Topological Insulators

A new state of matter with three dimensional topological electronic order

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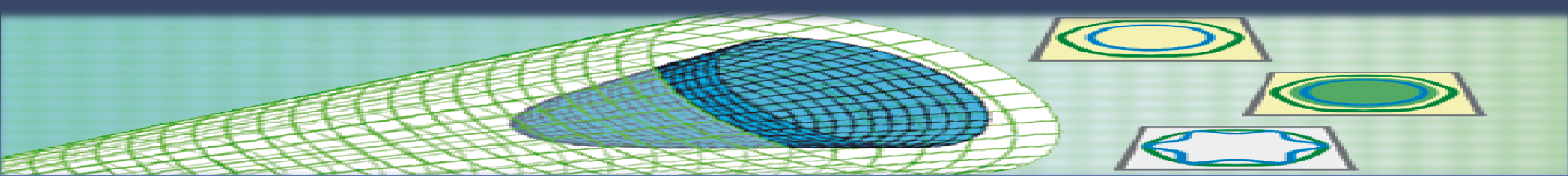
Princeton University

Surface States (Topological Order in 3D)
"Search & Discovery": PHYSICS TODAY 2009 (April)

REVIEWS

MZH & C.L. Kane, Rev. of Mod. Phys. 82, 3045 (2010)
MZH & J.E. Moore, Ann. Rev. of Cond-Mat. Phys. (2011)
X.L. Qi & S.C.Zhang, RMP (in press) 2011

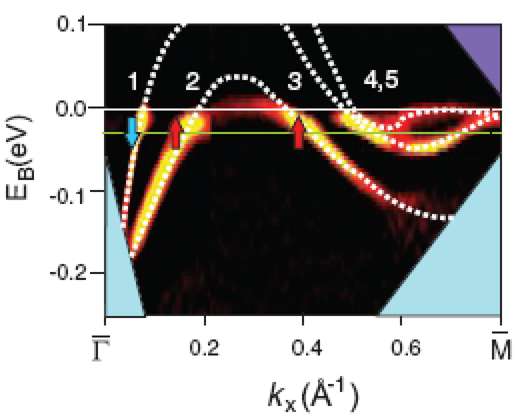


History of Z2 Topological Insulators

2005: Theoretical prediction of the Z2 TI phase (C.L. Kane and E.J. Mele *PRL 2005*)

2006-2007: Achievement of a 2D TI phase in HgTe (B.A. Bernevig, T.L. Hughes, S.-C. Zhang, *SCIENCE 2006*, M. König et al. *SCIENCE 2007*)

2007-2009: First discovery of a 3D TI ($Bi_{1-x}Sb_x$ alloy, L. Fu et al. PRL 2007, D. Hsieh *et al. NATURE 2008, SCIENCE 2009*)



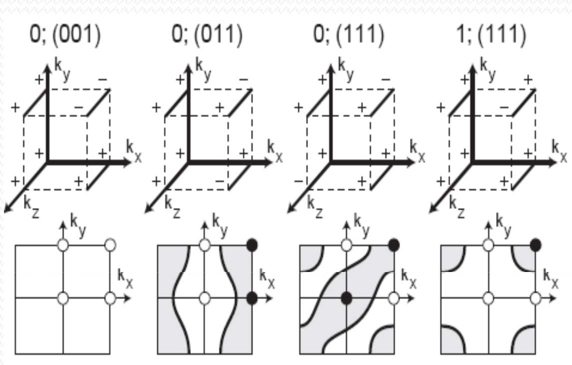
2008: Discovery of the M_2X_3 TI class (Y. Xia, arXiv 2008, H.-J. Zhang et al. NATURE 2009, D. Hsieh *et al. NATURE 2009*)

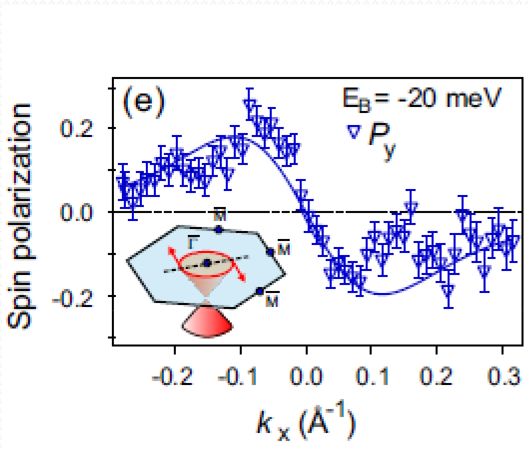
2010: **Symmetry breaking:** Observation of unconventional superconductivity in $Cu_xBi_2Se_3$, magnetism in $Mn_xBi_{2-x}Te_3$ (Wray et al. Nat. Phys. 2010, Hor et al. PRB 2010)

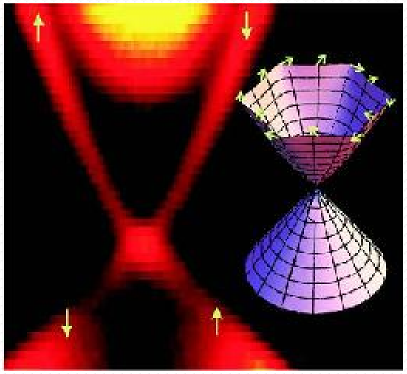
2010-2011: Many new ternary TIs, building of TI interfaces and nanodevices

Lecture Outline:

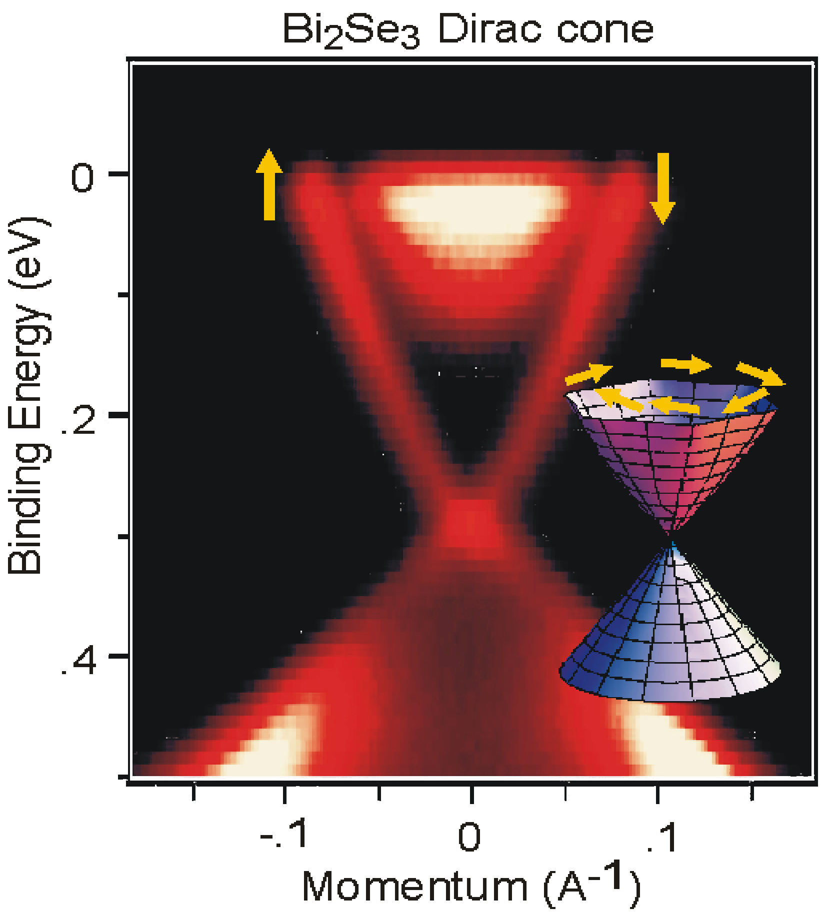
1. “Experimentally discovering” topological insulators
2. Understanding topological order
3. New material properties, new possibilities
4. Discussion

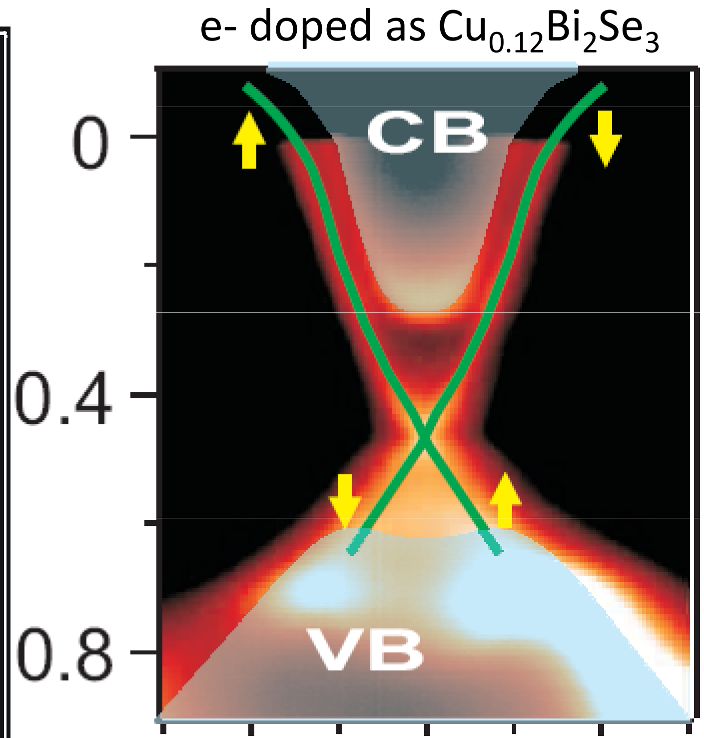


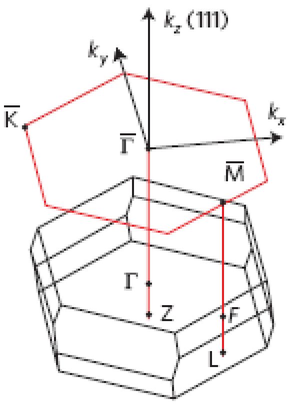




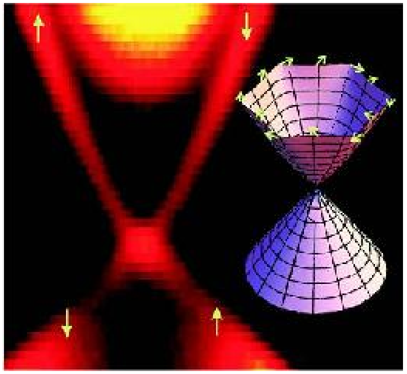
Bismuth Selenide





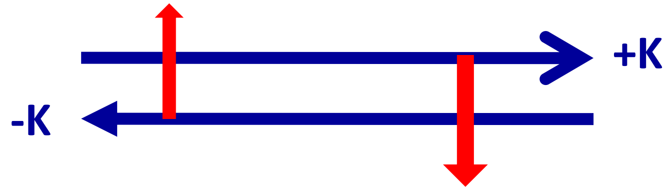


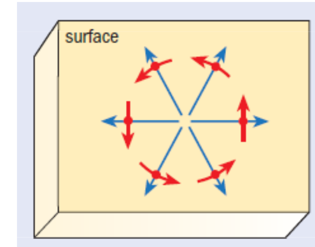
Y. Xia, arXiv 2008, D. Hsieh *et al. NATURE 2009,* L.A. Wray *et al. Nat. Phys. 2010*

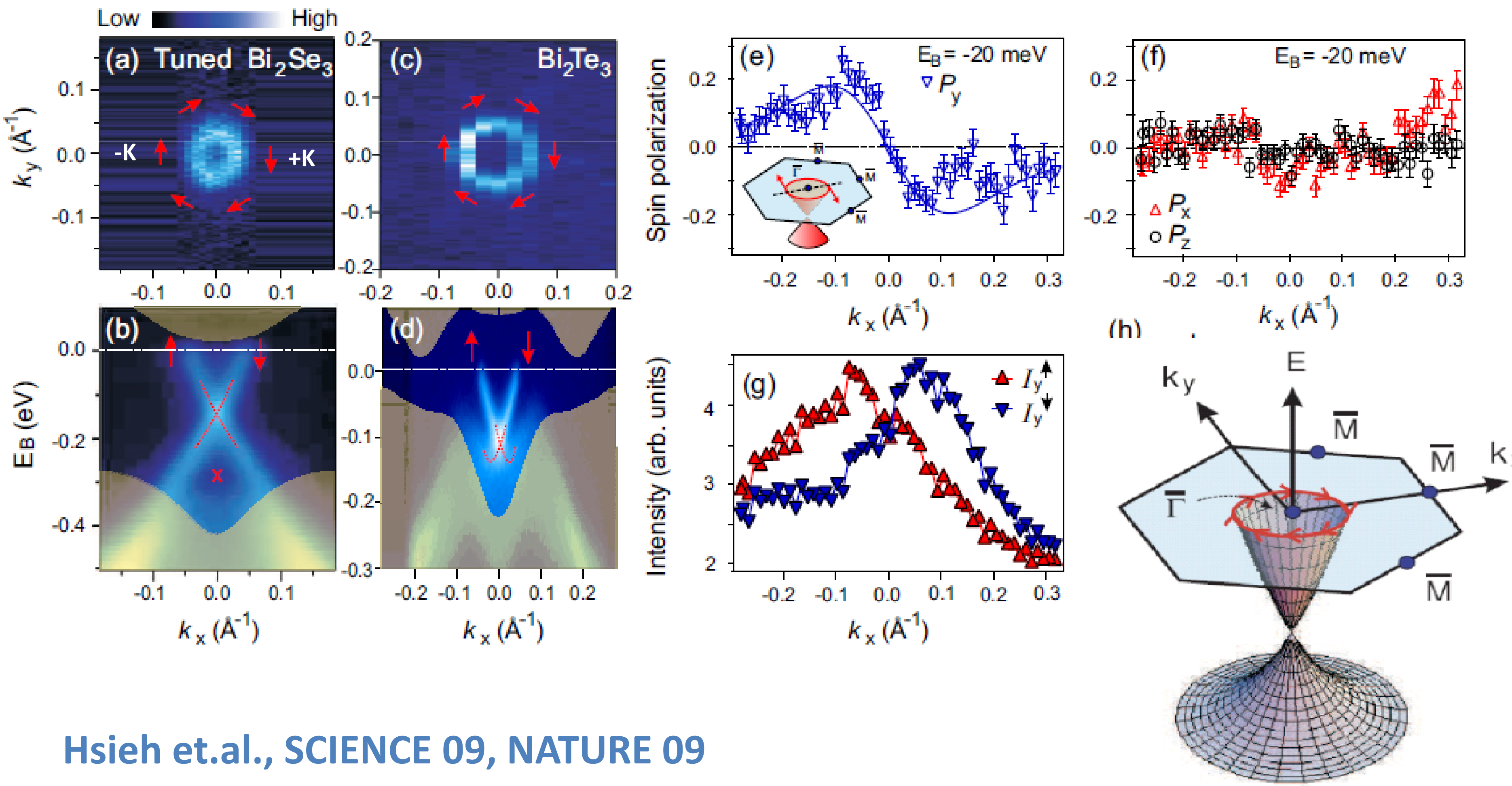


Helical Dirac fermions

One to One Spin-Linear Momentum Locking



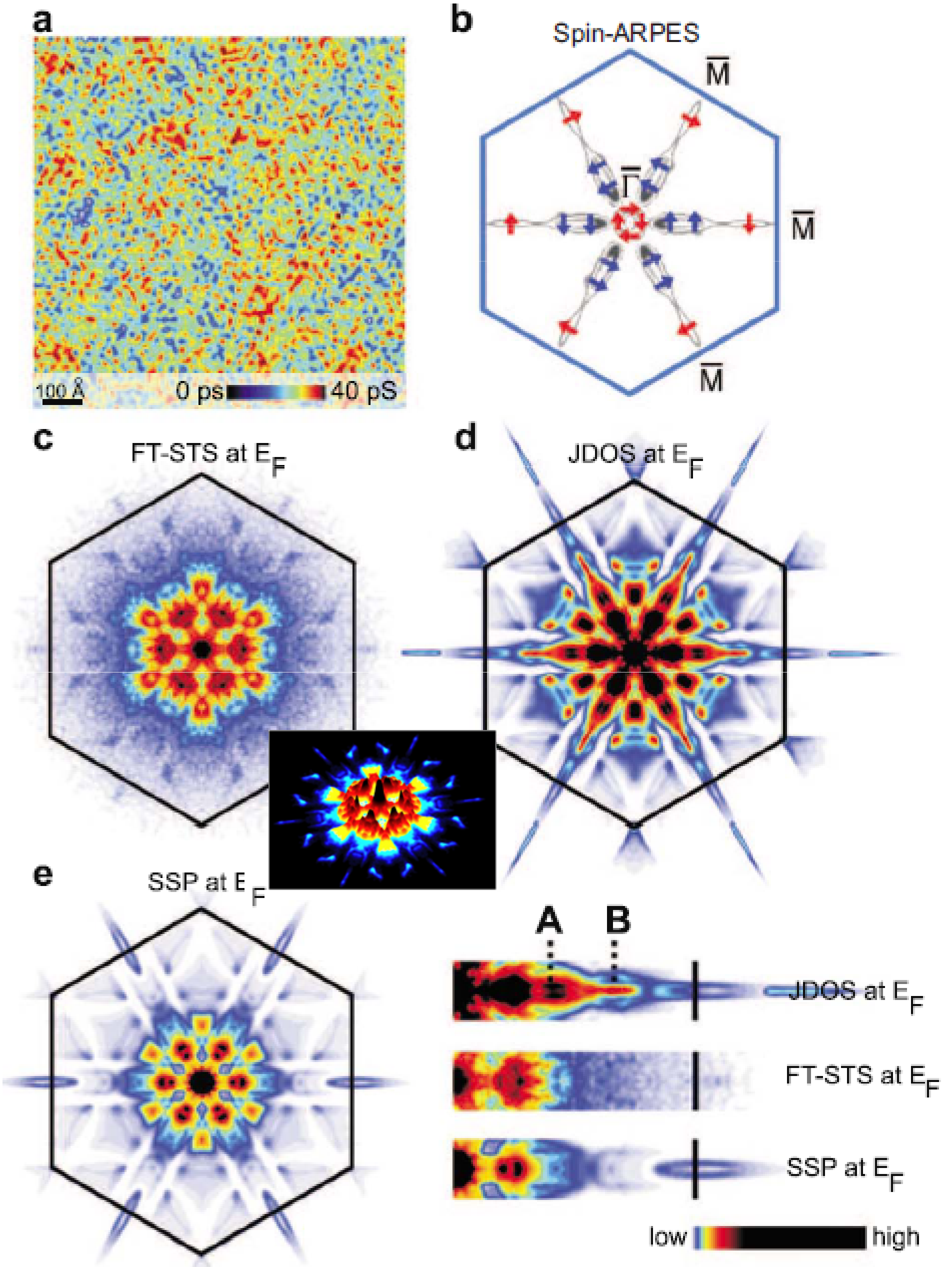


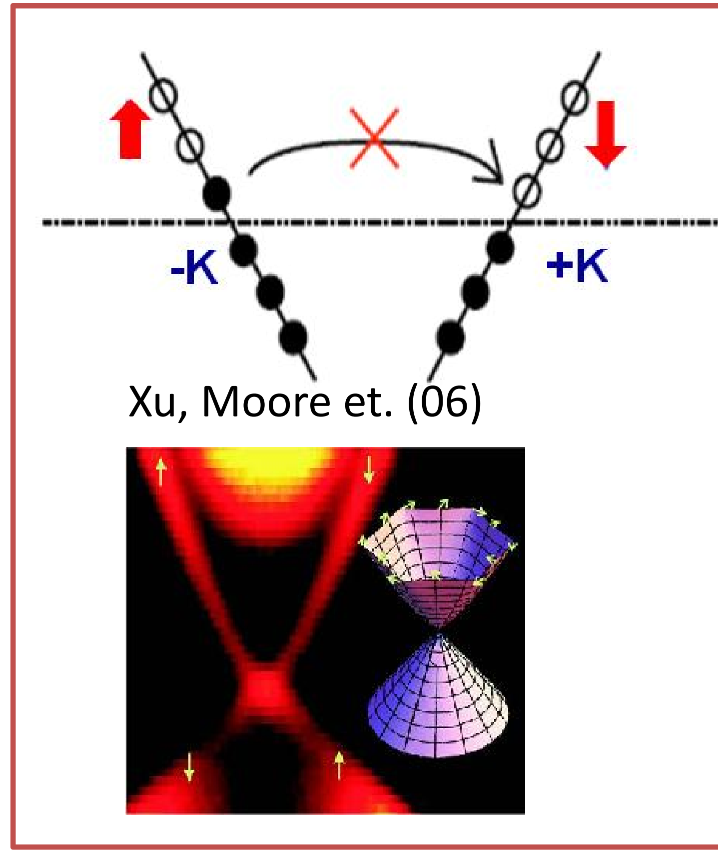


Hsieh et.al., SCIENCE 09, NATURE 09

Spin-texture → Absence of Backscattering

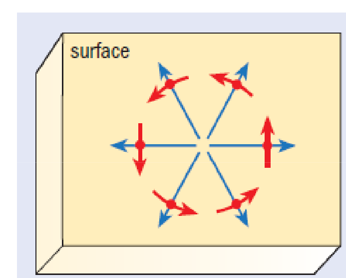
STM (Roushan et.al.) **Spin-ARPES (Hsieh et.al.)**





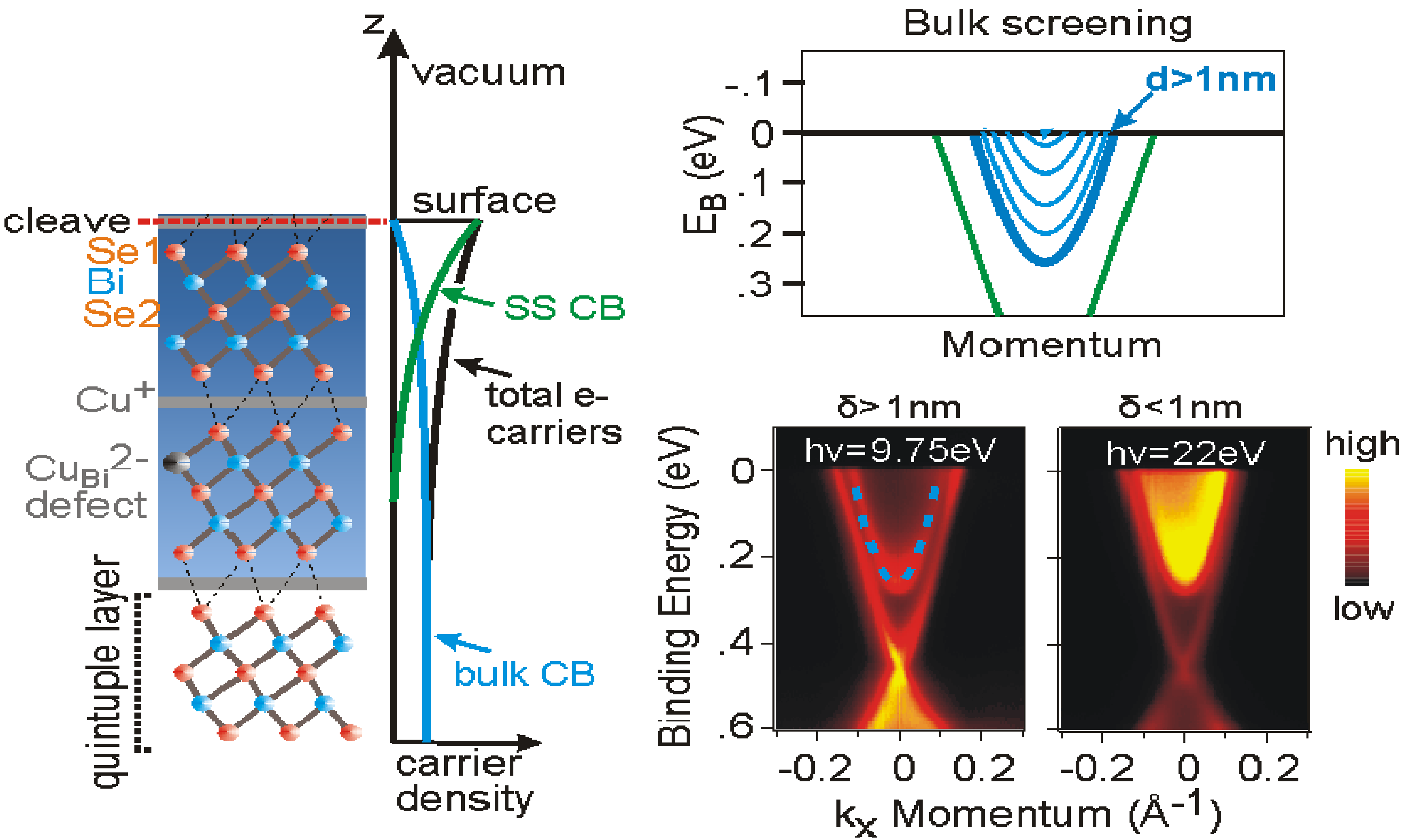
Spin-**Independent**

Spin-**Dependent**



Roushan et.al.,NATURE 09

Photoemission on a TI



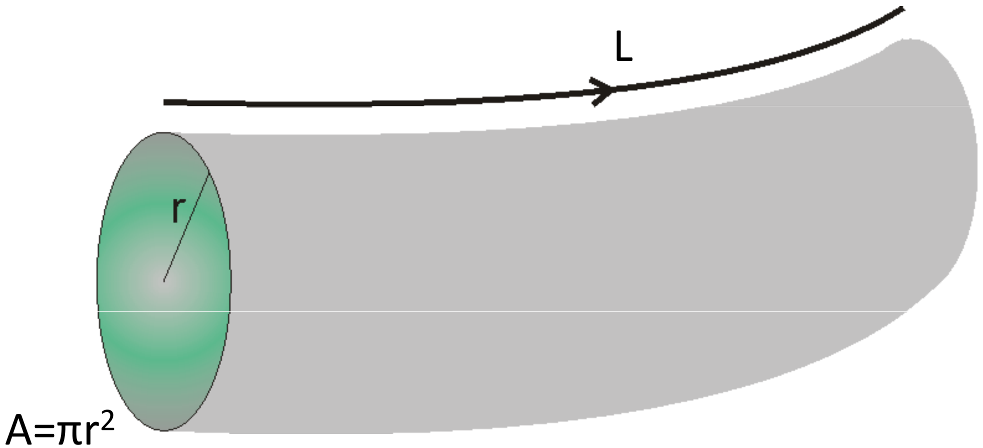
Wray PRB 2011

Macroscopic Effects

Conductivity by wire size:

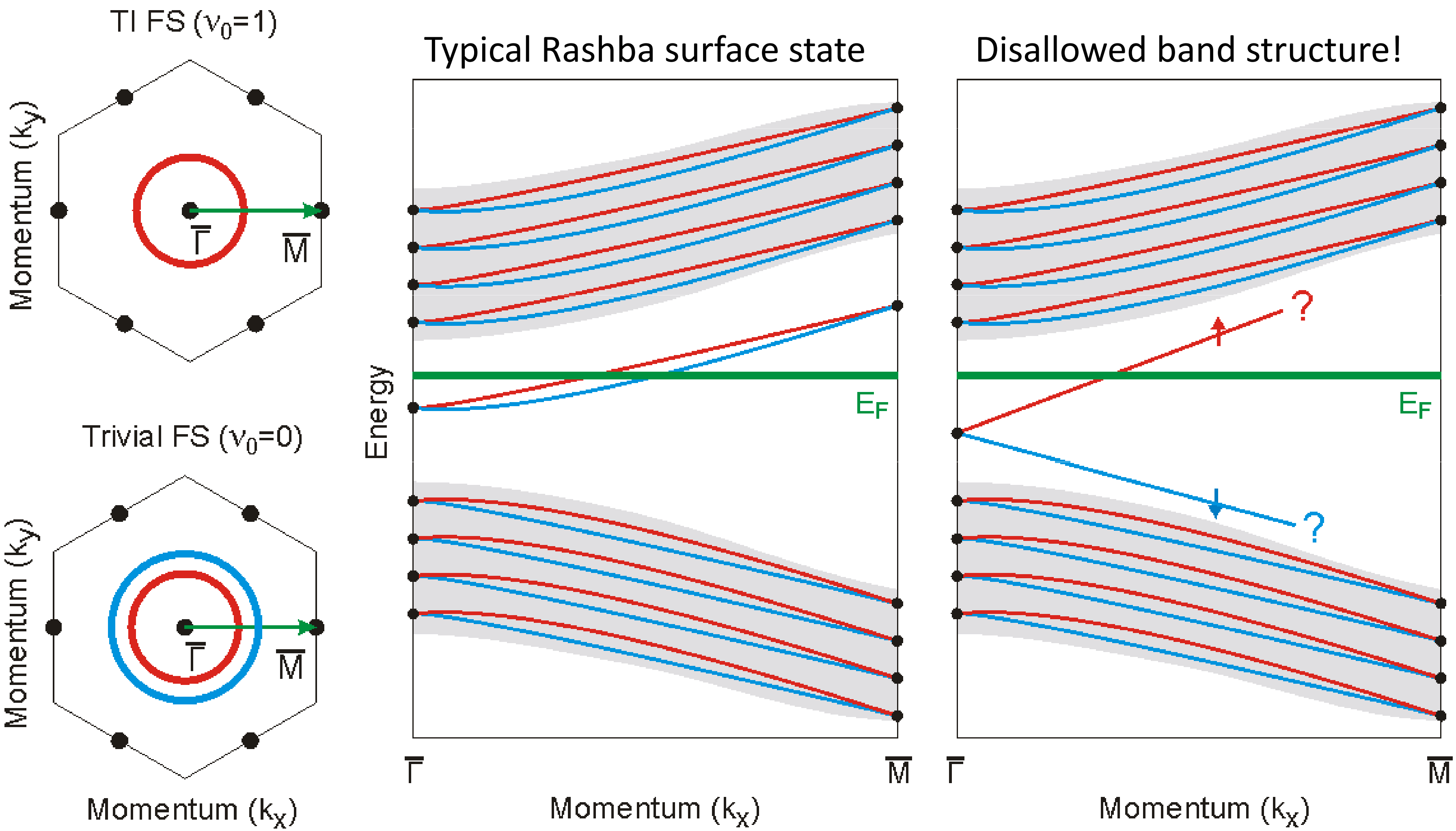
$\sigma_B \sim A/L$

$\sigma_S \sim r/L$



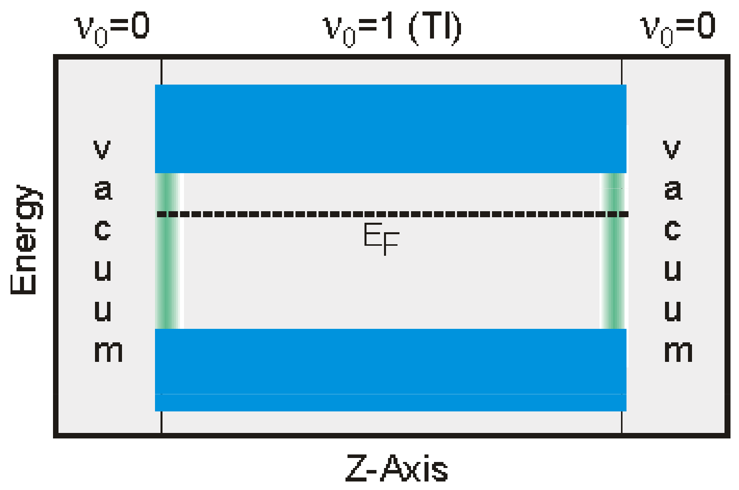
Critical crystal size for $\sigma_B \sim \sigma_S$ in Bi_2Te_2Se is ~1X1X0.1mm!!

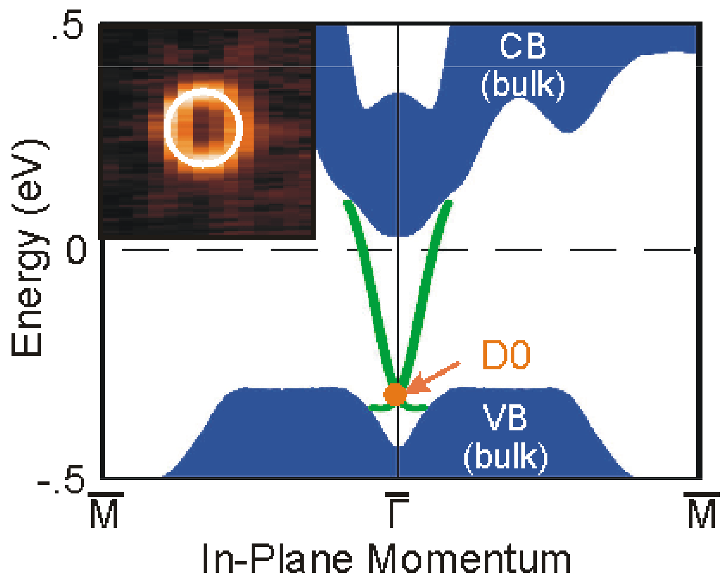
More than just a surface state

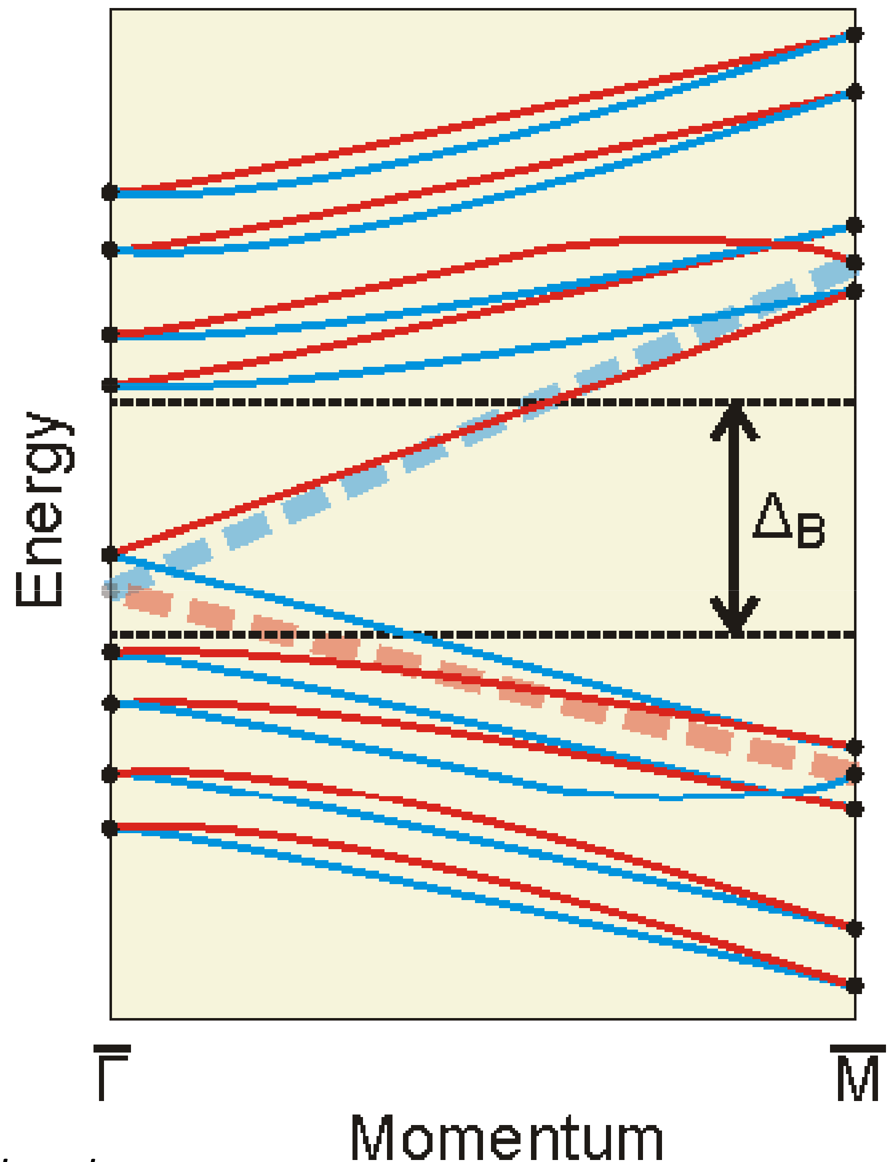


For visual clarity, 3D parity symmetry has been broken so that each band is singly degenerate away from the Kramers points

Tying a knot

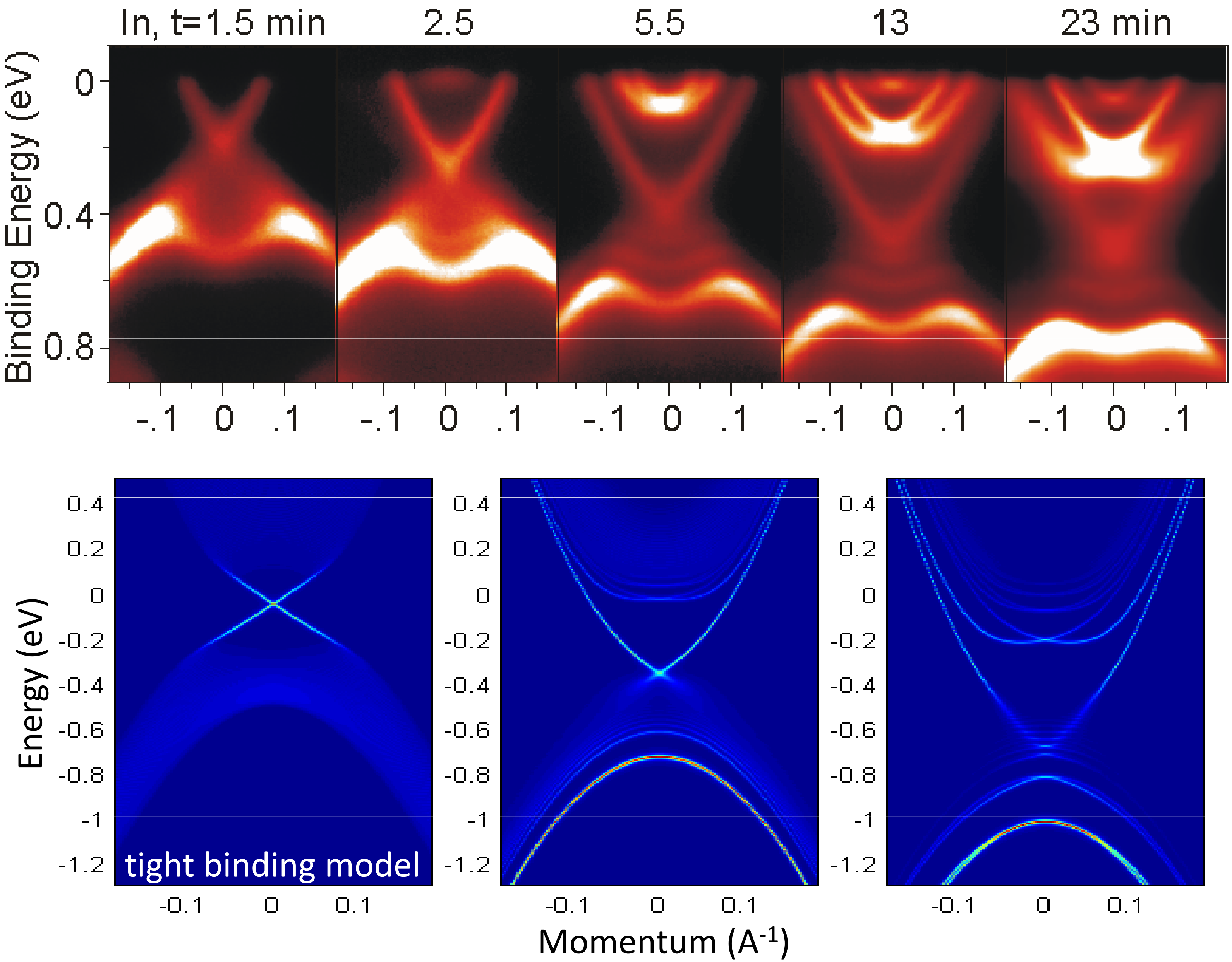




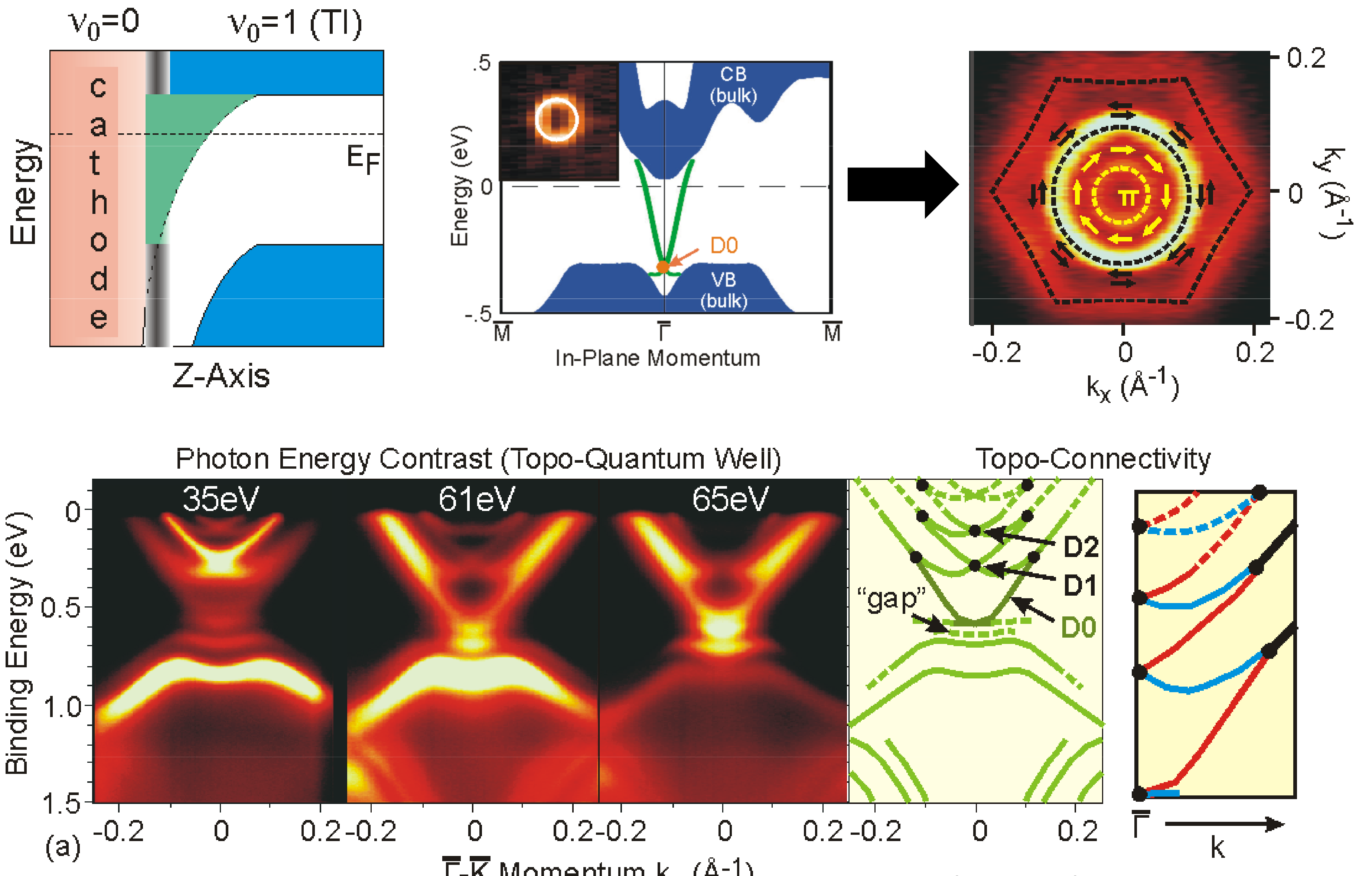


Note: this does not closely approximate a real band structure

How to see the topological connection: *band bending*

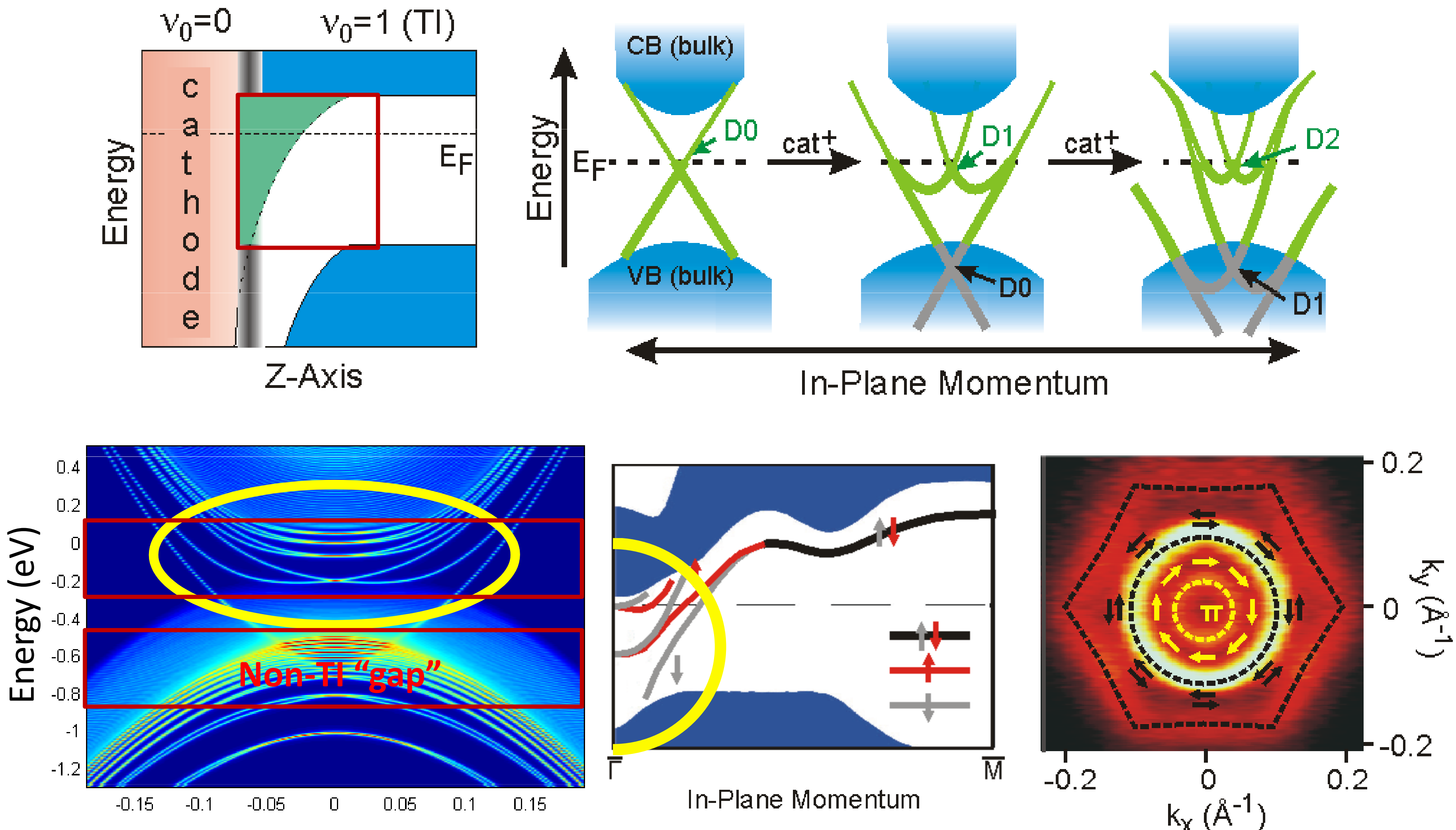


Band bending creates new topological surface states!



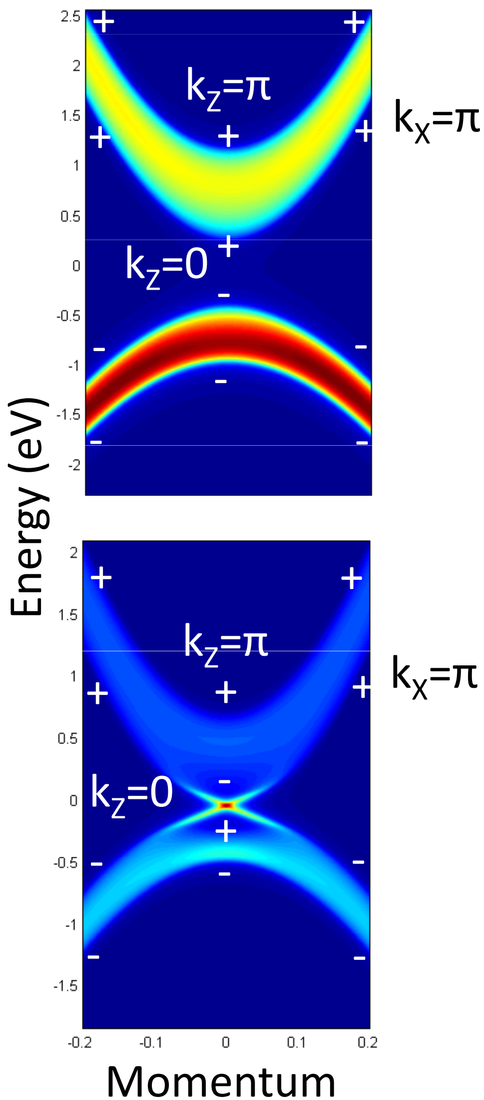
Wray *et al.*, Nat. Phys. 2010, arXiv 2011

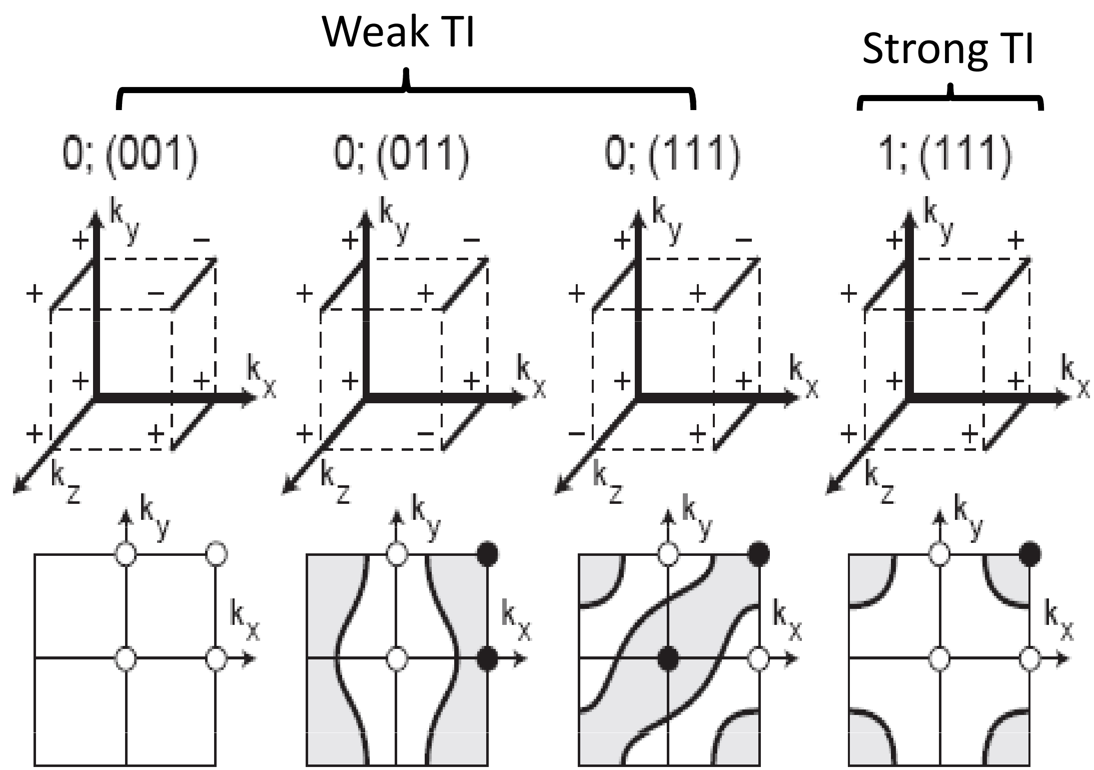
Partner exchange and symmetry inversion



Wray, Nature Physics 2010

Wray, arXiv:1105.4794 2011





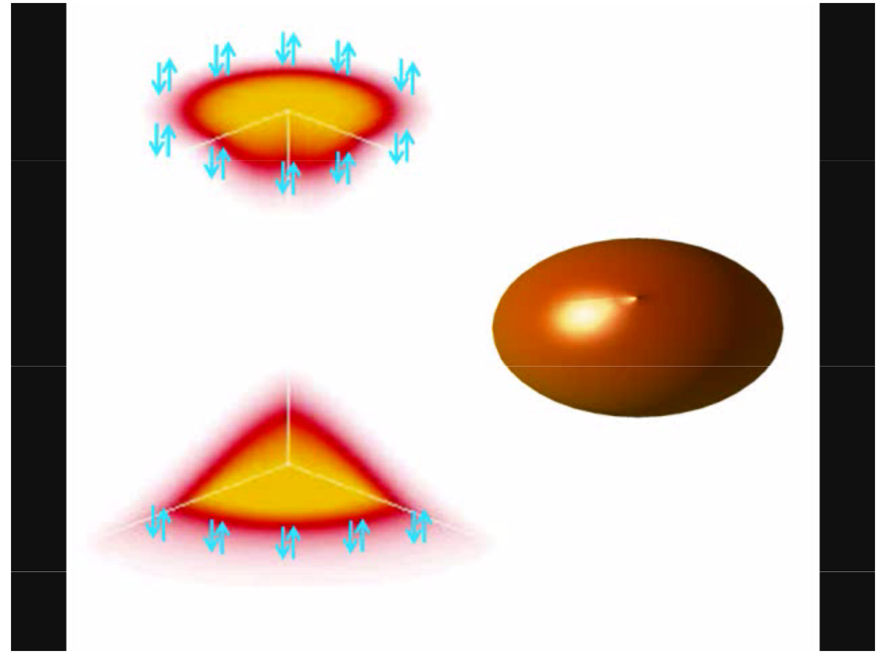
$$(-1)^{\nu_0} = \prod_{n_j=0,1} \delta_{n_1 n_2 n_3}$$

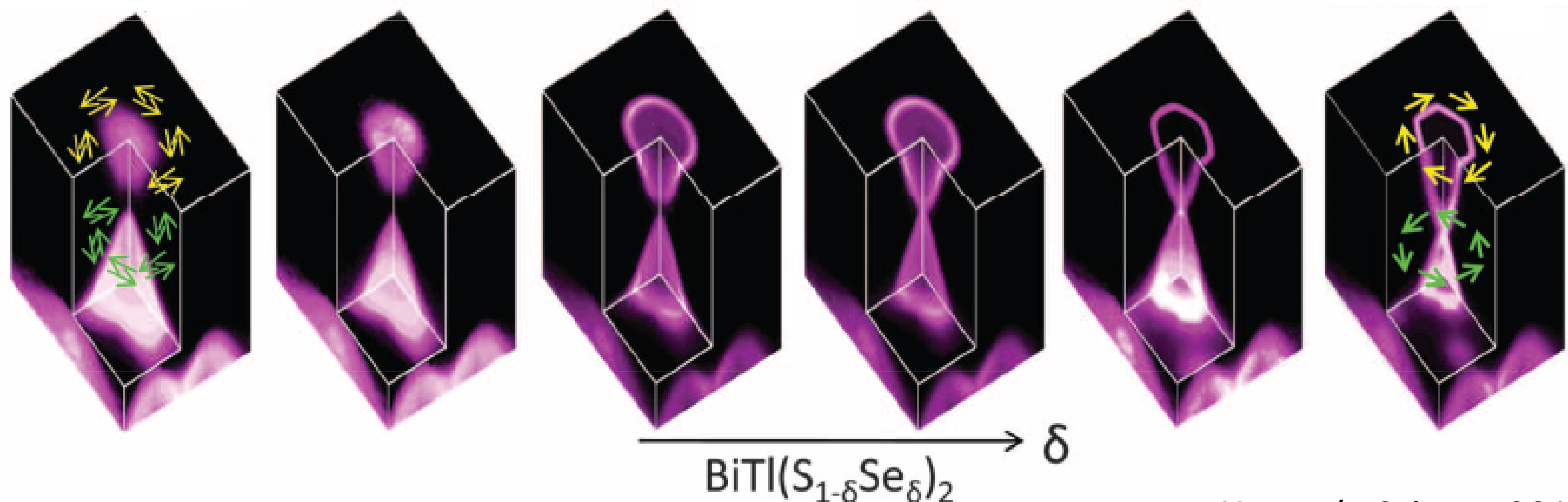
$$(-1)^{\nu_{i=1,2,3}} = \prod_{n_{j\neq i}=0,1;n_i=1} \delta_{n_1 n_2 n_3}.$$

L. Fu, PRL 2007

Inducing the topological state

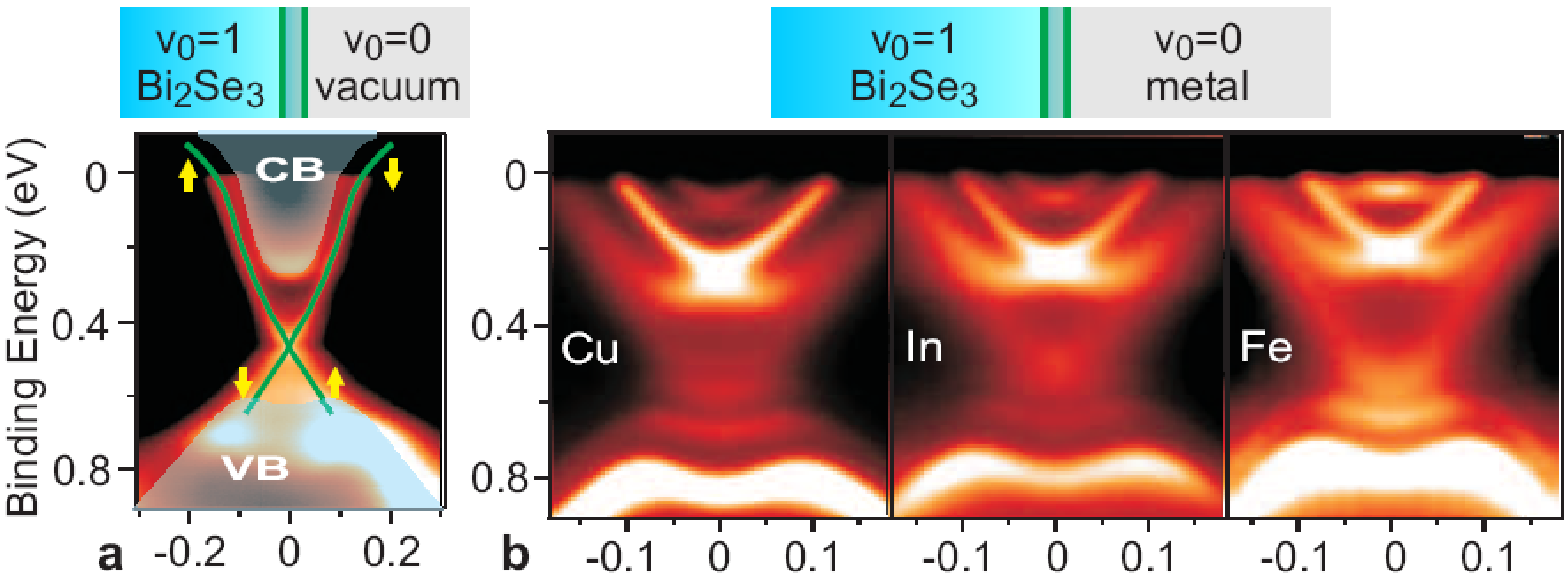
lattice strain and spin orbit coupling



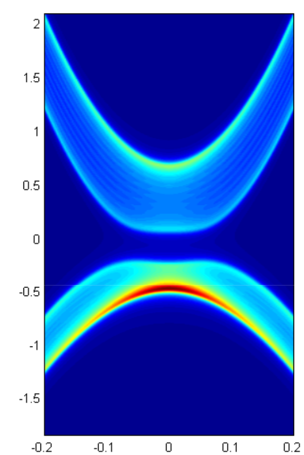


Xu et al., Science 2011

Topological Invariants Define Surface States



Fully gapped
$Bi_2Se_3 \rightarrow Bi_2Se_3$
“interface”



Topological Insulator

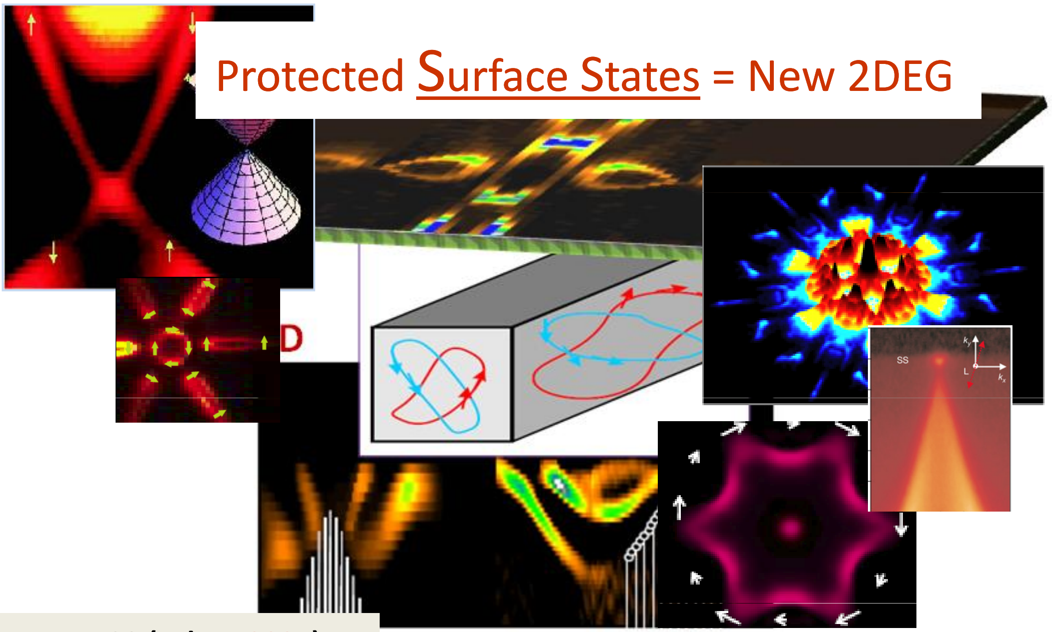
{ν_o} (Chern Parity invariants) Z_2

Quantum Hall Effect

ν (Chern Number): Thouless et.al.,

3D Topological Insulators

Protected Surface States = New 2DEG

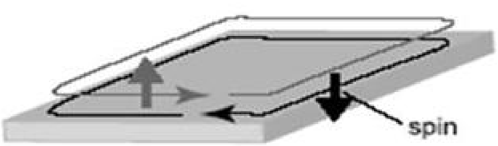


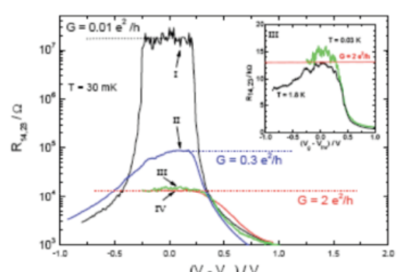
Nature **08 (subm. 2007)**

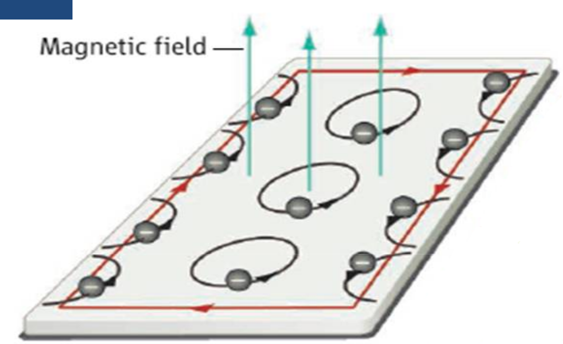
X

2D Topological Insulators

Science **07 (subm. 2007)**





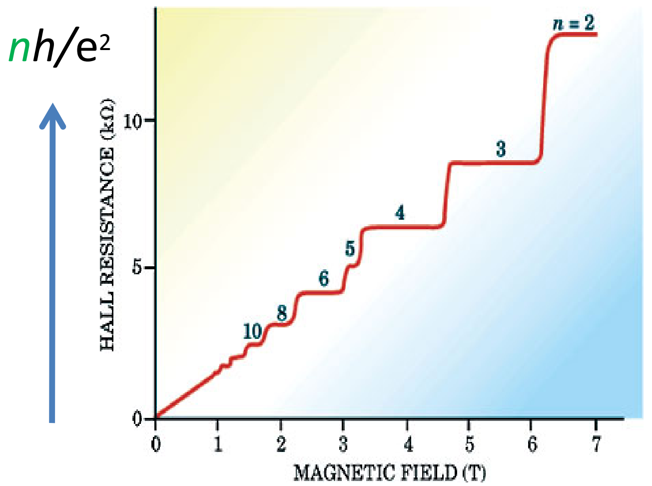


Edge States (1D) by TRS

Chiral Edge States (1D)

Topological Insulator in 2D: Quantum Hall State

Thouless et.al, ('82), (Berry Phase '84)



Hall conductance:

$$\sigma_{xy} = ne^2/h$$

n = Chern no. (Edge states)

Chern : Quantum version (Hilbert space) of Gauss-Bonnet formula

Topological Property

$$n = \frac{1}{2\pi}\int_{BZ}[\nabla_{\mathbf{k}} \times \mathbf{A}(k_x, k_y)]_z d^2\mathbf{k}$$

$$\mathbf{A} = -i\langle u_{\mathbf{k}}|\nabla_k|u_{\mathbf{k}}\rangle$$

TKNN invariant:
Topological Quantum Number

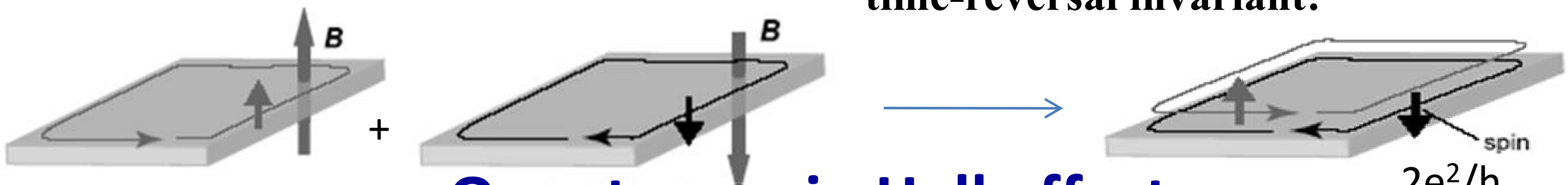
Electron-occupied **Bulk** bands

Finite n → **topologically "protected" edge-states**

Quantum Hall Effect (insulator) : 2D Topological insulator w/ LL

Haldane model (QAH) : 2D Topological insulator w/o LL

time-reversal invariant!



$2e^2/h$

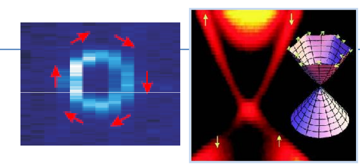
1 invariant → QSHE

Quantum spin Hall effect

Kane & Mele (05a), Kane & Mele (05b) [$\sigma_{spinHall}$ Not quantized]

Bernevig, Hughes, Zhang (06), Sheng, Haldane et.al., (06)

Expt: Molenkamp group HgCdTe-QWells, Science (2007)



4 Invariants → 3D TI

Distinct Topological state in 3D Topo Insulator

Moore & Balents(07), Fu & Kane(07), Fu, Kane &Mele(07), Roy (2009)

Expt: MZH group **Bi-based Semiconductors, KITP Proc.(2007)**

Nature 2008 [Submitted in 2007]

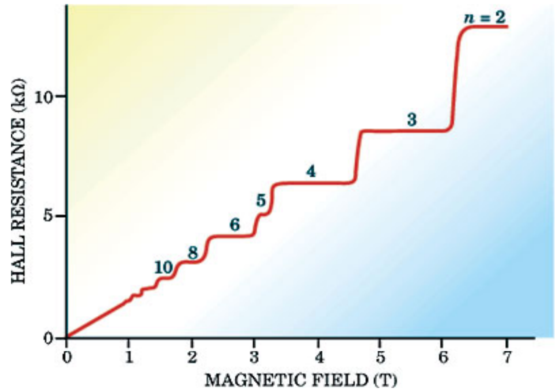
3D TIs -> Superconductors and Magnets (Tc)

Many others afterwards, ~ 800 papers on arXiv

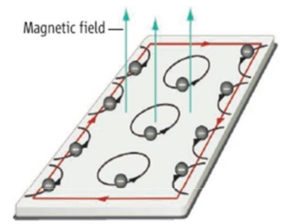
QHE phases

$\sigma_{xy} = \mathbf{n}e^2/h$

Topological quantum number



Transport



Topo Insulators

$\nu_o = \Theta/\pi$

$\Theta=\pi$ (odd)

$\Theta=2\pi$ (even)

No quantized transport

via : $\{\nu_i\}$

Topological quantum number

?

How to experimentally "measure" the topological quantum numbers (ν_i) ?

4 TQNs → **16** distinct insulators

$\{\nu_0, \nu_1\ \nu_2\ \nu_3\}$

Topological "Order Parameters"

Spin-sensitive

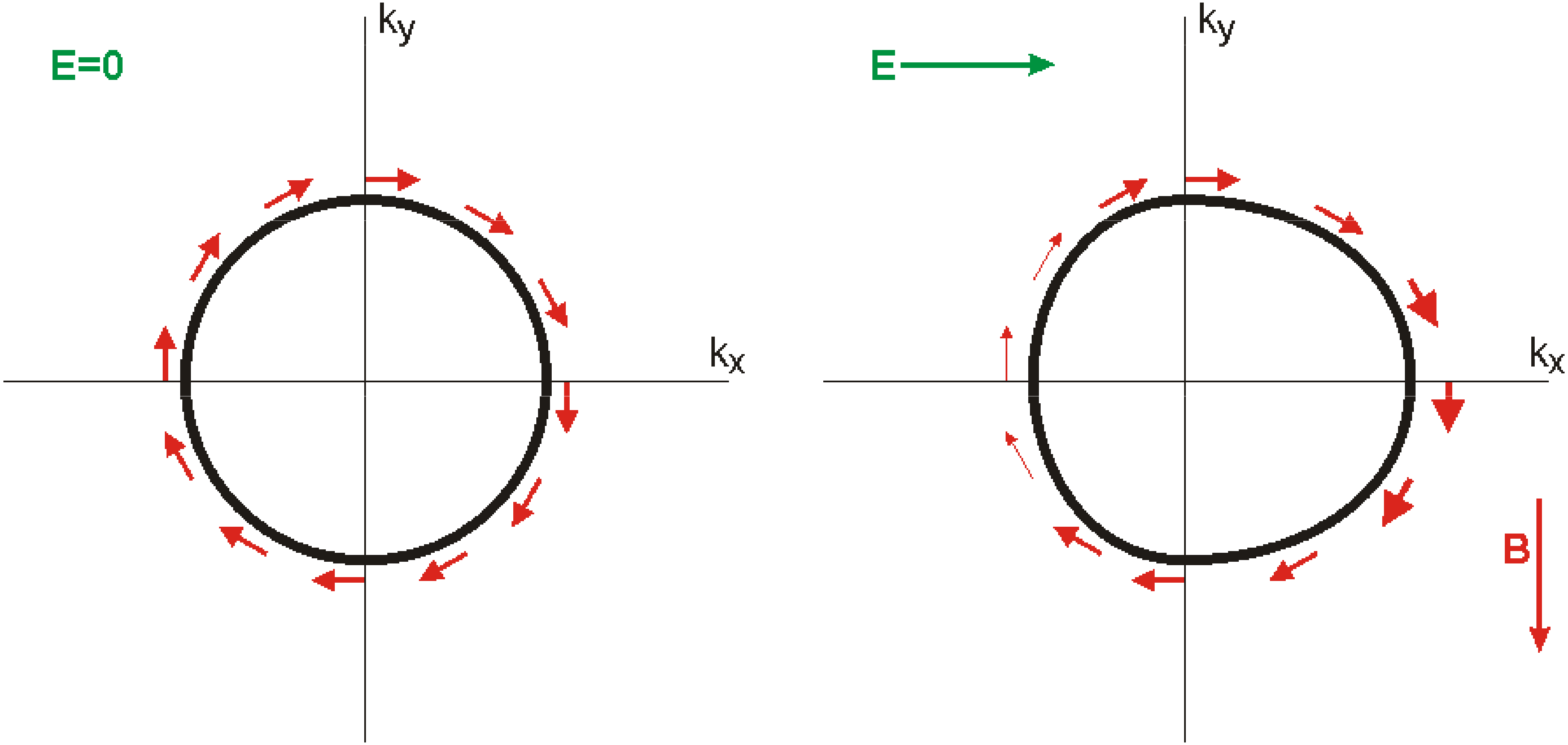
Momentum-resolved

Edge vs. Bulk

So what can they do?

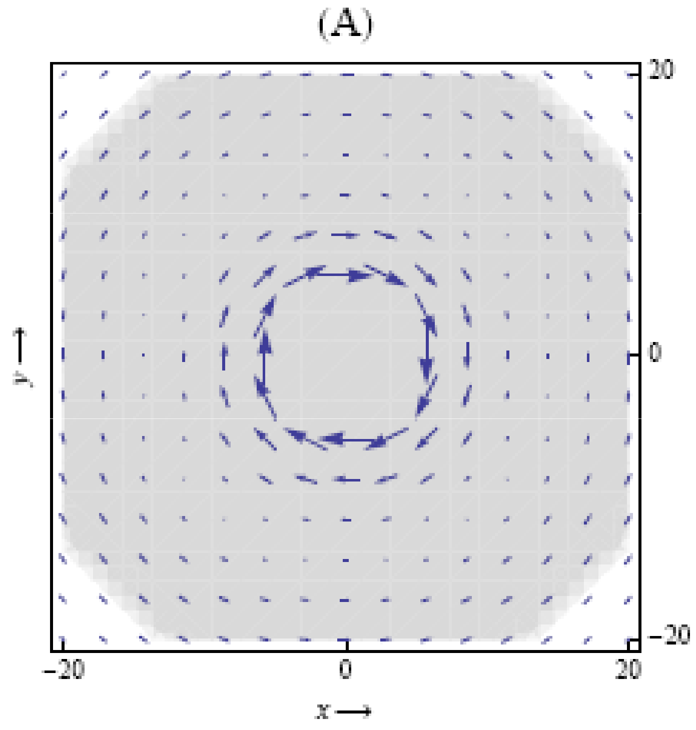
Magnetoelectric Effects

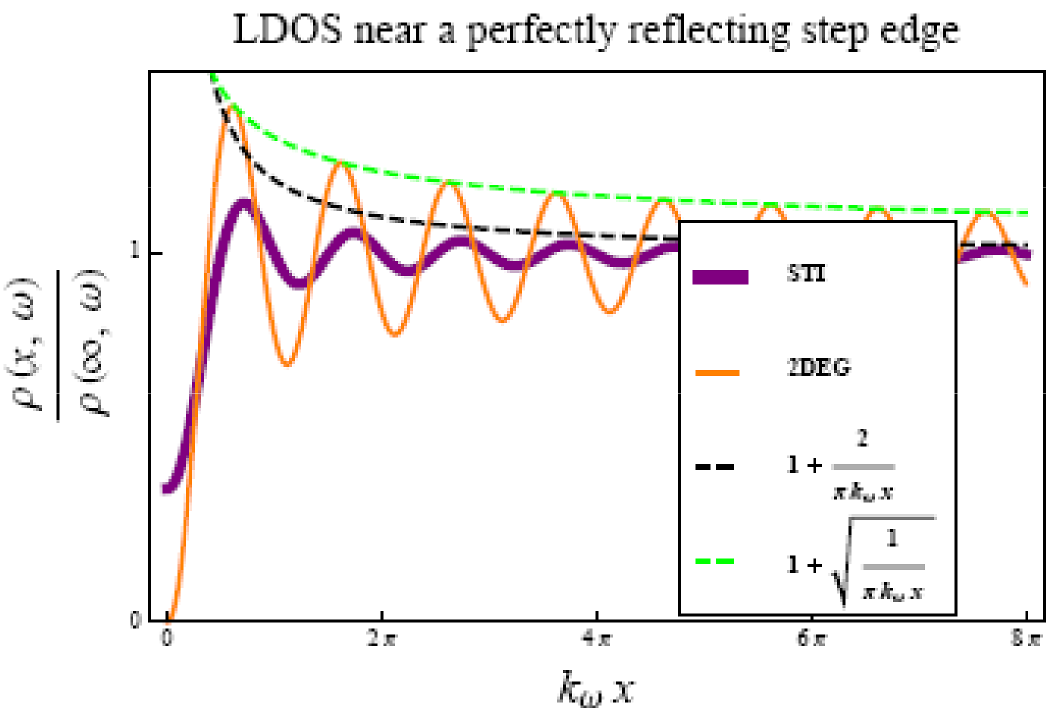
(not "Electromagnetic")



$$S_\theta = \frac{\theta}{2\pi}\frac{\alpha}{2\pi}\int d^3x dt \mathbf{E}\cdot\mathbf{B}$$

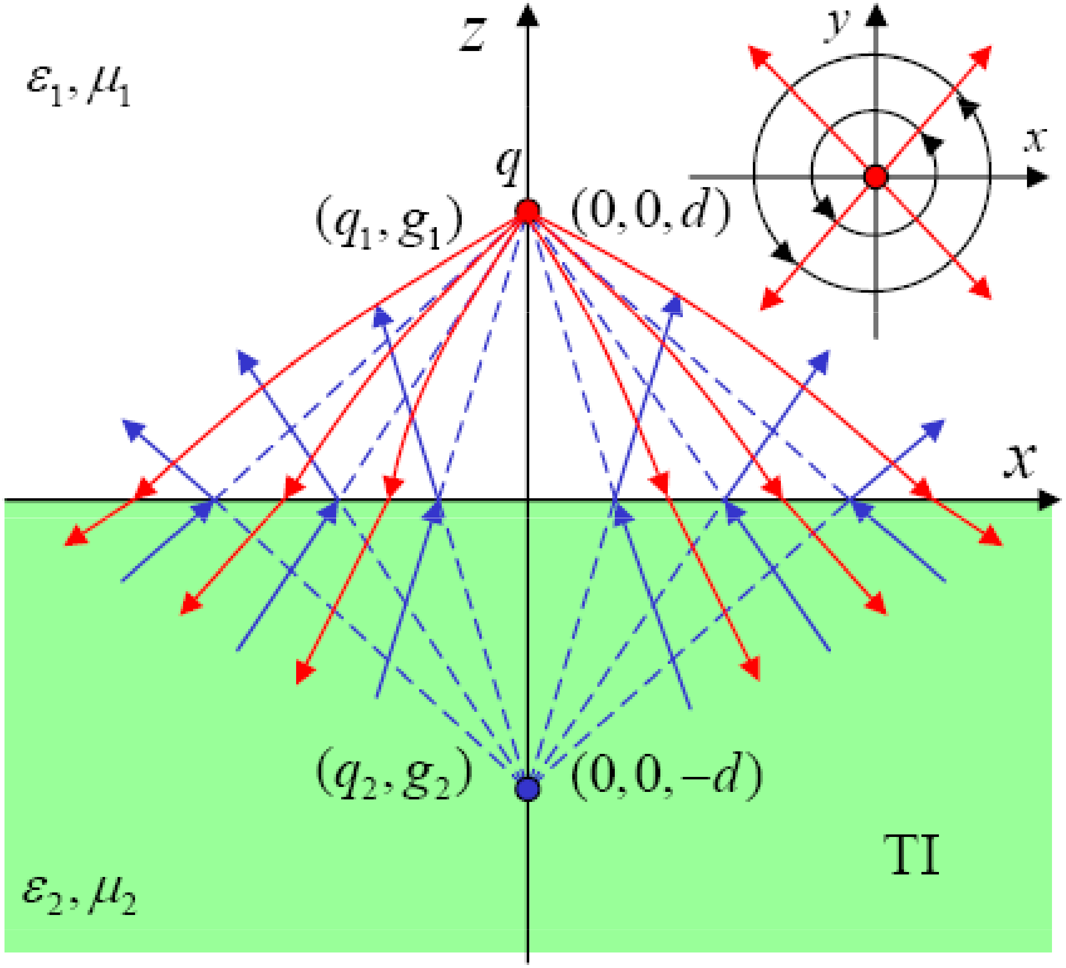
Spin helical states meeting defects

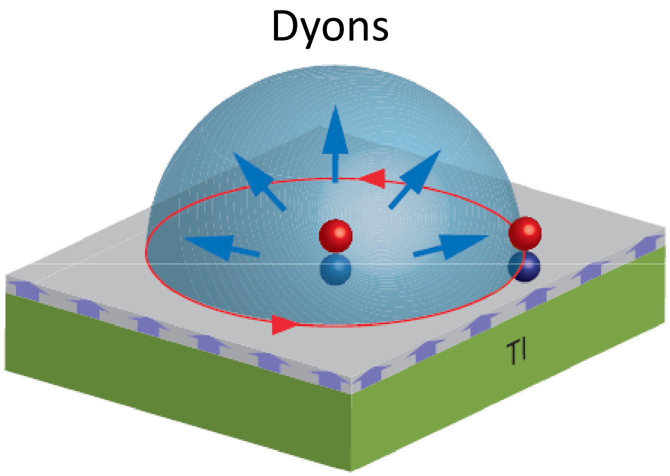




Biswas PRB 2010; arXiv 2010

Local Magnetic Monopoles

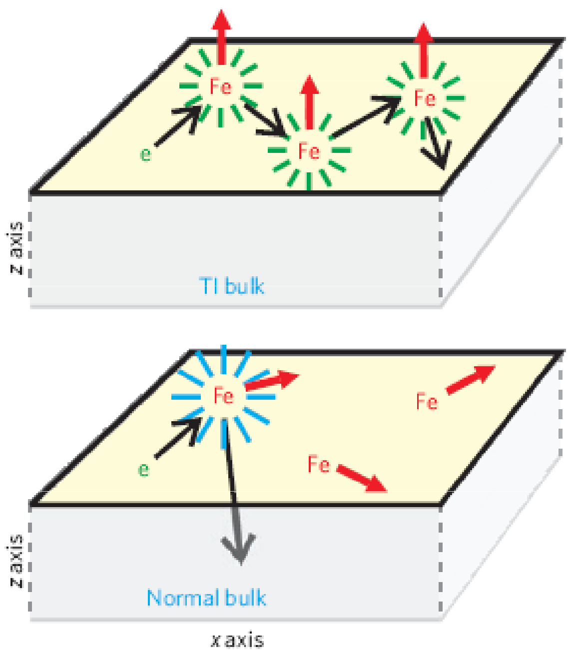


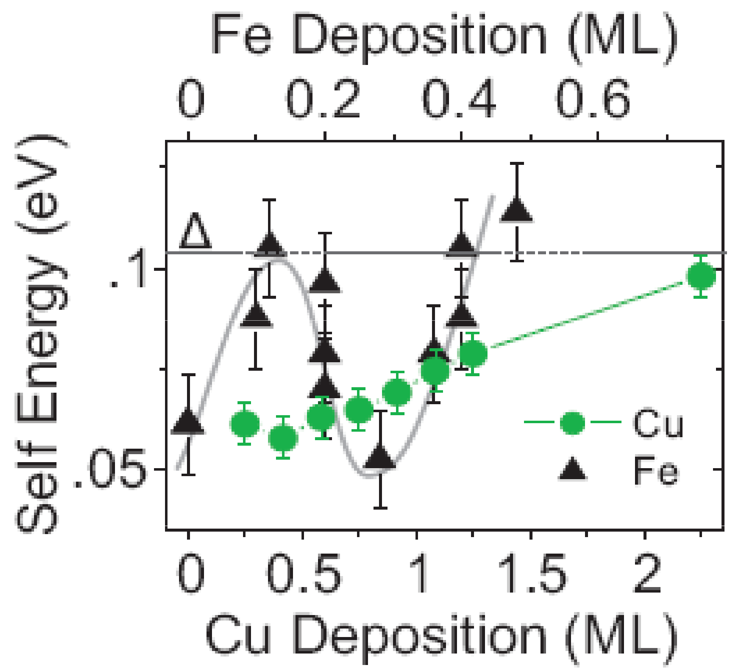


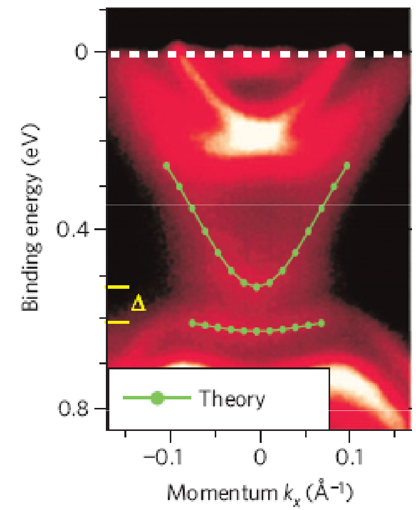
Magnetic order creates effective "axions"

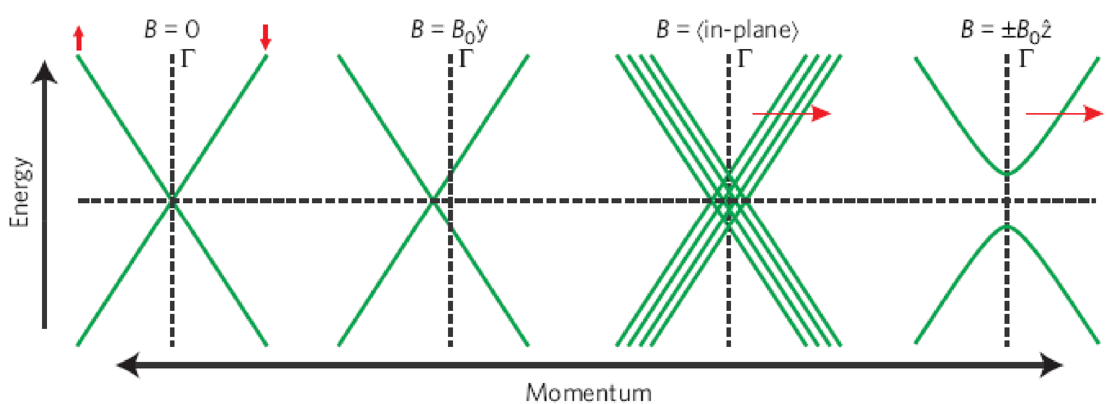
X.-L. Qi, Science 2009

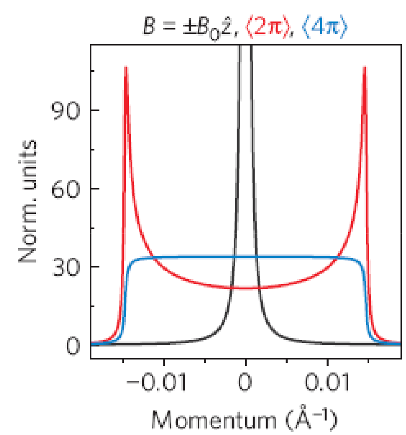
Possibilities for Surface Magnetism











$Cu_xBi_2Se_3$ (T_c ~ 3.8K) : **Hor et.al., PRL 2010**

A Majorana Platform

nature physics

LETTERS

PUBLISHED ONLINE: 19 SEPTEMBER 2010 | DOI: 10.1038/NPHYS1762

Topological Surface States: Superconductivity in doped topological insulators

Wray et.al., Nature Physics (2010)

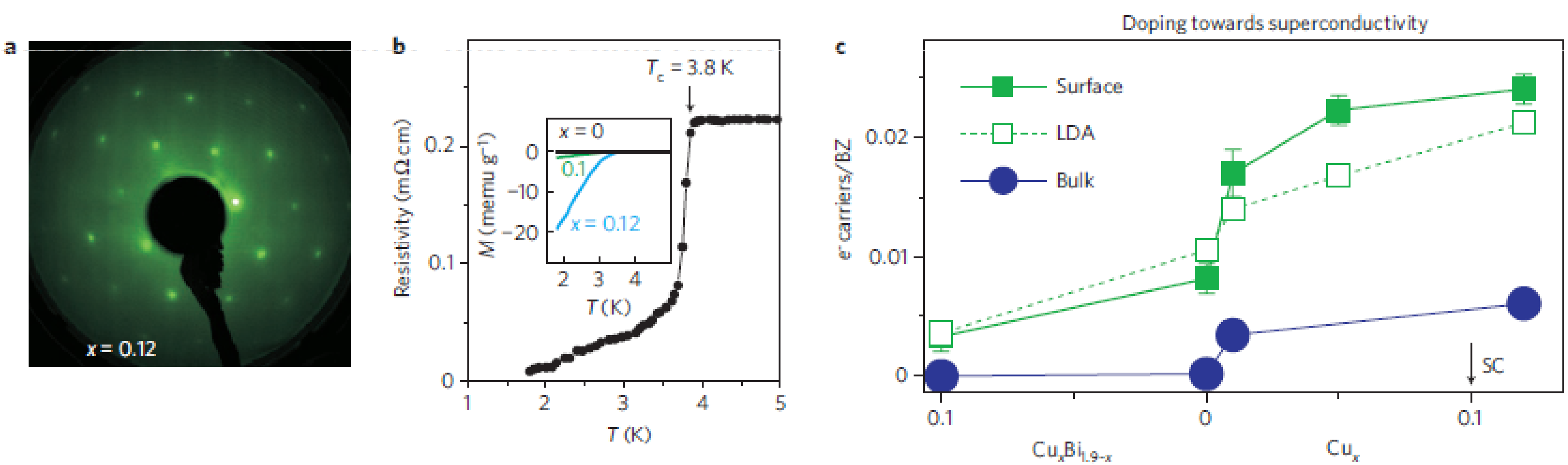
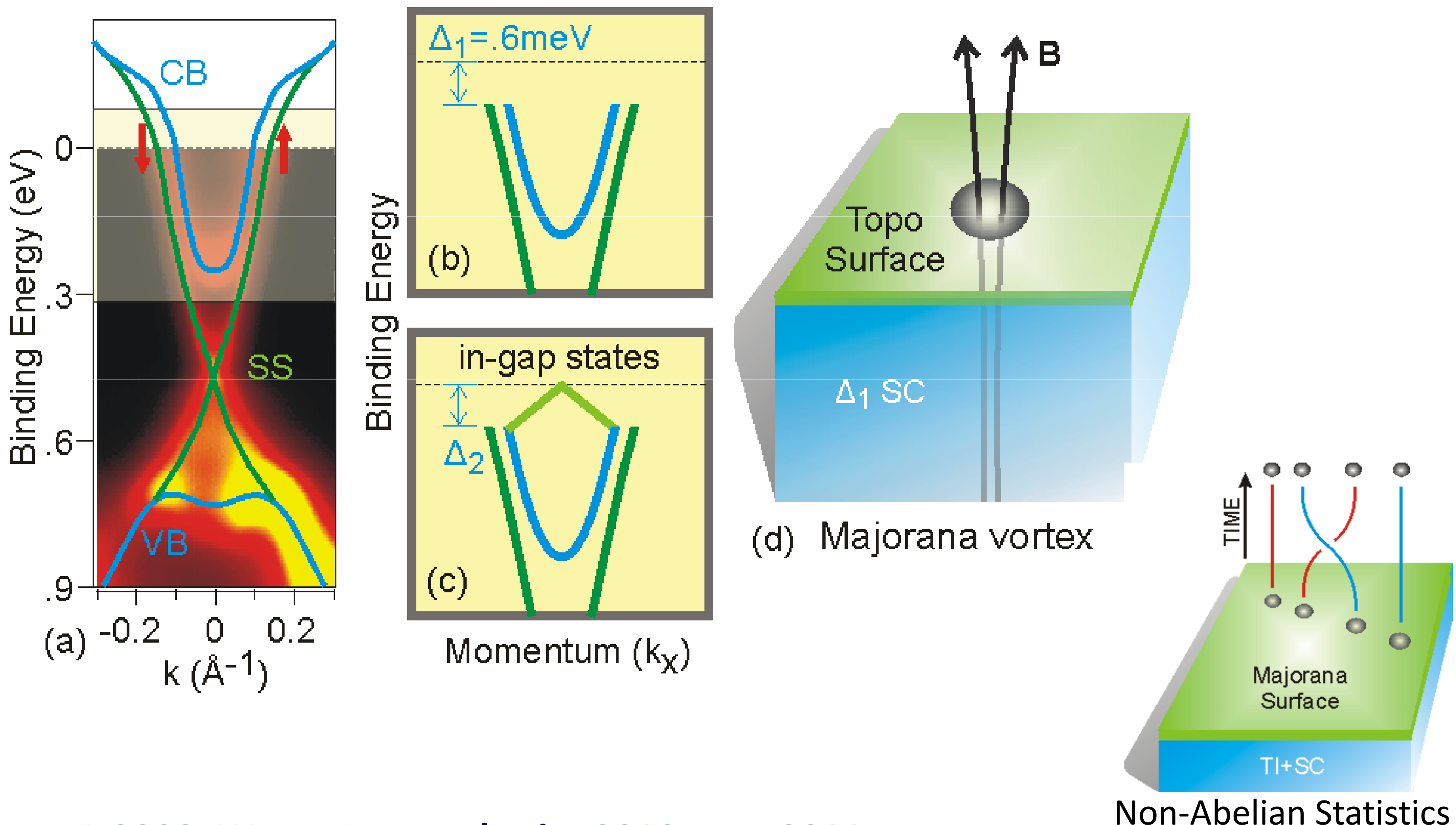


Figure 1 | Superconductivity in $Cu_xBi_2Se_3$ crystals. a, A low-energy electron diffraction image taken at 200 eV electron energy provides evidence for a well-ordered surface with no sign of superstructure modulation. **b**, Resistivity and magnetic susceptibility measurements for samples used in this study. Samples exhibit a superconducting transition at 3.8 K at optimal copper doping ($x = 0.12$). **c**, The number of charge carriers is calculated from the Luttinger count (Fermi surface area/Brillouin zone (BZ) area, ×2 for the doubly degenerate bulk band). Local density approximation (LDA) predictions show the

Majorana Fermions

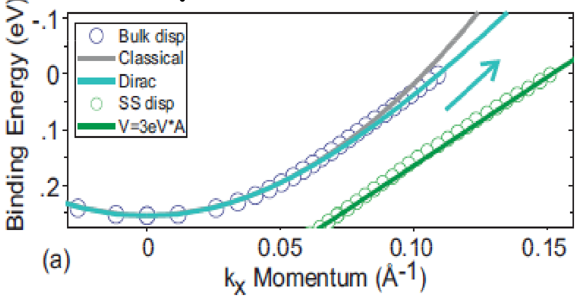


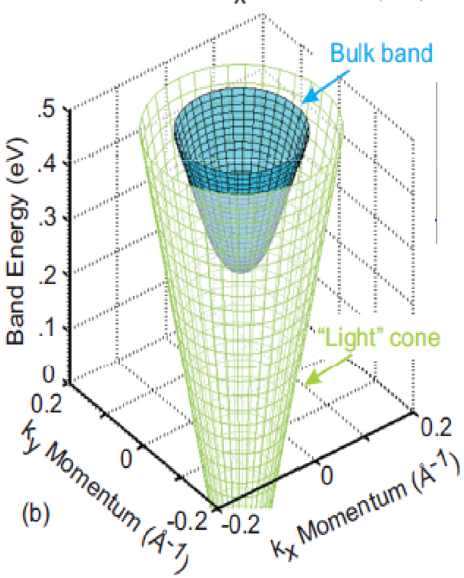
Fu, PRL 2008; Wray, Nature Physics 2010; PRB 2011

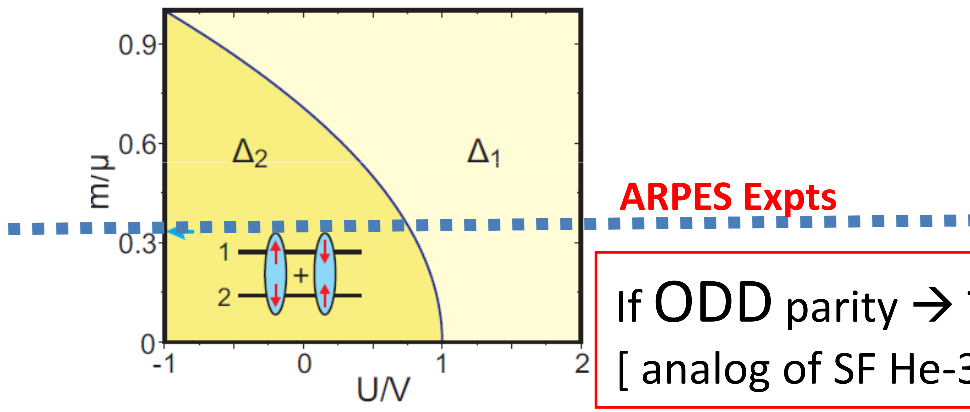
Topological Superconductor (TSC)?

Kitaev/Ludwig D3 class of TSC (proposed by Fu & Berg 09)

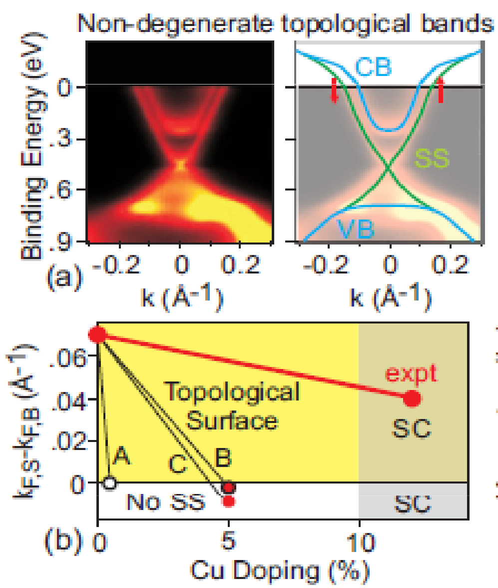
m/μ from ARPES

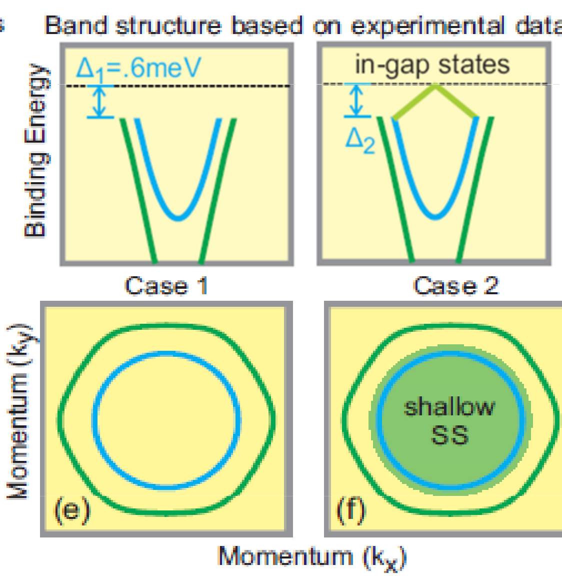






If **ODD** parity → TSC
[analog of SF He-3(B)]



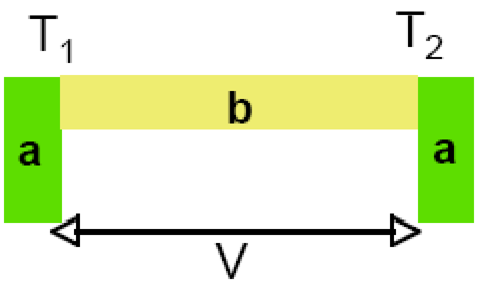


Wray et.al., Nature Physics (2010)

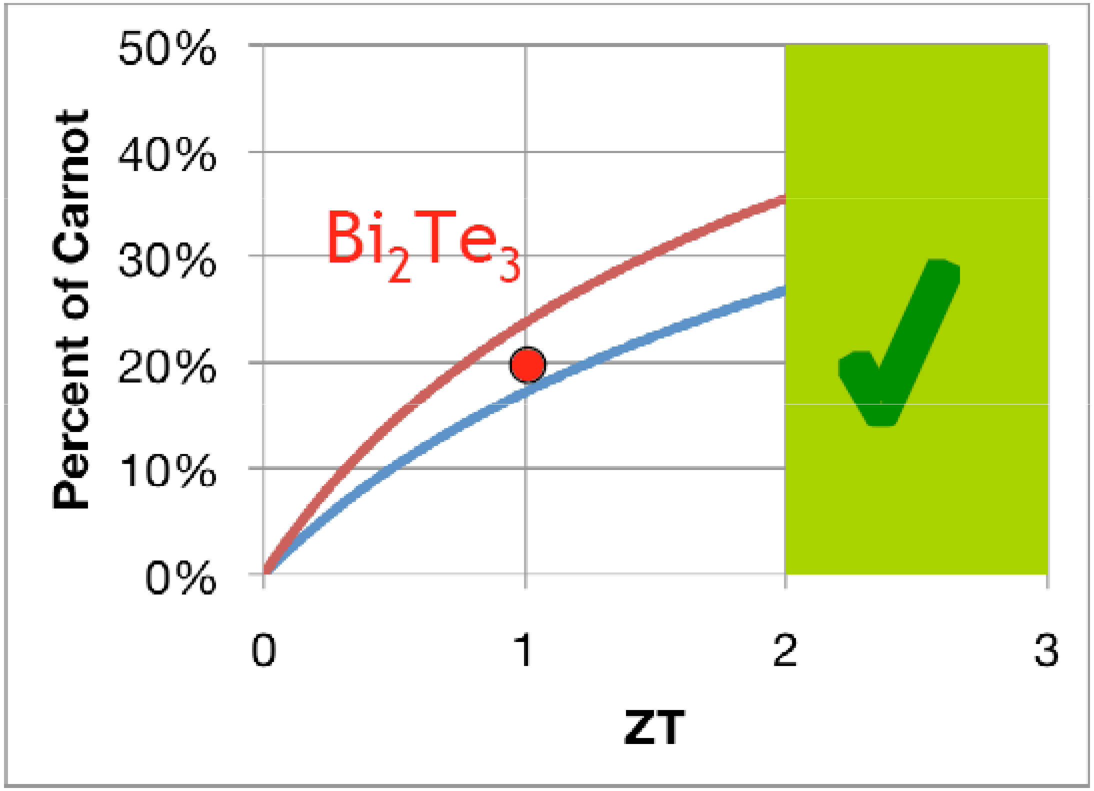
Topological insulators and energy

What makes a material a good thermoelectric?
The "thermoelectric figure of merit" *ZT* determines Carnot efficiency:

$$ZT = \frac{S^2 \sigma T}{k}$$



$S = V / \Delta T$
"Seebeck coefficent"



Slide from J. Moore

Topological insulators are:

Simple

- Exact non-interacting models (DFT, k.p)
- Most complexity reduces to 1D (much nicer than cuprates!!!)
- Surface is robust against non-magnetic scattering

Complicated

- Theory is difficult to learn, and few people know it
- Many surface instabilities, particularly because they occupy the same orbitals as bulk
- Lots of new phases and new physics to explore (Majorana Fermions, Dyons, magnetoelectric effect, unusual surface transport, unusual interface physics)
- Lots of different compounds! (Tl chalcogenides, Heuslers, M2X3, etc)
- Many "simple" issues are actually complicated:
  - -2nd order backscattering is allowed from Anderson impurities
  - -Self energy is poorly understood

Thanks !