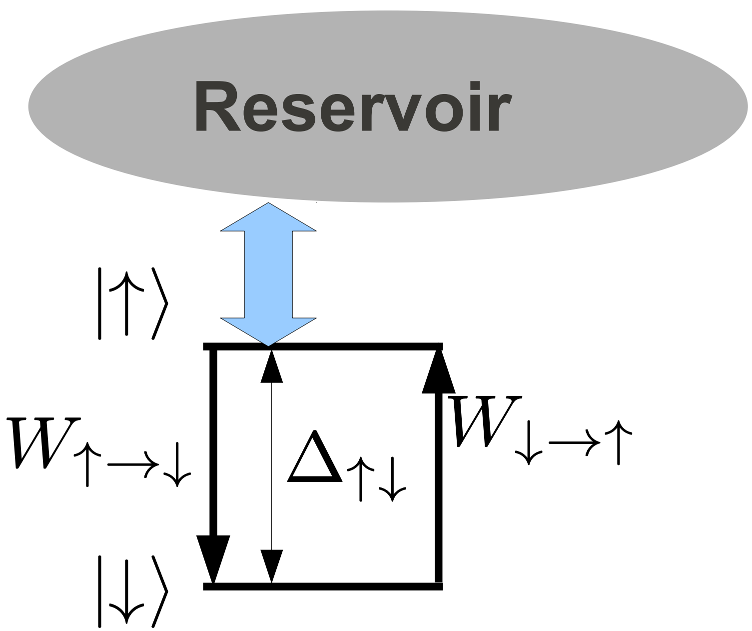
Energy Relaxation: "T_1"

$$\frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$$

$\rho_\uparrow : |\uparrow\rangle$

$\rho_\downarrow : |\downarrow\rangle$





Reservoir **in equilibrium:**

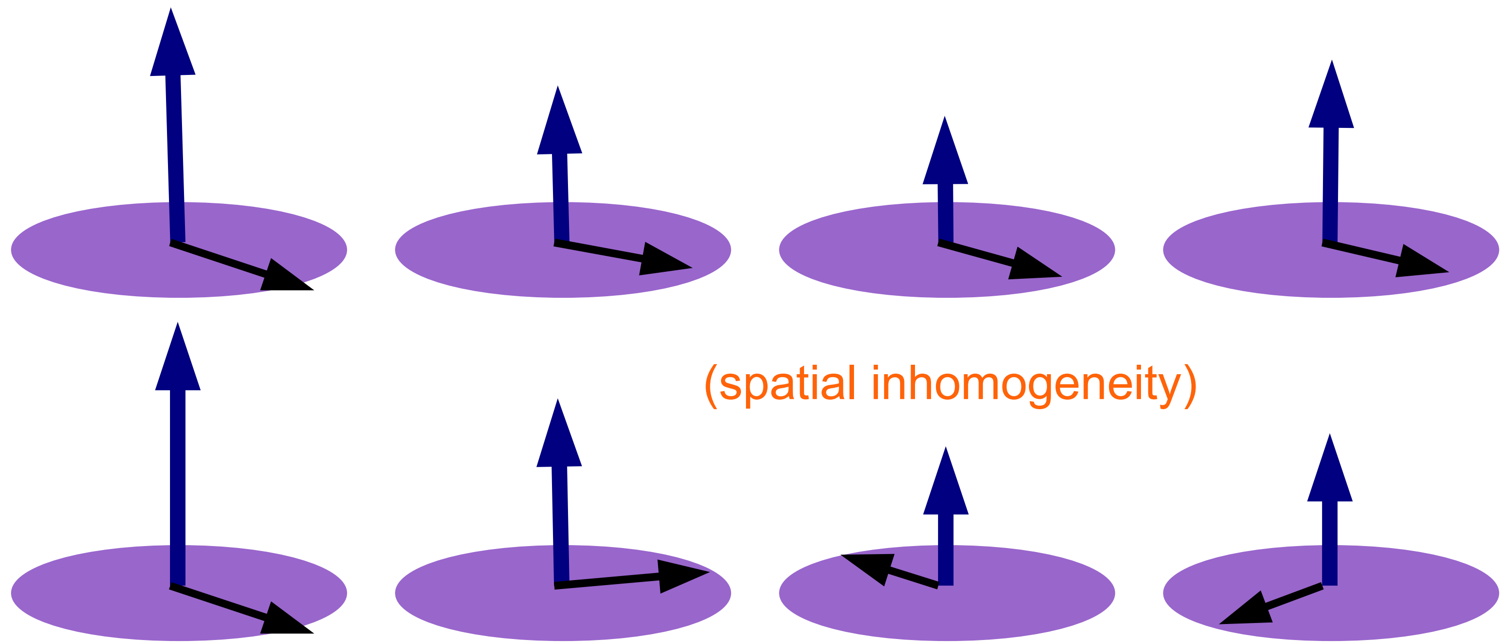
$$\frac{\rho_\uparrow}{\rho_\downarrow} = \frac{W_{\downarrow\to\uparrow}}{W_{\uparrow\to\downarrow}} = e^{-\Delta_{\uparrow\downarrow}/k_B T}$$

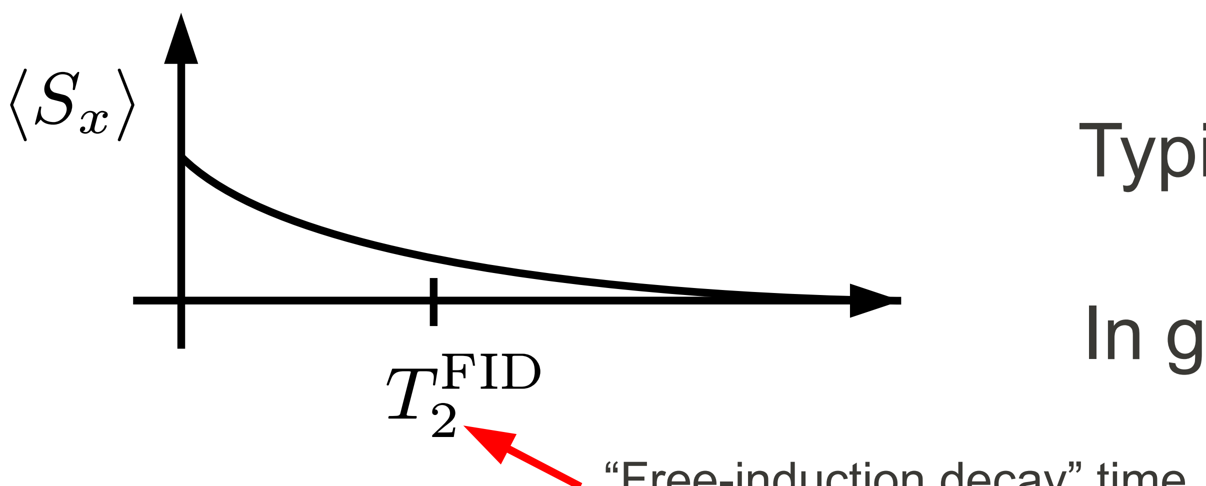
(Detailed Balance)

Dephasing ('decoherence'): "T_2"

$$\rho = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\rho' = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

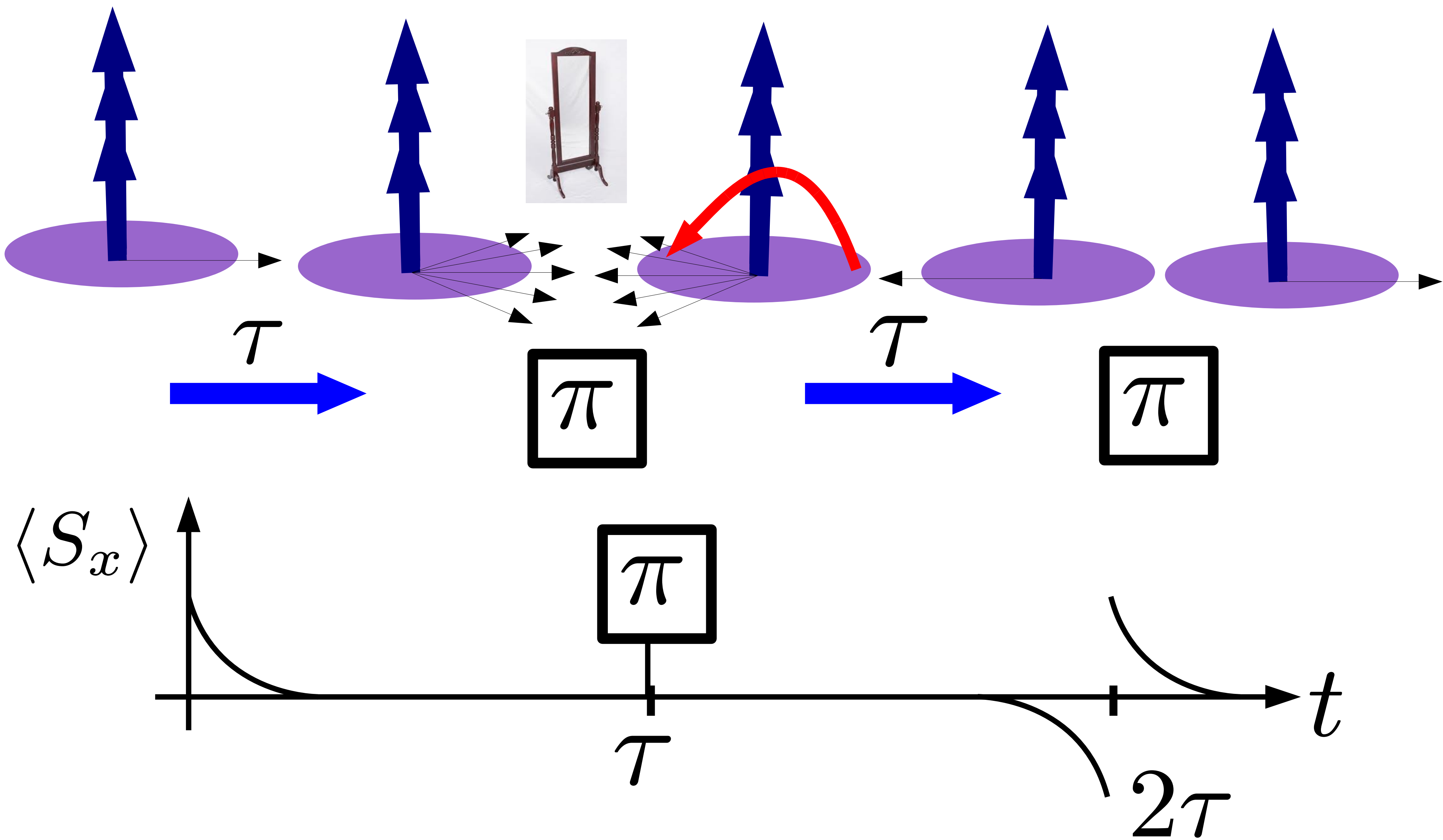




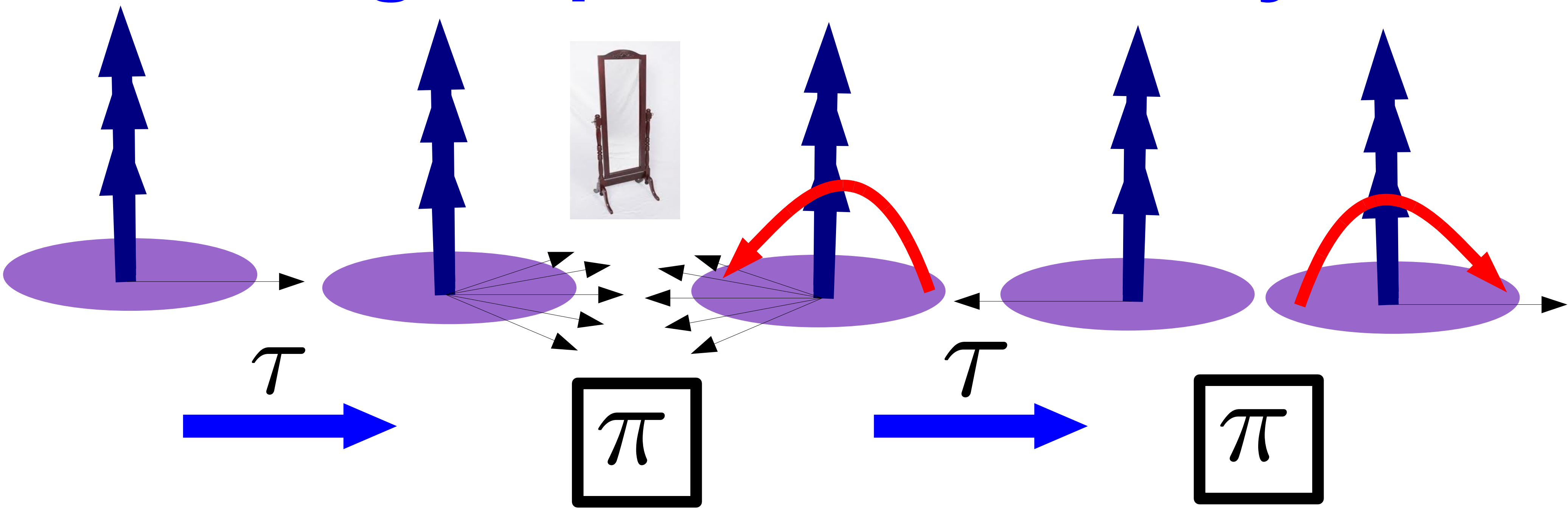
Typically: $T_2 \ll T_1$

In general: $T_2 \leq 2T_1$

Spin echo (static inhomogeneity):



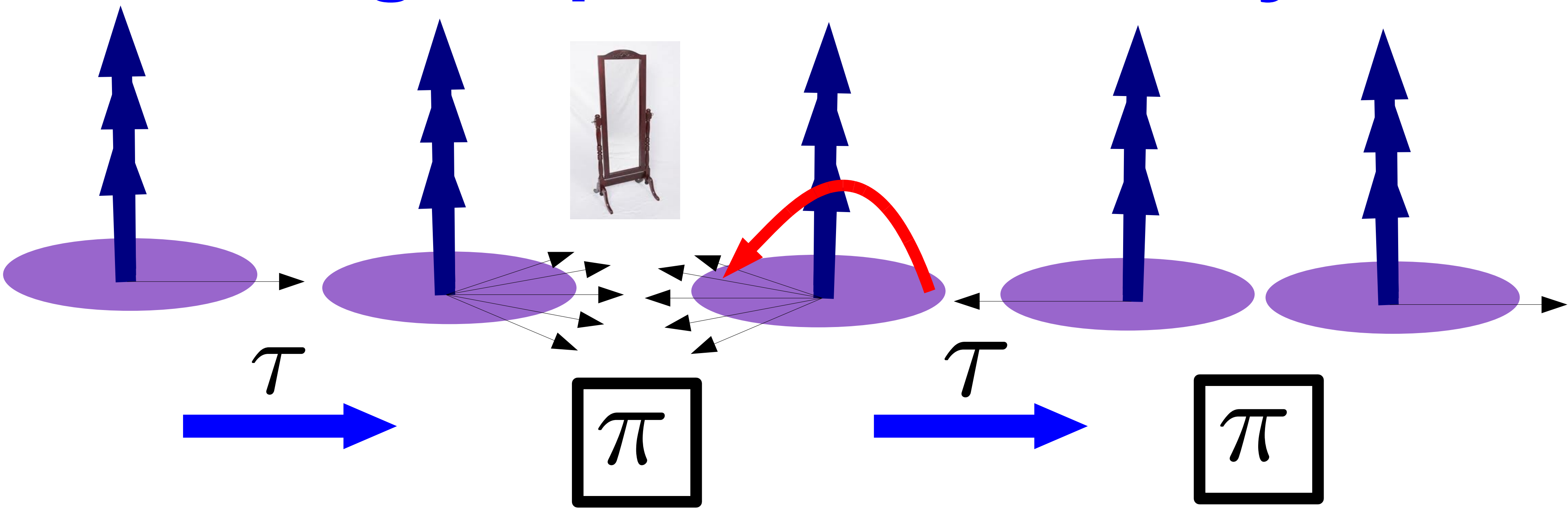
Semiclassical "belief": Single spin will not decay



But for a quantum environment, can still get decay:

$$|\psi_E\rangle = \sum_k c_k |\omega_k\rangle \qquad \langle S_x \rangle_t = \frac{1}{2}\sum_k |c_k|^2 \cos(\omega_k t) \simeq \frac{1}{2} e^{-t/\tau_{\text{dec.}}}$$

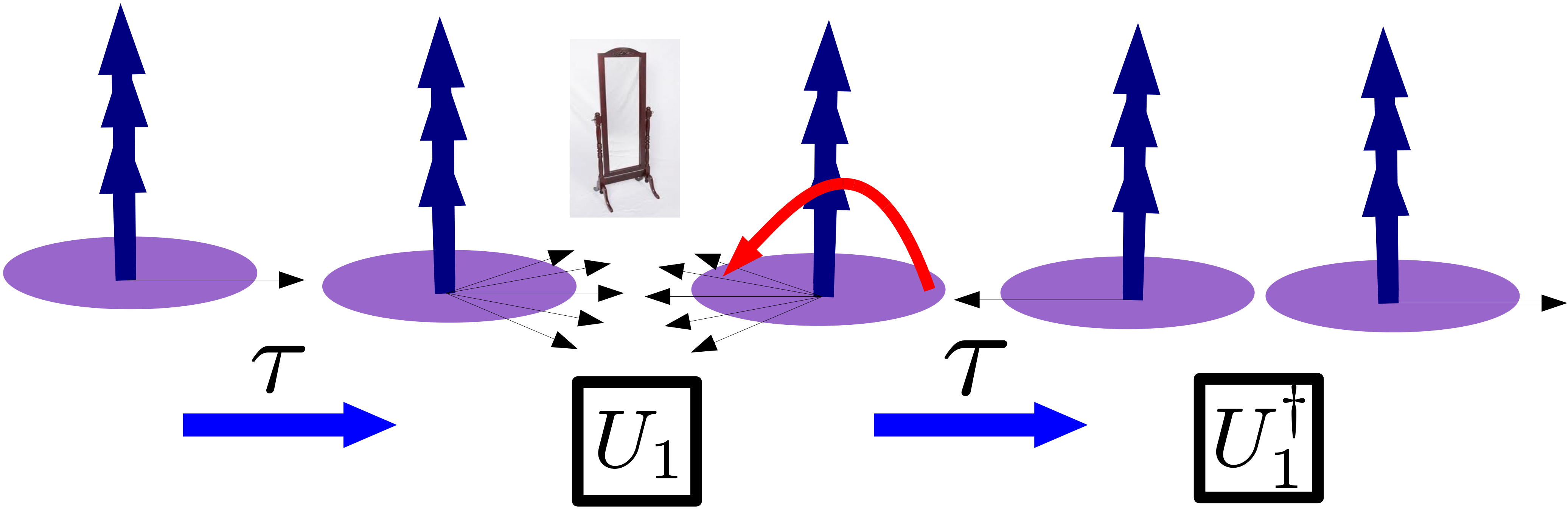
Semiclassical "belief": Single spin will not decay



Still reversible: $H = \hat{\omega} S_z; \quad \sigma_x H \sigma_x = -\hat{\omega} S_z$

$$|\psi(2t)\rangle = \sigma_x e^{-iHt} \sigma_x e^{-iHt} |\psi(0)\rangle = U^\dagger(t) U(t) |\psi(0)\rangle = |\psi(0)\rangle$$

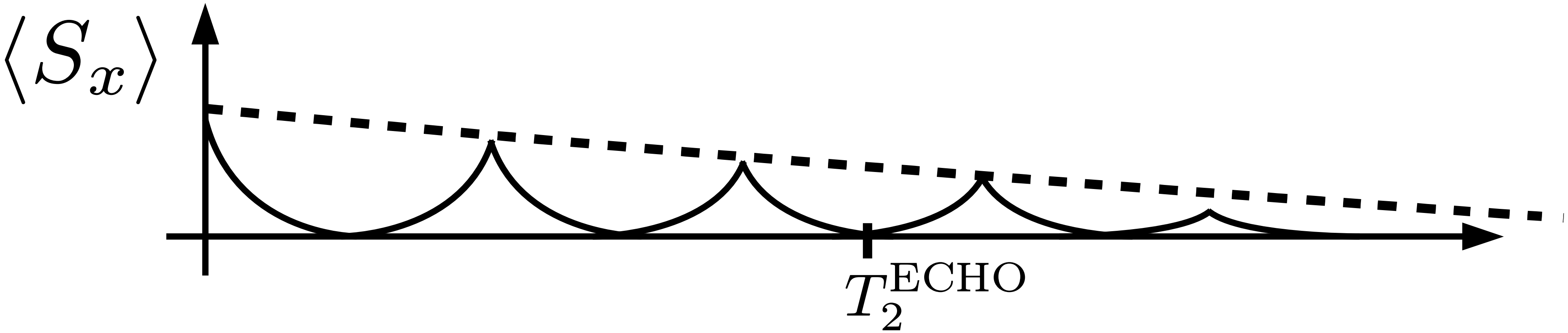
General principle: Time reversal



Ideally choose U_1 to be the time-reversal operation:

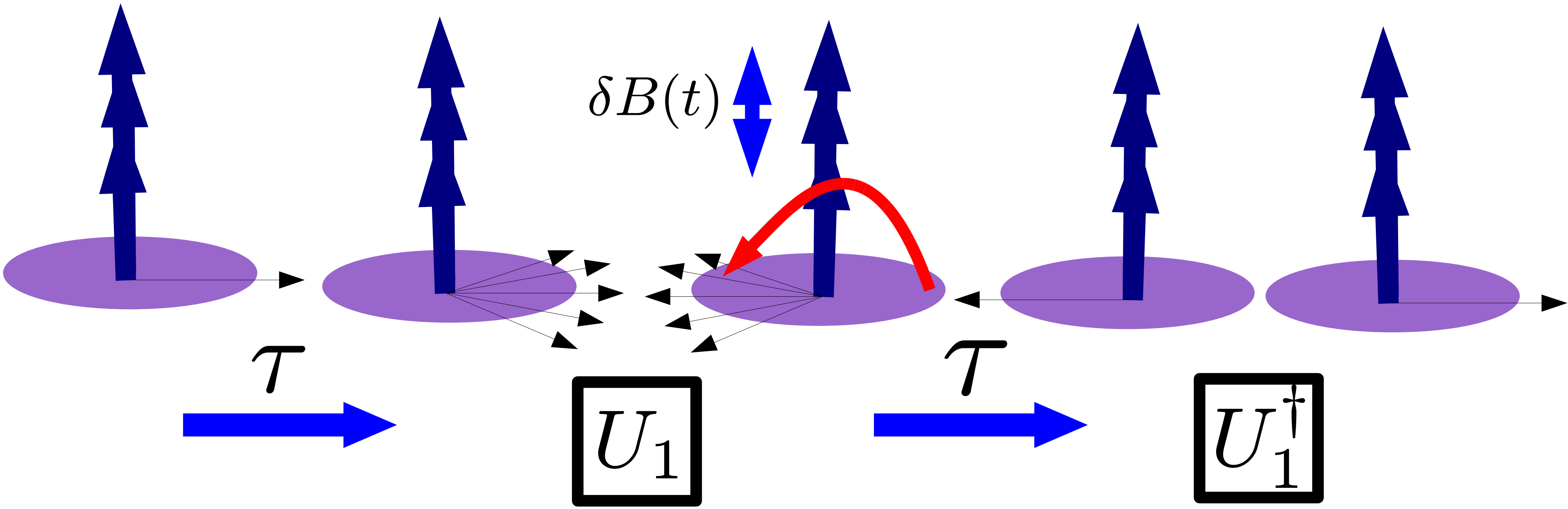
$$U_1^\dagger H U_1 = -H \qquad \Rightarrow |\psi(2t)\rangle = U(-t)U(t)\,|\psi(0)\rangle$$

Spin-echo envelope decay



Message: T_2^{ECHO} is not the 'intrinsic' single spin decay time.
The decay time is different for every pulse sequence!

Dynamic fluctuations; repeated pulses (dynamical decoupling)



$$|\psi(2t)\rangle = U_{2N}(-t/N)U_{2N-1}(t/N)\cdots U_2(-t/N)U_1(t/N)\,|\psi(0)\rangle$$

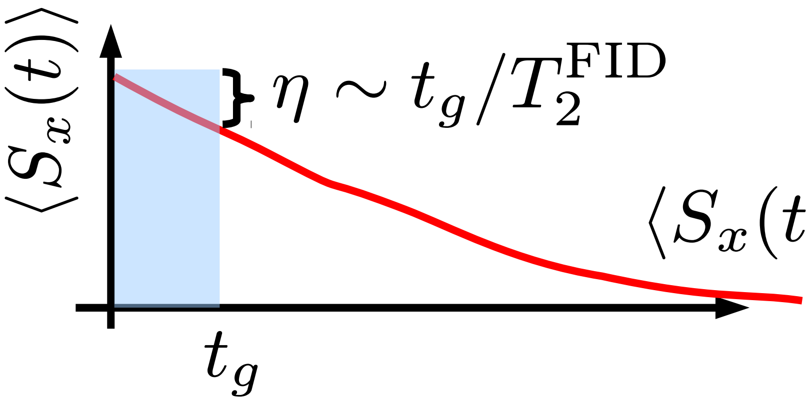
"Carr-Purcell-Meiboom-Gill" (CPMG) sequence

(with t/N < correlation time of B(t))

Types of error

In addition to initialization/readout error

Gate error (free-induction decay)



$$\langle S_x(t) \rangle \propto e^{-t/T_2^{\mathrm{FID}}}$$

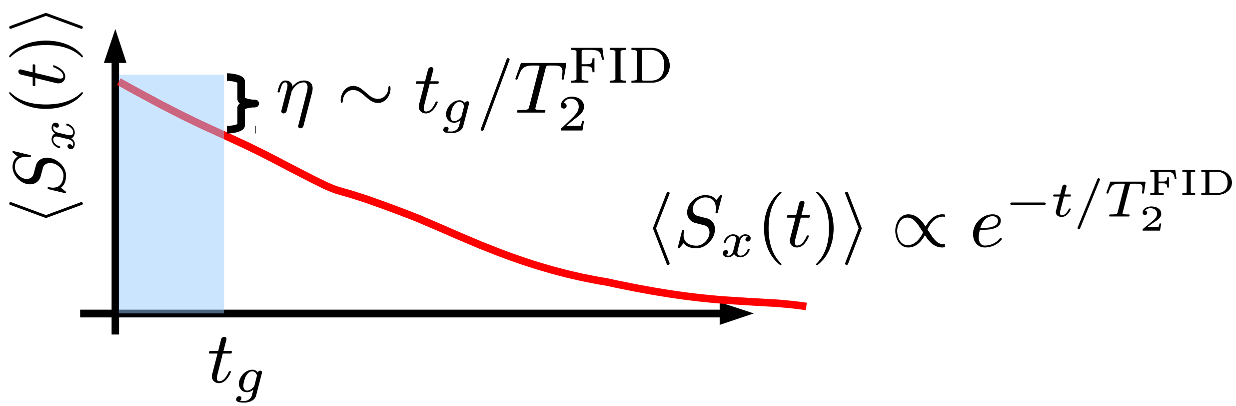
Error-correction threshold

$$\eta < \eta_c \sim 10^{-6} - 10^{-2}$$

Types of error

In addition to initialization/readout error

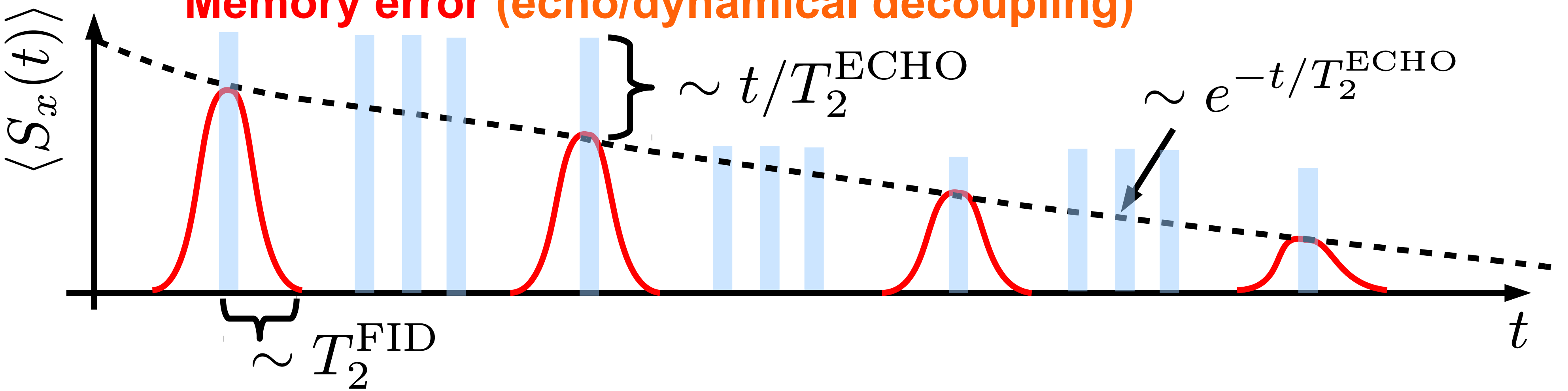
Gate error (free-induction decay)



Error-correction threshold

$\eta < \eta_c \sim 10^{-6} - 10^{-2}$

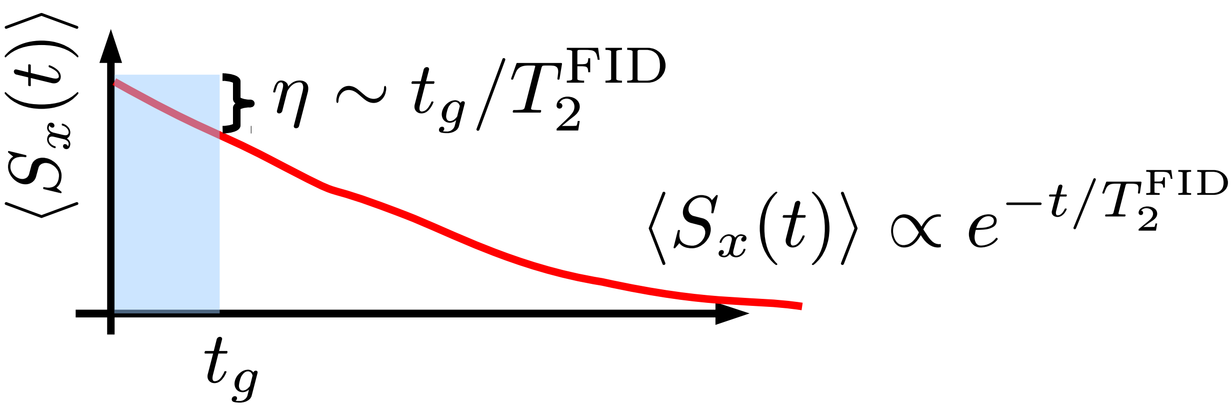
Memory error (echo/dynamical decoupling)



Types of error

In addition to initialization/readout error

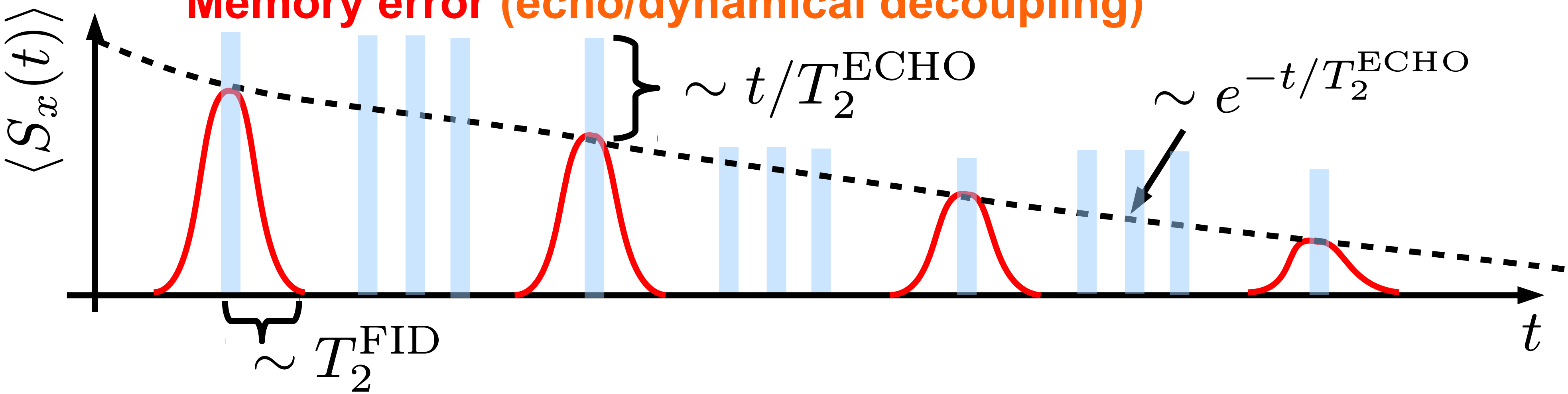
Gate error (free-induction decay)



Error-correction threshold

$$\eta < \eta_c \sim 10^{-6} - 10^{-2}$$

Memory error (echo/dynamical decoupling)

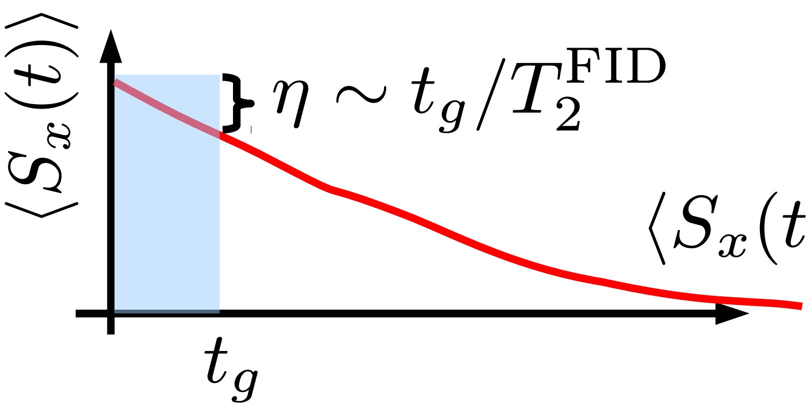


Even for single spin: $T_2^{\mathrm{ECHO}} \neq T_2^{\mathrm{FID}}$ **'Intrinsic' decay time is a myth!**

Types of error

In addition to initialization/readout error

Gate error (free-induction decay)



$$\eta \sim t_g / T_2^{\mathrm{FID}}$$

$$\langle S_x(t) \rangle \propto e^{-t/T_2^{\mathrm{FID}}}$$

Error-correction threshold

$$\eta < \eta_c \sim 10^{-6} - 10^{-2}$$

Focus on reducing gate error (increasing FID time): $T_2^{\mathrm{FID}} = T_2$

Caveat: Gating and decay not always independent (should really determine the gate fidelity).

e.g.:

$$F = \mathrm{Tr}\left\{ U^\dagger \tilde{U} \right\}$$

Some recent developments in dynamical decoupling

"UDD" (concatenated decoupling)

PRL 98, 100504 (2007) PHYSICAL REVIEW LETTERS week ending 9 MARCH 2007

Keeping a Quantum Bit Alive by Optimized π-Pulse Sequences

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(Received 26 September 2006; published 9 March 2007)

Science AAAS

Universal Dynamical Decoupling of a Single Solid-State Spin from a Spin Bath
G. de Lange, *et al.*
Science **330**, 60 (2010);
DOI: 10.1126/science.1192739

nature Vol 458|2

LETTERS

Optimized dynamical decoupling in a model quantum memory

Michael J. Biercuk[1,2]*, Hermann Uys[1,3]*, Aaron P. VanDevender[1], Nobuyasu Shiga[1]†, Wayne M. Itano[1] & John J. Bollinger[1]

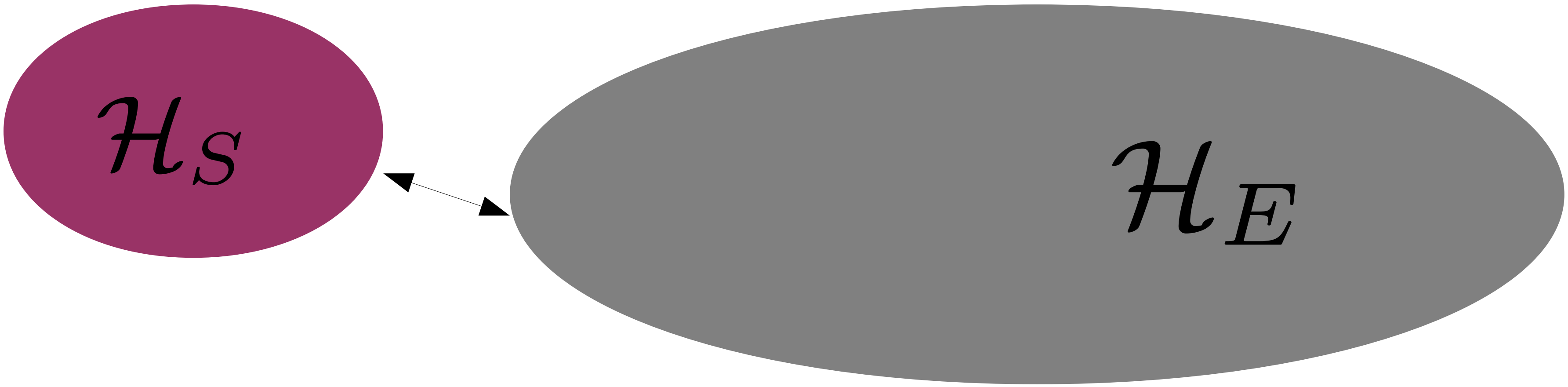
nature

LETTERS

Preserving electron spin coherence in solids by optimal dynamical decoupling

Jiangfeng Du[1], Xing Rong[1], Nan Zhao[2], Ya Wang[1], Jiahui Yang[1] & R. B. Liu[2]

Quantum Dynamics



Quantum Engineering (ideal):

$$|\psi(t)\rangle = U(t)\,|\psi(0)\rangle \qquad U(t) = \mathcal{T}e^{-i\int_0^t H(t')dt'} \qquad H \in \mathcal{H}_S$$

The Reality: $|\psi(0)\rangle \to \rho(0)$

$$U'(t) = \mathcal{T}e^{-i\int_0^t \left(H(t')+\delta H(t')\right)dt'} \qquad \delta H \in \mathcal{H}_S \otimes \mathcal{H}_E$$

What can go wrong?

(1) $\delta H(t)$ unknown.

➡ "Model" environment and coupling
(e.g., spin-boson, other phenomenological model)?

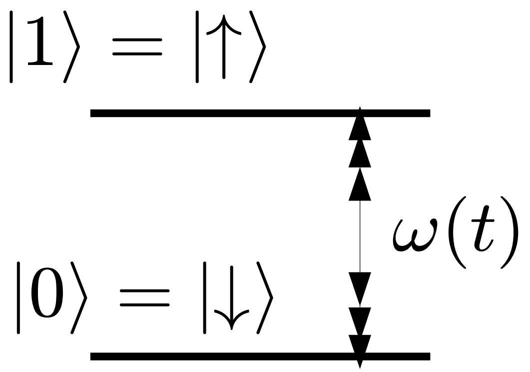
(2) Environment state unknown.

➡ Assume a thermal equilibrium state?

(3) $U'(t)$ too complicated.

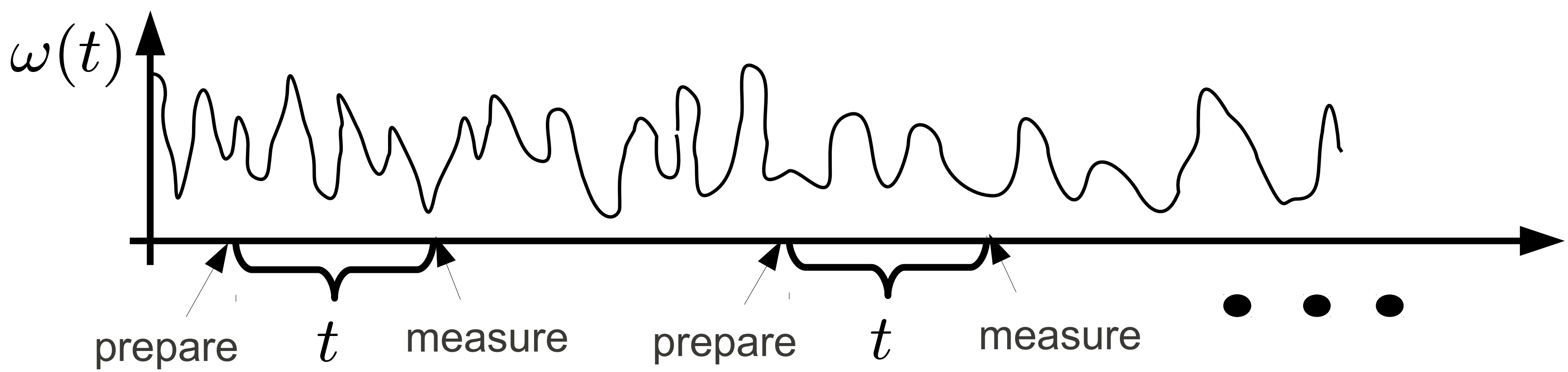
➡ Weak-coupling expansion?

Dephasing “decoherence”: Classical noise (phenomenological model)



$$H(t) = \omega(t)\sigma_z/2 \qquad \dot{\rho} = -i\left[H(t), \rho\right]$$

$$\langle \sigma_+(t) \rangle = e^{i\phi(t)} \langle \sigma_+(0) \rangle \qquad \phi(t) = \int_0^t dt' \omega(t')$$



Average over noise realizations:

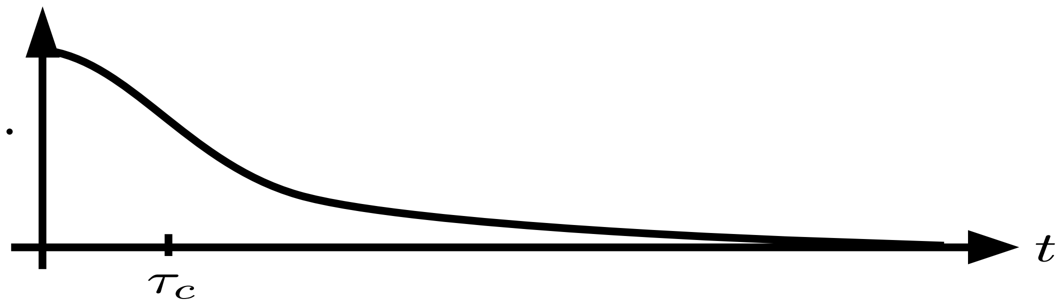
$$\langle \sigma_+(t) \rangle_{\text{av.}} = \left\langle e^{i\phi(t)} \right\rangle_{\text{av.}} \langle \sigma_+(0) \rangle$$

Dephasing "decoherence": Classical noise (phenomenological model)

$$\langle \sigma_+(t) \rangle_{\text{av.}} = \left\langle e^{i\phi(t)} \right\rangle_{\text{av.}} \langle \sigma_+(0) \rangle = e^{-\frac{1}{2}\left\langle \phi^2(t) \right\rangle_{\text{av.}}} \langle \sigma_+(0) \rangle$$

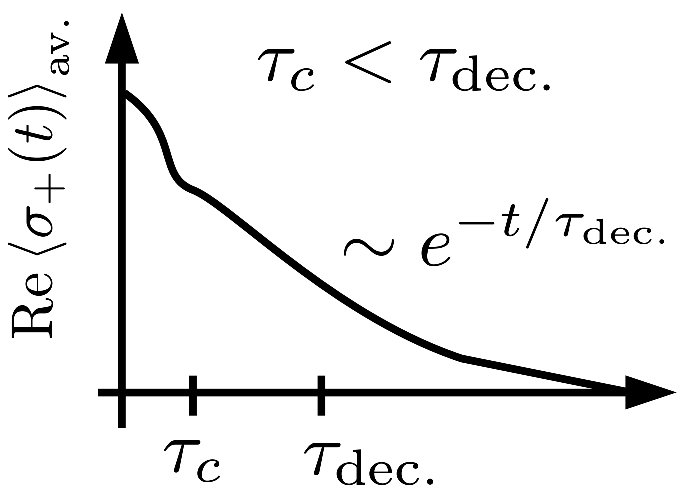
$$\left\langle \phi^2(t) \right\rangle_{\text{av.}} = \int_0^t dt'(t-t') \left\langle \delta\omega(t')\delta\omega(0) \right\rangle_{\text{av.}}$$ (Gaussian, stationary)

$\langle \delta\omega(t)\delta\omega(0) \rangle_{\text{av.}}$



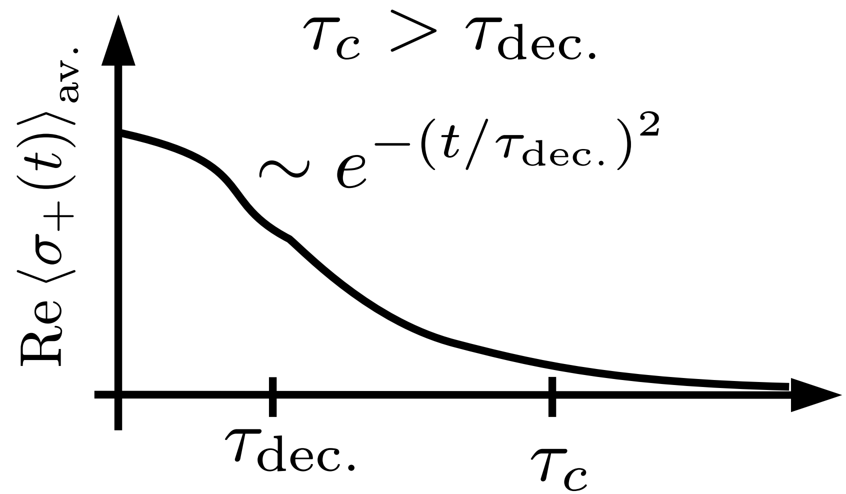
"Markovian limit"

$\tau_c < \tau_{\text{dec.}}$



"Non-Markovian limit"

$\tau_c > \tau_{\text{dec.}}$



A better approach?

(1) $\delta H(t)$ unknown.

➡ Figure it out!

(2) Environment state unknown.

➡ Measure it! (For a static environment)

(3) $U'(t)$ too complicated.

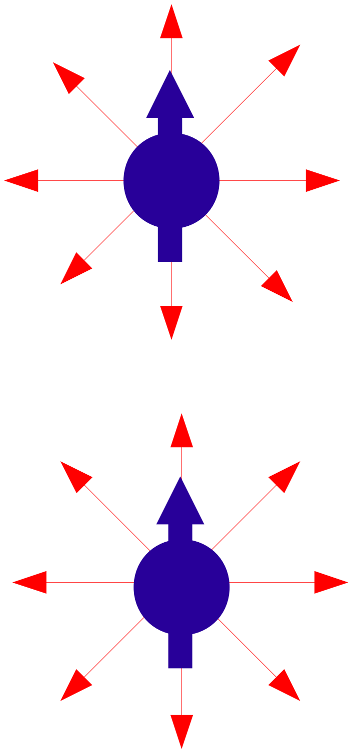
➡ Systematic expansion, not always weak-coupling.

A better approach?

(1) $\delta H(t)$ unknown.

→ Figure it out!

Charge vs. (Electron) Spin



$$E_{\text{Coul.}} = \frac{e^2}{4\pi\epsilon_0 a_B}$$

$$E^{\text{el.}}_{\text{Mag.}} = \frac{\mu_0}{4\pi}\frac{\mu_B^2}{a_B^3}$$

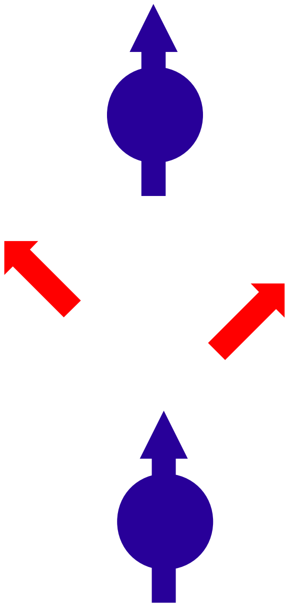
$$a_B = \frac{\hbar}{m_e c\alpha}$$

$$\mu_B = \frac{e\hbar}{2m_e}$$

$$c^2 = \frac{1}{\epsilon_0\mu_0}$$

$$\frac{E_{\text{Mag.}}}{E_{\text{Coul.}}} = \frac{\alpha^2}{4} \ll 1 \qquad \alpha \simeq \frac{1}{137}$$

Electron vs. Nuclear Spin



$$E^{\text{el.}}_{\text{Mag.}} = \frac{\mu_0}{4\pi}\frac{\mu_B^2}{a_B^3}$$

$$\mu_B = \frac{e\hbar}{2m_e}$$

$$E^{\text{nuc.}}_{\text{Mag.}} = \frac{\mu_0}{4\pi}\frac{\mu_B\mu_N}{a_B^3}$$

$$\mu_N = \frac{e\hbar}{2m_p}$$

$$\frac{E^{\text{nuc.}}_{\text{Mag.}}}{E^{\text{el.}}_{\text{Mag.}}} = \frac{m_e}{m_p} \sim 10^{-3}$$

Hierarchy of time scales

$$E_{\mathrm{Coul.}} \gg E^{\mathrm{el.}}_{\mathrm{Mag.}} \gg E^{\mathrm{nuc.}}_{\mathrm{Mag.}}$$

Typically,

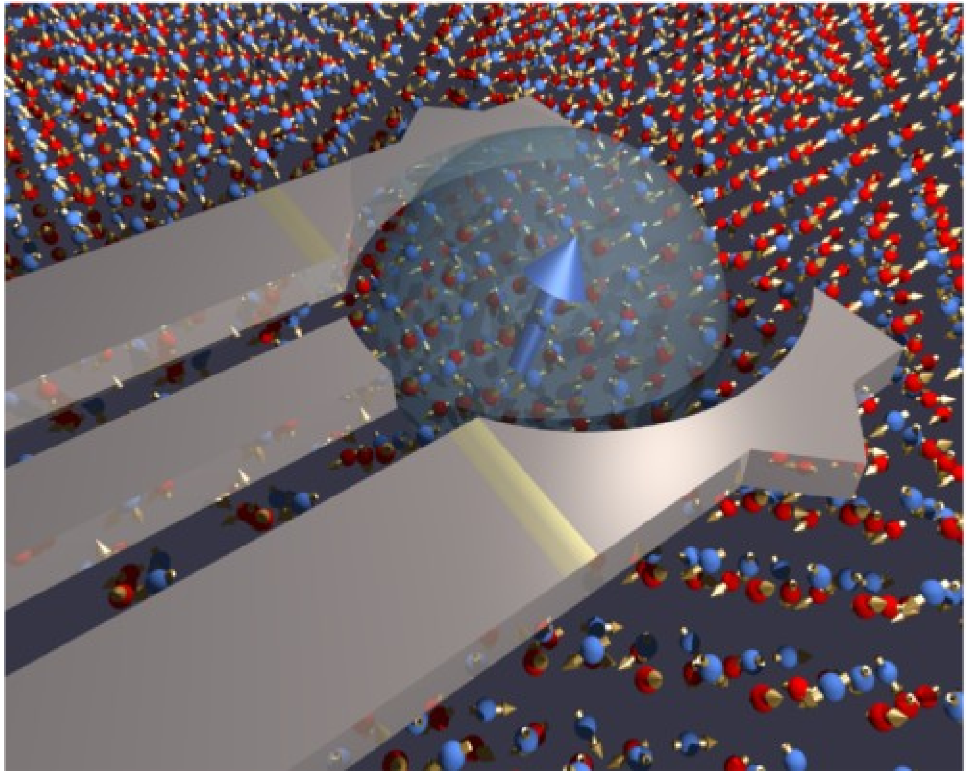
Long-lived coherence!

Longer-lived environment.

$$\tau_\phi^{\mathrm{charge}} \ll \tau_\phi^{\mathrm{el.\,spin}} \ll \tau_\phi^{\mathrm{nuc.\,spin}}$$

$\tau_c \sim \tau_\phi^{\mathrm{nuc.\,spin}} \ll \tau_\phi^{\mathrm{el.\,spin}}$ Non-exponential decay (typically)

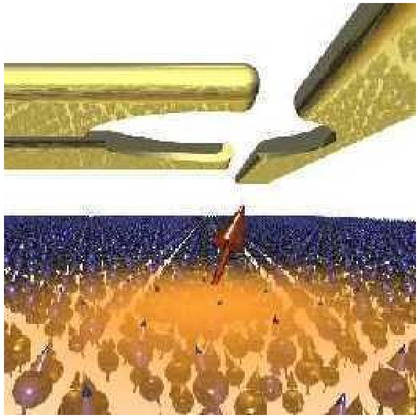
Good platform for quantum coherence?



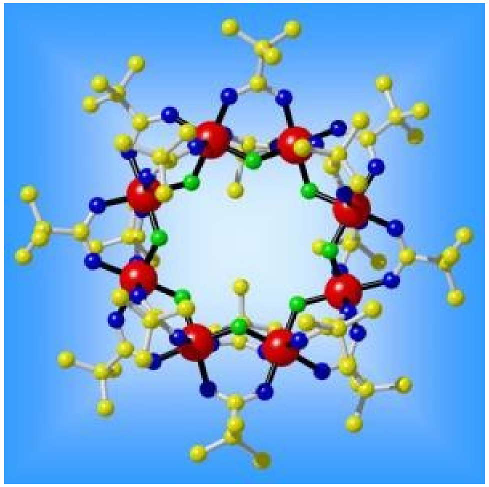
Nuclear spins are (almost) everywhere...

NV centers in diamond

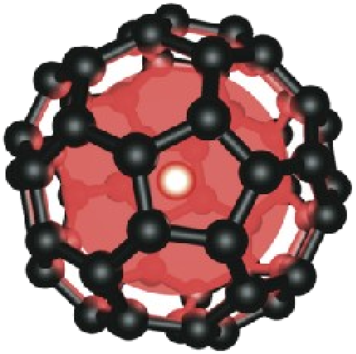
Quantum dots



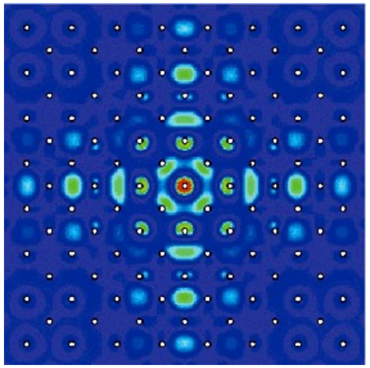
Molecular Magnets



N@C_{60}



Phosphorus donors



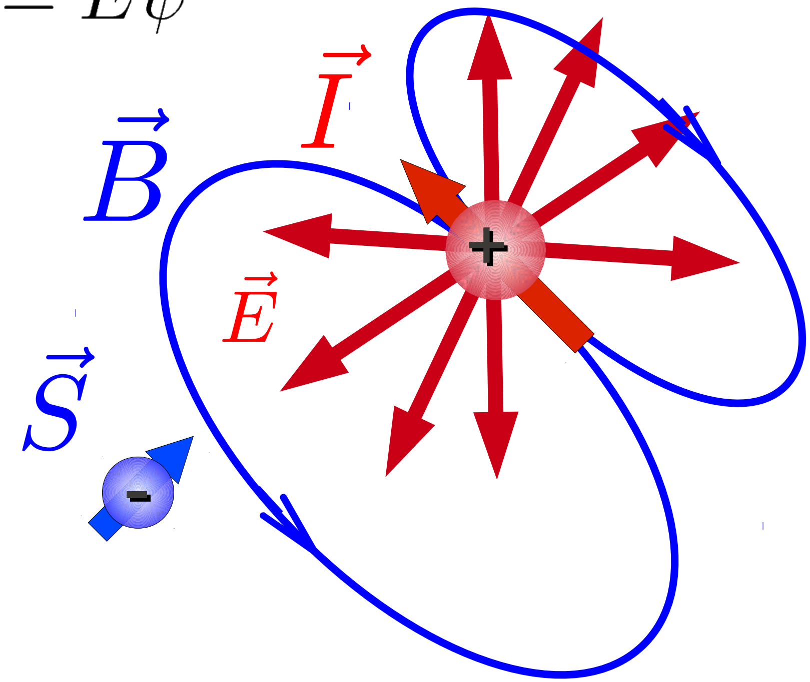
“Theory of everything” for spins in the solid state

$$\left(\boldsymbol{\alpha}\cdot\boldsymbol{\pi} + \beta mc^2 - |e|V(\mathbf{r})\right)\psi = E\psi$$

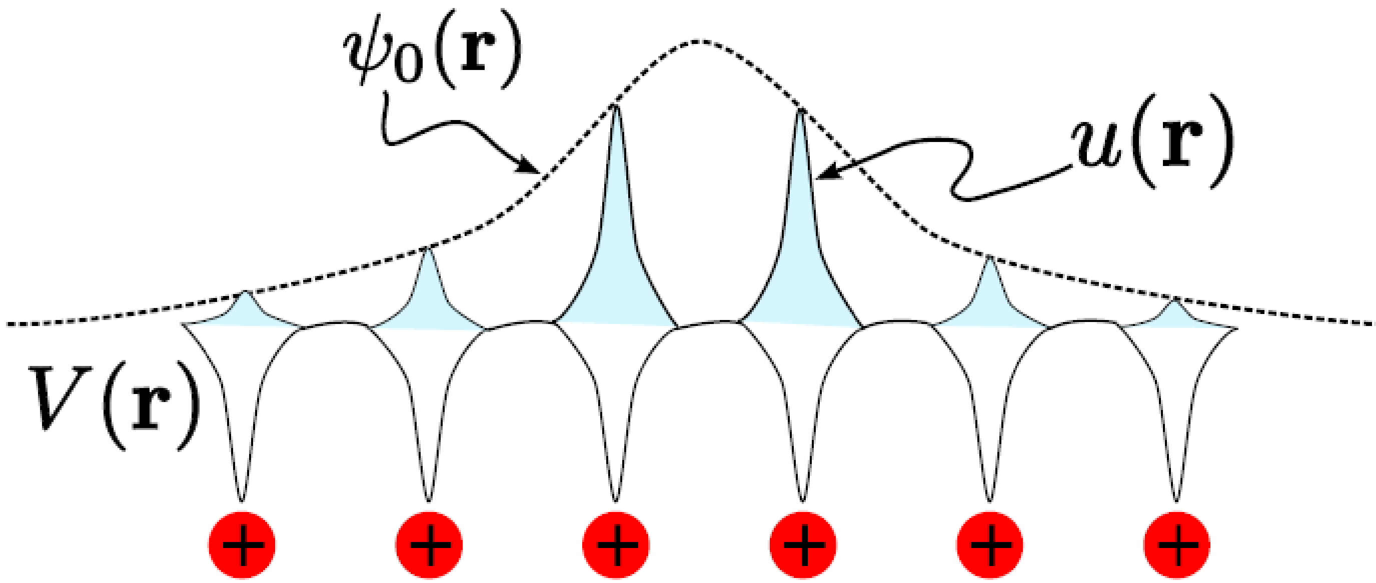
$$H_{\text{contact}} = \frac{8\pi}{3}\gamma_S\gamma_I\delta(\mathsf{r})\mathsf{S}\cdot\mathsf{I}$$

$$H_{\text{dip.}} = \gamma_S\gamma_I\frac{3(\mathsf{n}\cdot\mathsf{S})(\mathsf{n}\cdot\mathsf{I}) - \mathsf{S}\cdot\mathsf{I}}{r^3}$$

$$H_{\text{LI}} = \gamma_S\gamma_I\frac{\mathsf{L}\cdot\mathsf{I}}{r^3}$$



Confined electron

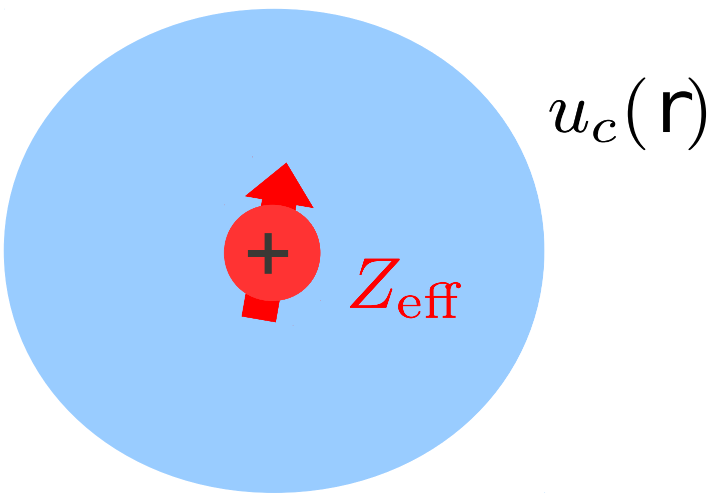


$$H_{\text{eff}} \simeq \langle\psi_0| H |\psi_0\rangle$$

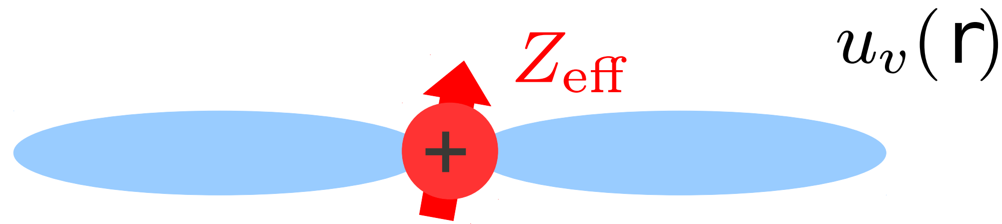
$$\langle \mathsf{r} |\psi_0\rangle \simeq u(\mathsf{r})\psi_0(\mathsf{r})$$

Interactions: s vs. p

s-state (electron)



p-state (hole)



$$\langle\psi^s_{\mathrm{orb}}|\,H_{\mathrm{contact}}\,|\psi^s_{\mathrm{orb}}\rangle \neq 0$$

$$\langle\psi^s_{\mathrm{orb}}|\,H_{\mathrm{dip.}}\,|\psi^s_{\mathrm{orb}}\rangle = 0$$

$$\langle\psi^s_{\mathrm{orb}}|\,H_{\mathrm{LI}}\,|\psi^s_{\mathrm{orb}}\rangle = 0$$

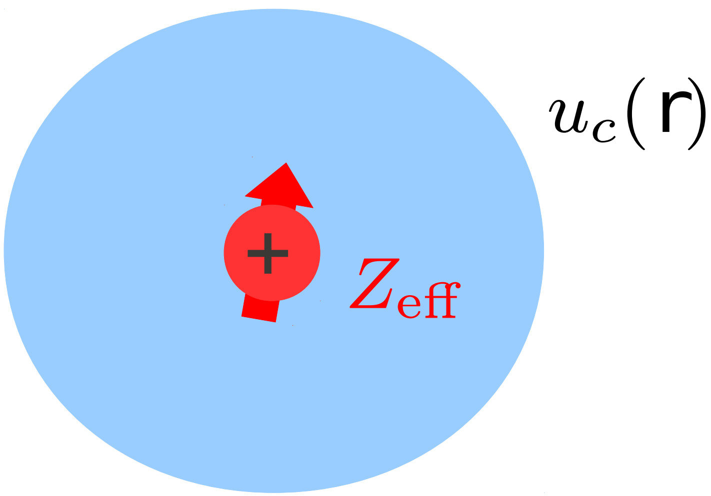
$$\langle\psi^p_{\mathrm{orb}}|\,H_{\mathrm{contact}}\,|\psi^p_{\mathrm{orb}}\rangle = 0$$

$$\langle\psi^p_{\mathrm{orb}}|\,H_{\mathrm{dip.}}\,|\psi^p_{\mathrm{orb}}\rangle \neq 0$$

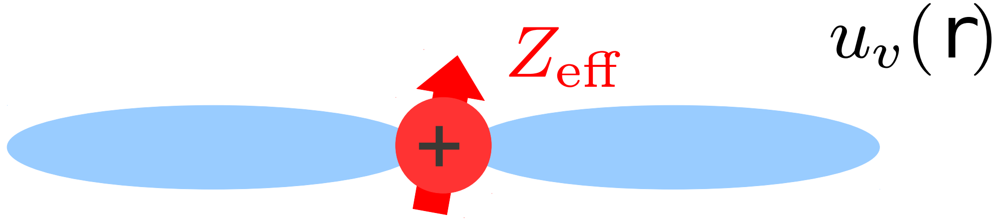
$$\langle\psi^p_{\mathrm{orb}}|\,H_{\mathrm{LI}}\,|\psi^p_{\mathrm{orb}}\rangle \neq 0$$

Interactions: s vs. p

s-state (electron)



p-state (hole)



$$\langle \psi^s_{\rm orb} | \, H_{\rm contact} \, | \psi^s_{\rm orb} \rangle \neq 0$$

$$\langle \psi^s_{\rm orb} | \, H_{\rm dip.} \, | \psi^s_{\rm orb} \rangle = 0$$

$$\langle \psi^s_{\rm orb} | \, H_{\rm LI} \, | \psi^s_{\rm orb} \rangle = 0$$

$$\langle \psi^p_{\rm orb} | \, H_{\rm contact} \, | \psi^p_{\rm orb} \rangle = 0$$

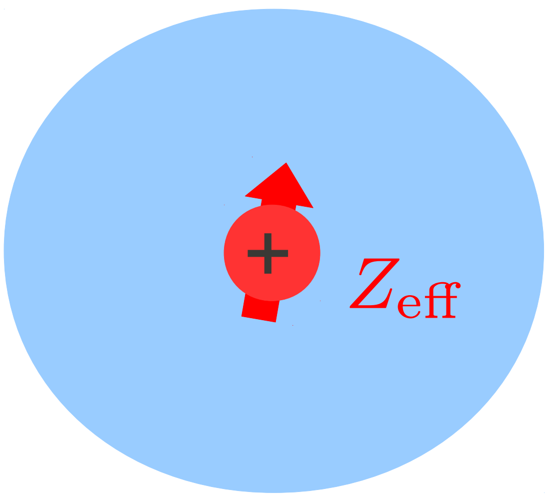
$$\langle \psi^p_{\rm orb} | \, H_{\rm dip.} \, | \psi^p_{\rm orb} \rangle \neq 0$$

$$\langle \psi^p_{\rm orb} | \, H_{\rm LI} \, | \psi^p_{\rm orb} \rangle \neq 0$$

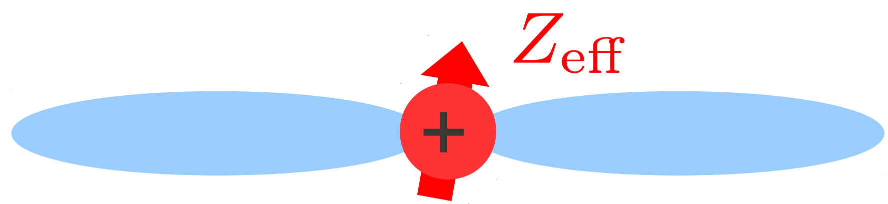
Anything else: NV Center, Nanotubes, graphene,...combination

Interactions: s vs. p

s-state



p-state



Project onto $m_J = \pm\frac{3}{2} \Rightarrow s_z = \pm\frac{1}{2}$

$$H_s^{\mathrm{eff}} = A_s \mathbf{S} \cdot \mathbf{I}$$

$$H_p^{\mathrm{eff}} = A_p s_z I_z$$

For 4s, 4p Hydrogen-like atomic orbitals (valence states of Ga, As):

$$\frac{A_p}{A_s} = \frac{1}{5}\left(\frac{Z_{\mathrm{eff}}(4p)}{Z_{\mathrm{eff}}(4s)}\right)^3$$

The two coupling strengths are comparable!

Fischer, WAC, Bulaev, Loss, PRB (2008)

Hyperfine Hamiltonian: Electron

$$H_{\mathrm{hf}} = bS^z + \mathsf{h} \cdot \mathsf{S}$$

Electron Zeeman energy (bS^z)

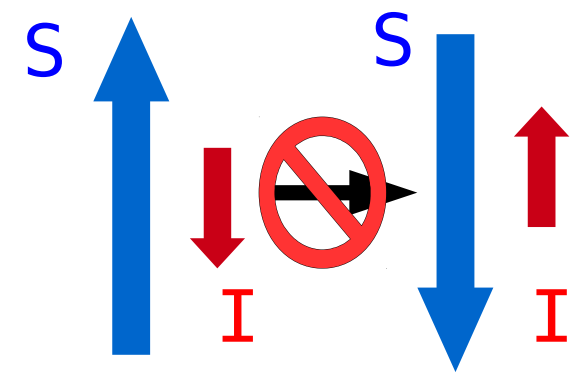
Coupling to nuclear field ($\mathsf{h} \cdot \mathsf{S}$)

$$\mathsf{h} = \sum_k A_k \mathtt{I}_k$$

$$A = \sum_k A_k$$

$$\mathsf{h} \cdot \mathsf{S} = h^z S^z + \frac{1}{2}\left(h^+ S^- + h^- S^+\right)$$

V_{ff} does not conserve energy for large b



→ Perturbation theory in $\frac{A}{b} \ll 1$ $\quad b/g^* \mu_B \gtrsim 3.5\,\mathrm{T}\ (\mathrm{GaAs})$

WAC and Loss, PRB (2004)

A better approach?

(1) $\delta H(t)$ unknown.

➡ Figure it out!

(2) Environment state unknown.

➡ Measure it! (For a static environment)

Initial Conditions

Fast initialization:

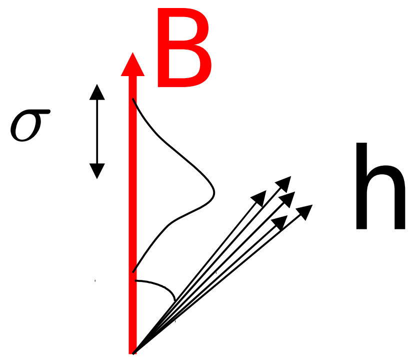
$$\rho(0) = \rho_S(0) \otimes \rho_I(0)$$

Sufficient condition: $\tau_{\text{init}} \lesssim 1/A \simeq 50\,\text{ps}$

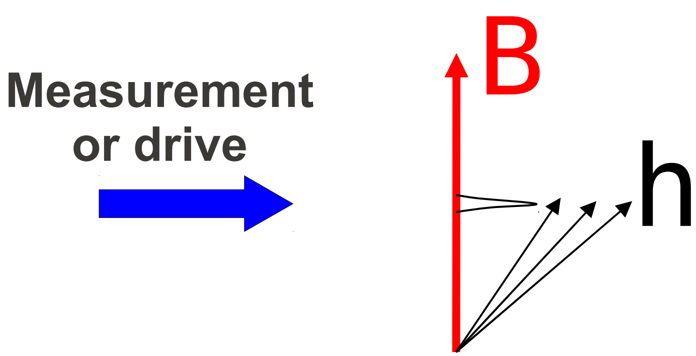
Nuclear Bath:

$$\rho_I(0) = ??$$

State Narrowing



$$\Rightarrow \langle S_x \rangle_t \propto e^{-(t/\tau)^2} \quad \tau \sim \frac{1}{\sigma} \sim \mathrm{ns}$$



$$\Rightarrow \langle S_x \rangle_t \propto e^{i\omega t}$$

(narrowed state)

Theory: WAC and Loss, PRB (2004), Klauser, WAC and Loss, PRB (2006,2008), ...

Experiments: Latta et al., Nature Phys. (2009), Vink et al., Nature Phys. (2009), Xu et al., Nature (2009), ...

A better approach?

(1) $\delta H(t)$ unknown.

➡ Figure it out!

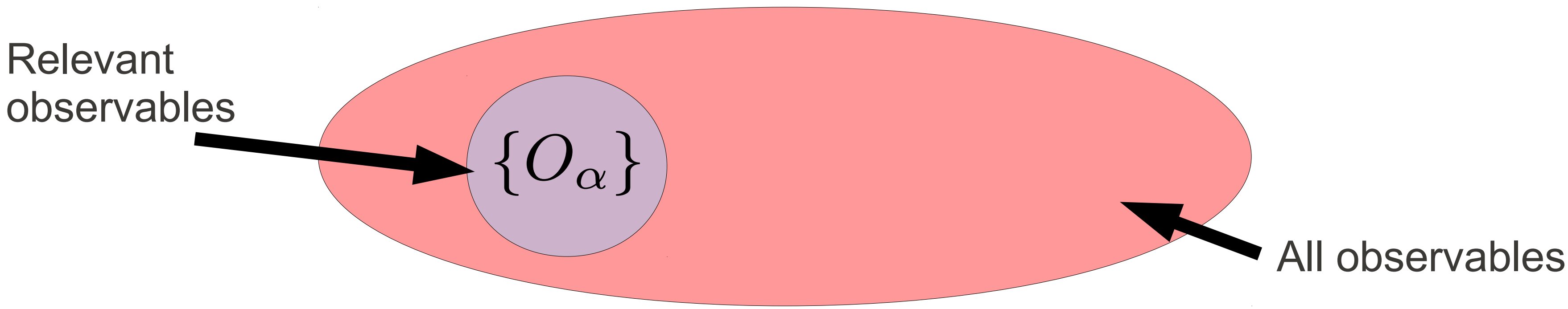
(2) Environment state unknown.

➡ Measure it! (For a static environment)

(3) $U'(t)$ too complicated.

➡ Systematic expansion, not always weak-coupling.

New approach: A general theory of coherent quantum dynamics



Von Neumann: $\dot{\rho} = -i\,[H,\rho]$ $\quad \langle O_\alpha \rangle_t = \mathrm{Tr}\,\{\mathrm{O}\rho(\mathrm{t})\}$

Nakajima-Zwanzig Generalized Master Equation

$$\left\langle \dot{O}_\alpha \right\rangle_t = -i \sum_\beta \omega_{\alpha\beta} \left\langle O_\beta \right\rangle_t - i \sum_\beta \int_0^t dt' \Sigma_{\alpha\beta}(t-t') \left\langle O_\beta \right\rangle_{t'}$$

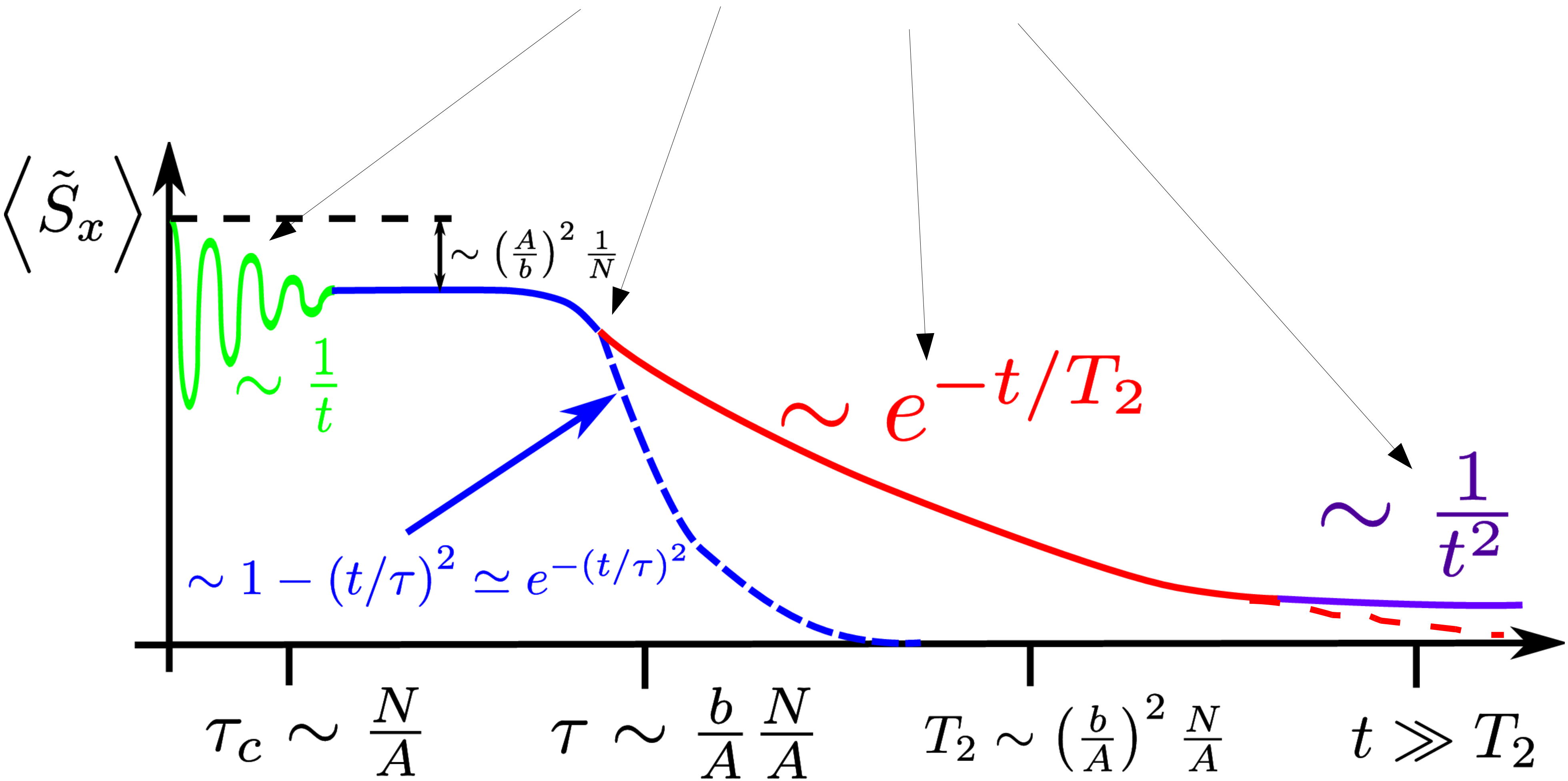
$$H = H_0 + V \qquad \Sigma(t) = \sum_n \Sigma^{(n)}(t) \qquad \Sigma^{(n)}(t) = O\left(V^n\right)$$

WAC and Loss, PRB (2004); WAC, Fischer and Loss, PRB (2008); WAC, Fischer and Loss, PRB (2010)

Free-induction decay

Generalized Master Equation, Higher order.

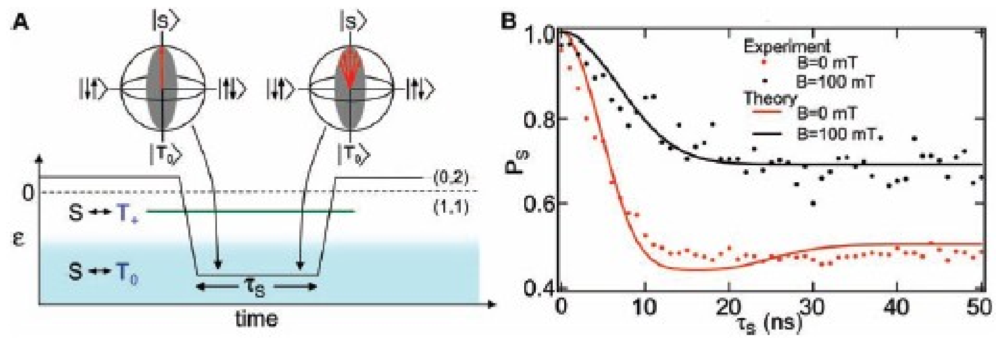
WAC, Fischer, Loss, PRB (2010)



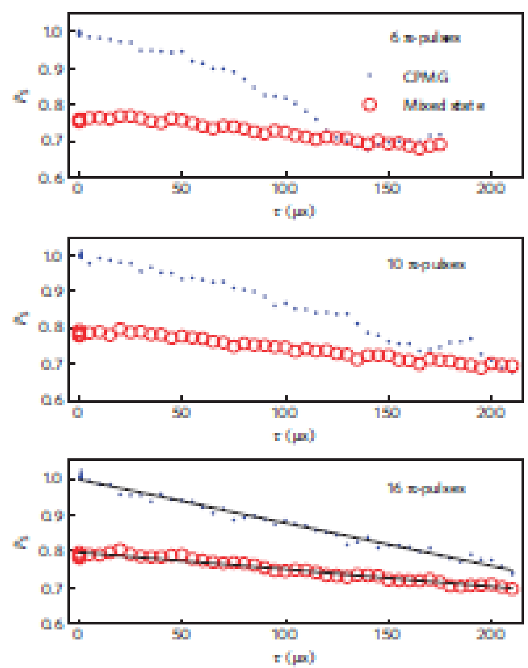
Spin coherence in quantum dots: How far have we come?

Petta et al., Science (2005)

$$T_2^{\rm FID} = 10\,{\rm ns}$$



(Free-induction decay-- no echo)



Bluhm et al., Nature Physics (2010)

$$T_2^{\rm CPMG} = 200\,\mu{\rm s}$$

(Dynamical decoupling)

Quantum Information Processing with spins

0. Historical overview

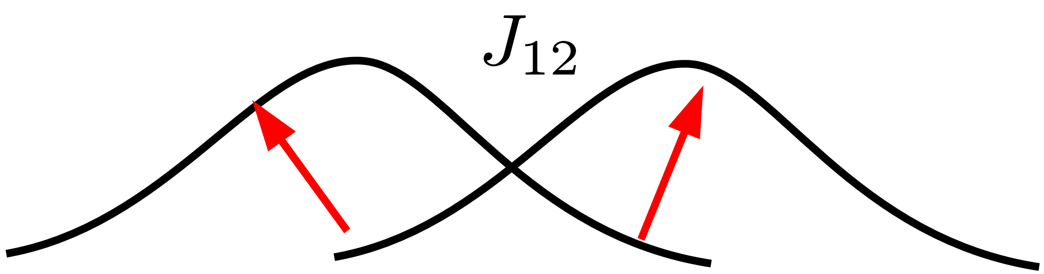
I. Physical requirements

II. Errors and spin coherence

III. Scaling up

Scalability: Why electron spins?

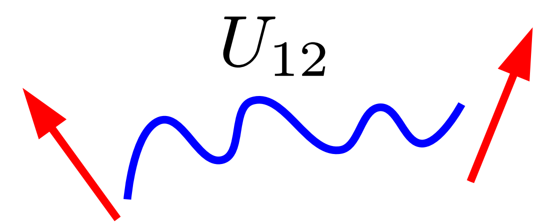
Electrons



$$J \propto \int d^3r' \left|\psi_1(r')\psi_2(r')\right|^2 \sim e^{-r_{12}/r_0}$$

Exchange is local

NMR, Charge, etc....

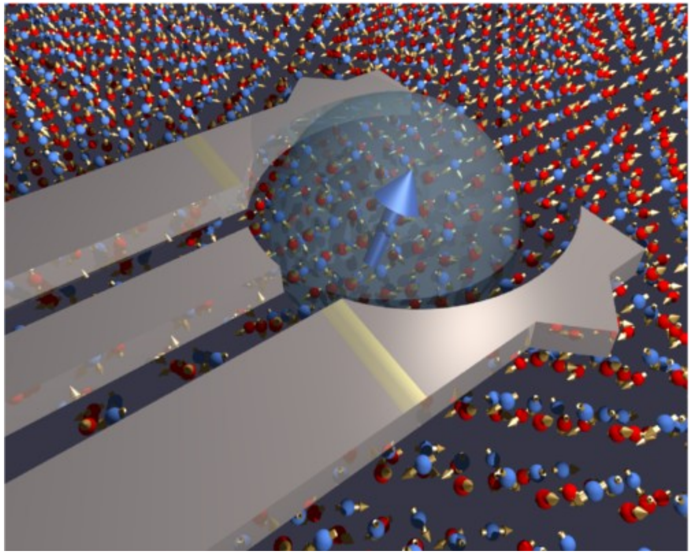


$$U_{12} \sim \frac{1}{r_{12}^3}, \sim \frac{1}{r_{12}}$$

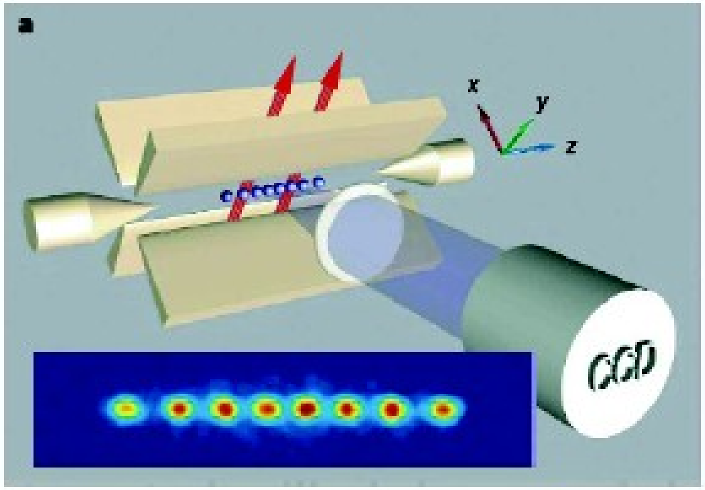
Dipolar, Coulomb interactions long-ranged

Scaling up: What can we learn from other implementations?

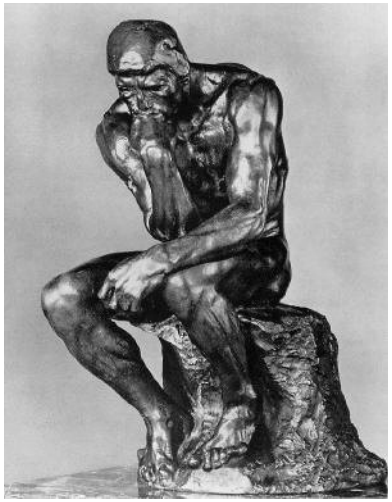
Quantum coherence and dynamics in spin qubits



Quantum coherence in ion traps



General Philosophy



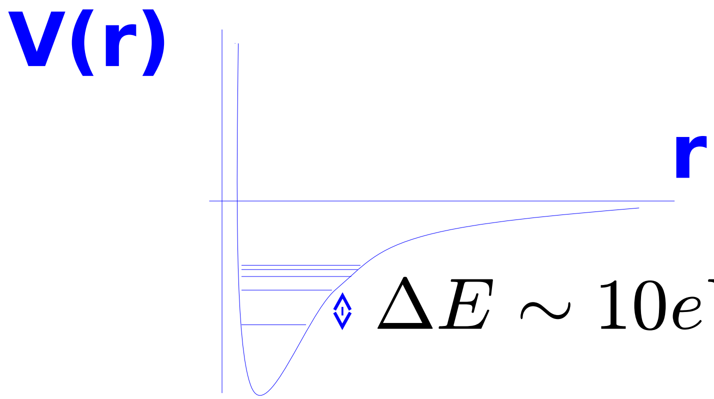
Abstract models are excellent for fast progress, well defined questions...

BUT: too many to choose from.

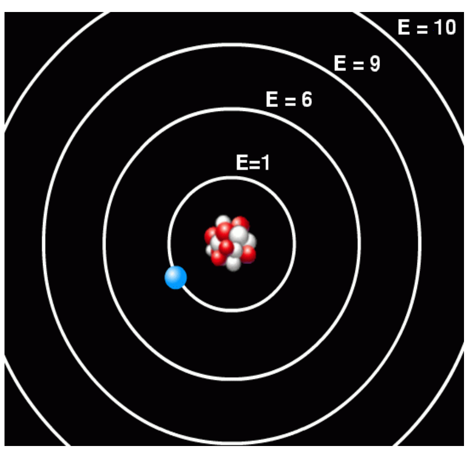
Physical considerations often show the way to go.

True dynamics/decoherence often **more complex** than initial models suggest.

Qubit encoding: Single ions ($^{40}Ca^{+}$)

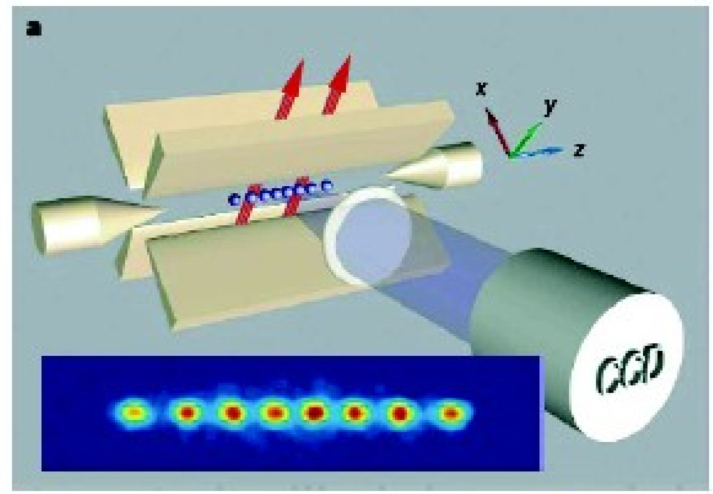


$\Delta E \sim 10 eV$



$\sim 10^{-10}$ m

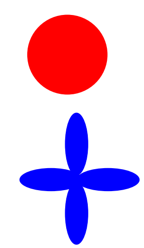
R. Blatt and D. Wineland, Nature (2008)



encoding:

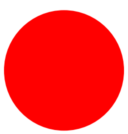
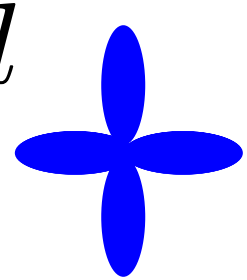
$|s\rangle \rightarrow |0\rangle$

$|d\rangle \rightarrow |1\rangle$



Sources of dephasing in ion traps

- Global magnetic field fluctuations (slow)

s  $= |0\rangle$ d  $= |1\rangle$ (orbital Zeeman)

- Fluctuating global phase reference (laser stability, also slow)

AMO Physics: Usually assume fast, local noise.

Gaussian dephasing model:

$$H(t) = \delta B(t) \sum_k S_k^z \qquad S_k^z = \left(|0\rangle\langle 0|_k - |1\rangle\langle 1|_k\right)/2$$

$$\langle \delta B(t) \delta B(0) \rangle = \left\langle \delta B^2 \right\rangle e^{-t/\tau_c}$$

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}} \left(\underbrace{|0000...\rangle}_{\text{N qubits}} + |1111...\rangle \right)$$

(GHZ/Schroedinger's cat)

Gaussian dephasing model:

$$H(t) = \delta B(t) \sum_k S_k^z \qquad S_k^z = \left(|0\rangle\langle 0|_k - |1\rangle\langle 1|_k\right)/2$$

$$\langle \delta B(t)\delta B(0)\rangle = \left\langle \delta B^2\right\rangle e^{-t/\tau_c}$$

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left(\underbrace{|0000\ldots\rangle}_{\text{N qubits}} + |1111\ldots\rangle\right)$$

(GHZ/Schroedinger's cat)

$$F(t) = \overline{|\langle\psi(0)|\,\psi(t)\rangle\,|^2} = \frac{1}{2}\left(1+\exp\left[-2\epsilon(N,t)\right]\right) \simeq 1-\epsilon(N,t)$$

$$\epsilon(N,t) = \frac{N^2}{2}\int_0^t d\tau(t-\tau)\,\langle\delta B(t)\delta B(0)\rangle$$

“Superdecoherence”

(Palma et al., Proc. Roy. Soc. Lond.A (1996))

Problem for short-term scalability??

T. Monz et al., PRL (2011)

Big questions in physical quantum information processing

- Quantum simulation: What is the simplest problem that would see a real advantage?

- Quantum-enhanced precision measurements (are they practical)? Effects of realistic decoherence?

- Quantum networks: Spin-photon coupling, Swapping between microwave and optical frequencies.

- Long-range (~cm) distributed entanglement between electron-spin quantum bits.

Spintronics and Qm. Information Processing: Joined at the hip

- Fast and accurate single-spin rotation and readout (gated lateral dots) **combined** with long memory time.

- Materials/Architechtures:
  - III-V dots; electrons vs. holes? Nanowires vs. vertical/lateral/self-assembled dots?
  - Nuclear-spin free?: Silicon, Carbon Nanotubes, Graphene, NV Centers in Diamond

- Theory: Controlled theory of spin echoes in a spin bath

- Coherence in dynamic nuclear polarization, diffusion

- Hybrid structures (spin-photon coupling; coupling to superconductors)

Summary: Electron spins as qubits

Demonstrated:

- Fast two-qubit gates
- Long (potential) coherence times
- Selective single-spin rotations

Still needed:

- Fast high-fidelity single-qubit gates (single-spin rotations)
- Long coherence times (for a single-spin qubit)
- Transfer of information from stationary to flying qubits
- Long-range distributed entanglement

Method 1: Magnus expansion

$$U(t) = \mathcal{T} e^{-i \int_0^t dt' \mathcal{H}(t')} = e^{-iH(t)}$$

$$H(t) = H^{(0)}(t) + H^{(1)}(t) + H^{(2)}(t) + \ldots$$

$$H^{(0)}(t) = \int_0^t \mathcal{H}(t') dt'$$

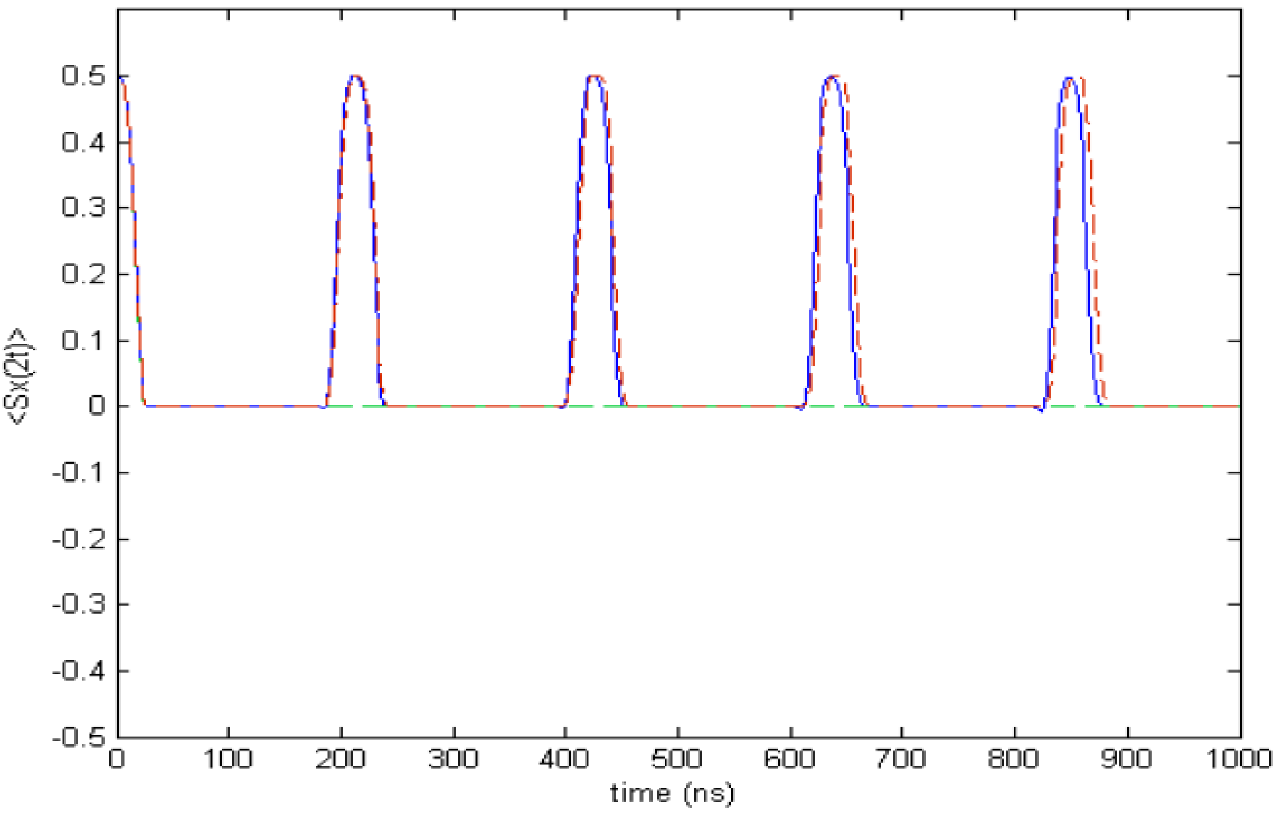
$$H^{(1)} = \frac{-i}{2} \int_0^t dt_2 \int_0^{t_2} dt_1 \left[\mathcal{H}(t_2), \mathcal{H}(t_1)\right]$$

• • •

Magnus expansion: Spin echoes in a dynamic environment

$$H = S_z \sum_k A_k I_k^z + B\gamma_I I^x$$

"Heavy-hole spin echoes", with X. Wang



$$B = 0.5\,\mathrm{T}$$

$$N = 2000\,\mathrm{nuclei}$$

Magnus (dashed red)

Exact (solid blue)