Topological Insulators
A new state of matter with three dimensional topological electronic order

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Surface States (Topological Order in 3D)
“Search & Discovery”: PHYSICS TODAY 2009 (April)

REVIEWS

MZH & C.L. Kane, Rev. of Mod. Phys. 82, 3045 (2010)
History of Z2 Topological Insulators

**2005:** Theoretical prediction of the Z2 TI phase (C.L. Kane and E.J. Mele *PRL 2005*)

**2006-2007:** Achievement of a 2D TI phase in HgTe (B.A. Bernevig, T.L. Hughes, S.-C. Zhang, *SCIENCE 2006*, M. König et al. *SCIENCE 2007*)

**2007-2009:** First discovery of a 3D TI (Bi$_{1-x}$Sb$_x$ alloy, L. Fu et al. *PRL 2007*, D. Hsieh *et al. NATURE 2008*, *SCIENCE 2009*)


**2010:** Symmetry breaking: Observation of unconventional superconductivity in Cu$_x$Bi$_2$Se$_3$, magnetism in Mn$_x$Bi$_{2-x}$Te$_3$ (Wray et al. Nat. Phys. 2010, Hor et al. *PRB 2010*)

**2010-2011:** Many new ternary TIs, building of TI interfaces and nanodevices
Lecture Outline:

1. “Experimentally discovering” topological insulators
2. Understanding topological order
3. New material properties, new possibilities
4. Discussion
Bismuth Selenide

Bi$_2$Se$_3$ Dirac cone

e- doped as Cu$_{0.12}$Bi$_2$Se$_3$

Helical Dirac fermions
One to One Spin-Linear Momentum Locking

Hsieh et al., SCIENCE 09, NATURE 09
Spin-texture $\rightarrow$ Absence of Backscattering

STM (Roushan et al.)  Spin-ARPES (Hsieh et al.)

Spin-Independent

Spin-Dependent

Xu, Moore et al. (06)
Photoemission on a TI

Bulk screening

Momentum

Binding Energy (eV)

high

low

carrier density

Vacuum

Surface

Bulk CB

total e-carriers

SS CB

Cu²⁺

Cu²⁺ defect

Se₁

Bi

Se₂

cleave

quintuple layer

Wray PRB 2011
Macroscopic Effects

Conductivity by wire size:
\[ \sigma_B \sim \frac{A}{L} \]
\[ \sigma_S \sim \frac{r}{L} \]

Critical crystal size for \( \sigma_B \sim \sigma_S \) in Bi\(_2\)Te\(_2\)Se is \(~1\times1\times0.1\)mm!!
More than just a surface state

For visual clarity, 3D parity symmetry has been broken so that each band is singly degenerate away from the Kramers points.
Tying a knot

Note: this does not closely approximate a real band structure
How to see the topological connection: \textit{band bending}
Band bending creates new topological surface states!

Partner exchange and symmetry inversion

Wray, Nature Physics 2010


\[ (-1)^{\nu_0} = \prod_{n_j=0,1} \delta_{n_1 n_2 n_3} \]

\[ (-1)^{\nu_{i=1,2,3}} = \prod_{n_j \neq i=0,1; n_i=1} \delta_{n_1 n_2 n_3}. \]

L. Fu, PRL 2007
Inducing the topological state
lattice strain and spin orbit coupling

Xu et al., Science 2011
Topological Invariants Define Surface States

Fully gapped
$\text{Bi}_2\text{Se}_3 \rightarrow \text{Bi}_2\text{Se}_3$
“interface”
Topological Insulator
\{\nu_0\} (Chern Parity invariants) \mathbb{Z}_2

Quantum Hall Effect
\nu (Chern Number): Thouless et al.,

3D Topological Insulators

Protected Surface States = New 2DEG

Nature 08 (subm. 2007)

2D Topological Insulators

Science 07 (subm. 2007)

Edge States (1D) by TRS

Chiral Edge States (1D)

Magnetic field
Topological Insulator in 2D: Quantum Hall State

Thouless et.al, ('82), (Berry Phase ‘84)

Hall conductance:

$$\sigma_{xy} = n e^2 / h$$

$$n = \text{Chern no. (Edge states)}$$

Chern : Quantum version (Hilbert space) of Gauss-Bonnet formula

$$n = \frac{1}{2\pi} \int_{BZ} [\nabla_k \times A(k_x, k_y)]_z d^2 k$$

$$A = -i \langle u_k | \nabla_k | u_k \rangle$$

Electron-occupied Bulk bands

Finite $n \rightarrow$ topologically “protected” edge-states
Quantum Hall Effect (insulator): 2D Topological insulator w/ LL
Haldane model (QAH): 2D Topological insulator w/o LL

1 invariant → QSHE

Quantum spin Hall effect
Kane & Mele (05a), Kane & Mele (05b) [\(\sigma_{\text{spinHall}}\) Not quantized]
Bernevig, Hughes, Zhang (06), Sheng, Haldane et.al., (06)

Expt: Molenkamp group HgCdTe-QWells, Science (2007)

4 Invariants → 3D TI

Distinct Topological state in 3D Topo Insulator
Moore & Balents(07), Fu & Kane(07), Fu, Kane & Mele(07), Roy (2009)

Expt: MZH group Bi-based Semiconductors, KITP Proc.(2007)
Nature 2008 [Submitted in 2007]

3D TIs -> Superconductors and Magnets (Tc)

Many others afterwards, ~ 800 papers on arXiv
QHE phases
\[ \sigma_{xy} = n e^2 / h \]

Transport

Topological quantum number

How to experimentally “measure” the topological quantum numbers (\(\nu_i\))?

4 TQNs \(\rightarrow\) 16 distinct insulators

Topo Insulators
\[ \nu_0 = \Theta / \pi \]
\[ \Theta = \pi \text{ (odd)} \]
\[ \Theta = 2\pi \text{ (even)} \]

No quantized transport via:
\[ \{\nu_i\} \]

Topological quantum number

Spin-sensitive
Momentum-resolved
Edge vs. Bulk

{\nu_0, \nu_1, \nu_2, \nu_3}
Topological “Order Parameters”
So what can they do?
Magnetoelectric Effects
(not “Electromagnetic”)

\[ S_\theta = \frac{\theta}{2\pi} \frac{\alpha}{2\pi} \int d^3 x dt \mathbf{E} \cdot \mathbf{B} \]
Spin helical states meeting defects

(A)

LDOS near a perfectly reflecting step edge

Biswa PRB 2010; arXiv 2010
Local Magnetic Monopoles

Magnetic order creates effective “axions”
Possibilities for Surface Magnetism

![Diagram showing various states and their corresponding energy levels under different magnetic fields.](image)
A Majorana Platform

Cu$_x$Bi$_2$Se$_3$ ($T_c \sim 3.8$K) : Hor et.al., PRL 2010

Topological Surface States: Superconductivity in doped topological insulators

Wray et.al., Nature Physics (2010)
Majorana Fermions

Fu, PRL 2008; Wray, Nature Physics 2010; PRB 2011
Topological Superconductor (TSC)?

Kitaev/Ludwig D3 class of TSC (proposed by Fu & Berg 09)

$m/\mu$ from ARPES

If ODD parity $\rightarrow$ TSC

[ analog of SF He-3(B) ]

Wray et.al., Nature Physics (2010)
Topological insulators and energy

What makes a material a good thermoelectric?
The “thermoelectric figure of merit” $ZT$ determines Carnot efficiency:

$$ZT = \frac{S^2 \sigma T}{k}$$

$S = V / \Delta T$
“Seebeck coefficient”

$\text{Bi}_2\text{Te}_3$
Topological insulators are:

**Simple**

- Exact non-interacting models (DFT, k.p)
- Most complexity reduces to 1D (much nicer than cuprates!!!)
- Surface is robust against non-magnetic scattering

**Complicated**

- Theory is difficult to learn, and few people know it
- Many surface instabilities, particularly because they occupy the same orbitals as bulk
- Lots of new phases and new physics to explore (Majorana Fermions, Dyons, magnetoelectric effect, unusual surface transport, unusual interface physics)
- Lots of different compounds! (TI chalcogenides, Heuslers, M2X3, etc)
- Many “simple” issues are actually complicated:
  - 2\textsuperscript{nd} order backscattering is allowed from Anderson impurities
  - Self energy is poorly understood
Thanks!