Introduction to microscopic theory of spin transport

多々良 源
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Outline

1. Field description, Green's function method
2. Spin current
3. Spin 'conservation' law
4. Spin relaxation
5. Spin transport equation (semiclassical)
6. Semiclassical vs. quantum
7. Spin pumping + Inverse spin Hall effect
8. Maxwell's equation in spin transport
9. Monopole in spintronics
10. Summary

Classical particle to a field

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  - $p = mv$
  - Hamiltonian $\boxed{H = \frac{p^2}{2m} + V(r)}$

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    $\hbar = 1.1 \times 10^{-34}$Js
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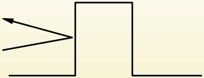
  - Annihilation and creation operators
    $a(r,t)$ and $a^\dagger(r,t)$
  - Particle number $n = a^\dagger a$ $[n, a^\dagger] = 1$
  - $p \to a^\dagger(-i\hbar\nabla)a$
  -

$$\boxed{H = \int d^3r a^\dagger(r,t)\left(-\frac{\hbar^2\nabla^2}{2m} + V(r)\right)a(r,t)}$$

Classical particle to a field



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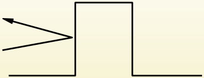

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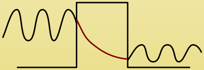

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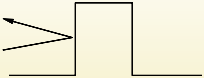

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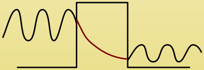

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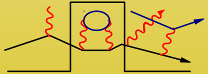

Field description

- Bose field
  - Commutation relation
    $[a, a^\dagger] = aa^\dagger - a^\dagger a = 1$
  - n-particle state : $|n\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}}|0\rangle$ $\quad n = 0, 1, 2, \cdots$
  - Photon, phonon, Spin wave, $\cdots$
- Fermi field
  - Anticommutation relation
    $\{c, c^\dagger\} = cc^\dagger + c^\dagger c = 1,\ a^2 = (a^\dagger)^2 = 0$
  - states : $|0\rangle, |1\rangle = c^\dagger|0\rangle$ $\quad n = 0, 1$
  - Electron

Field description

- Interaction
  - Photon emission in solid

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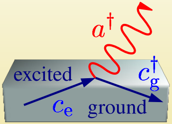
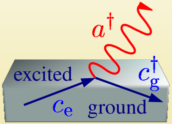
    Electron state changes excited → ground state



Field description

- Interaction
  - Photon emission in solid
    Electron state changes excited $\rightarrow$ ground state
  - Interaction Hamiltonian g: coupling constant



$$\widehat{H}_{\mathrm{e-p}} = \int d^3r\, g \left(c_{\mathrm{g}}^\dagger c_{\mathrm{e}} a^\dagger + c_{\mathrm{e}}^\dagger c_{\mathrm{g}} a \right)$$

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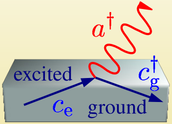

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- Time evolution of state $|\Psi\rangle$
  - "Schrödinger's equation"

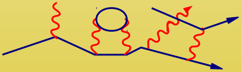
$$i\hbar \frac{\partial}{\partial t}|\Psi\rangle = \widehat{H}|\Psi\rangle$$

  - Solution

$$|\Psi(t)\rangle = \mathrm{T} e^{-\frac{i}{\hbar}\int_0^t dt' \widehat{H}(t')}|\Psi(0)\rangle$$

$$|\Psi(t)\rangle = e^{-\frac{i}{\hbar}\widehat{H}t}|\Psi(0)\rangle$$

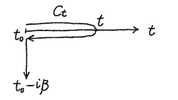
(Time-independent H)



Physical observable

- Observable

Quantum and thermal average



$$O(t) = \frac{1}{Z}\sum_\alpha e^{-\beta E_\alpha}\langle\alpha(t)|\hat{O}|\alpha(t)\rangle$$

$$= \frac{1}{Z}\sum_\alpha \langle\alpha|e^{-\beta\hat{H}}e^{\frac{i}{\hbar}\hat{H}t}\hat{O}e^{-\frac{i}{\hbar}\hat{H}t}|\alpha\rangle = \frac{1}{Z}\sum_\alpha\langle\alpha|e^{-\beta\hat{H}}\hat{O}_{\mathrm{H}}(t)|\alpha\rangle$$

$\hat{O}$: Operator

Electron density $c^\dagger c$, spin density $c^\dagger\sigma c$...

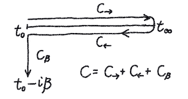
$\hat{O}_{\mathrm{H}}(t) = e^{i\hat{H}t}Oe^{-i\hat{H}t}$: Heisenberg representation

$e^{-\beta\hat{H}}$: Boltzmann weight $\quad \beta = \frac{1}{k_B T}$

α: label of energy eigenstate

- Path ordering

$$O(t) = \frac{1}{Z}\sum_\alpha\langle\alpha|\mathrm{T}_C e^{-\frac{i}{\hbar}\int_C dt'\hat{H}}\hat{O}(t)|\alpha\rangle$$



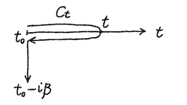
T_C: Path ordering along time contour C

Systematic equation (Dyson's equation)

Physical observable

- Electron charge $\rho = \langle c^\dagger c \rangle$

$$\rho(t) = \frac{1}{Z} \sum_\alpha \langle \alpha | \mathrm{T}_C e^{-\frac{i}{\hbar}\int_C dt' \widehat{H}} c^\dagger(t) c(t) | \alpha \rangle$$



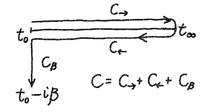
- Extension to different time Path-ordered Green's function

$$G(\tau, \tau') \equiv -i \frac{1}{Z} \sum_\alpha \langle \alpha | \mathrm{T}_C e^{-\frac{i}{\hbar}\int_C d\tau_1 \widehat{H}} c(\tau) c^\dagger(\tau') | \alpha \rangle$$

$$\equiv -i \langle\!\langle \mathrm{T}_C e^{-\frac{i}{\hbar}\int_C d\tau_1 \widehat{H}} c(\tau) c^\dagger(\tau') \rangle\!\rangle = -i \langle\!\langle \mathrm{T}_C c_{\mathrm{H}}(\tau) c^\dagger_{\mathrm{H}}(\tau') \rangle\!\rangle$$

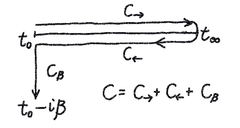
$\tau, \tau' \in C$

$c_{\mathrm{H}}(\tau) = e^{i\widehat{H}t} c e^{-i\widehat{H}t}$: Heisenberg operator



Keldysh (non-equilibrium) Green's function

- Ordering of the operators on C



$$G(\tau, \tau') = -i\langle\langle \mathrm{T}_C c_{\mathrm{H}}(\tau) c_{\mathrm{H}}^\dagger(\tau') \rangle\rangle$$

$$\Rightarrow \begin{cases} -i\langle\langle \mathrm{T} c_{\mathrm{H}}(t) c_{\mathrm{H}}^\dagger(t') \rangle\rangle = G^{\mathrm{t}}(t,t') & (\tau, \tau' \in C_{\rightarrow}) \\ \text{Time-ordered} & \\ -i\langle\langle \overline{\mathrm{T}} c_{\mathrm{H}}(t) c_{\mathrm{H}}^\dagger(t') \rangle\rangle = G^{\bar{\mathrm{t}}}(t,t') & (\tau, \tau' \in C_{\leftarrow}) \\ \text{Anti time-ordered} & \\ -i\langle\langle c_{\mathrm{H}}^\dagger(t') c_{\mathrm{H}}(t) \rangle\rangle = G^{<}(t,t') & (\tau \in C_{\rightarrow}, \tau' \in C_{\leftarrow}) \\ \text{Lesser (particle density)} & \\ -i\langle\langle c_{\mathrm{H}}(t) c_{\mathrm{H}}^\dagger(t') \rangle\rangle = G^{>}(t,t') & (\tau \in C_{\leftarrow}, \tau' \in C_{\rightarrow}) \\ \text{Greater (hole)} & \end{cases}$$

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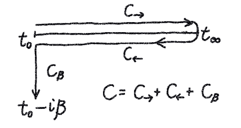

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- $G^{<}(t,t)$: One-particle observable (Electron density) Useful!

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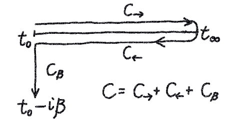

$$G(\tau,\tau') = -i\langle\langle \mathrm{T}_C c_\mathrm{H}(\tau) c_\mathrm{H}^\dagger(\tau')\rangle\rangle$$

$$\Rightarrow \begin{cases} -i\langle\langle \mathrm{T} c_\mathrm{H}(t) c_\mathrm{H}^\dagger(t')\rangle\rangle = G^\mathrm{t}(t,t') & (\tau,\tau' \in C_\rightarrow) \\ \text{Time-ordered} & \\ -i\langle\langle \overline{\mathrm{T}} c_\mathrm{H}(t) c_\mathrm{H}^\dagger(t')\rangle\rangle = G^{\bar{\mathrm{t}}}(t,t') & (\tau,\tau' \in C_\leftarrow) \\ \text{Anti time-ordered} & \\ -i\langle\langle c_\mathrm{H}^\dagger(t') c_\mathrm{H}(t)\rangle\rangle = G^<(t,t') & (\tau \in C_\rightarrow, \tau' \in C_\leftarrow) \\ \text{Lesser (particle density)} & \\ -i\langle\langle c_\mathrm{H}(t) c_\mathrm{H}^\dagger(t')\rangle\rangle = G^>(t,t') & (\tau \in C_\leftarrow, \tau' \in C_\rightarrow) \\ \text{Greater (hole)} & \end{cases}$$

- $G^<(t,t)$: One-particle observable (Electron density) Useful!
- Retarded and advanced Green's functions

$$G_k^\mathrm{r}(t,t') = -i\theta(t-t')\langle\langle\{c_{k',\mathrm{H}}^\dagger(t'), c_{k,\mathrm{H}}(t)\}\rangle\rangle$$

$$G_k^\mathrm{a}(t,t') = i\theta(t'-t)\langle\langle\{c_{k',\mathrm{H}}^\dagger(t'), c_{k,\mathrm{H}}(t)\}\rangle\rangle$$

Free electron Green's function

- Hamiltonian: $$H_0 = -\frac{\hbar^2 \nabla^2}{2m} - \epsilon_F$$
  - Heisenberg equation: $$\frac{\partial}{\partial t} c_{\mathrm{H}}(k,t) = -\frac{i}{\hbar}[H_0, c_{\mathrm{H}}]$$
  - Solution: $$c_{\mathrm{H}}(k,t) = e^{-\frac{i}{\hbar}\epsilon_k t} c(k,0)$$ $$\epsilon_k = \frac{\hbar^2 k^2}{2m} - \epsilon_F$$

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We can calculate anything!

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- Free Green's functions

$$g^<_{k,k'}(t,t') = i\langle\langle c^\dagger_{k',\mathrm{H}}(t') c_{k,\mathrm{H}}(t)\rangle\rangle$$

$$= ie^{-\frac{i}{\hbar}\epsilon_k(t-t')} f_k \delta_{k,k'} \qquad f_k = \frac{1}{e^{\beta\epsilon_k}+1}$$ Fermi distribution

$$g^<_k(\omega) = f_k(g^{\mathrm{a}}_k(\omega) - g^{\mathrm{r}}_k(\omega)) = 2\pi i f_k \delta(\hbar\omega - \epsilon_k)$$ Equilibrium

$$g^{\mathrm{r}}_k(\omega) = \frac{1}{\hbar\omega - \epsilon_k + i0} = [g^{\mathrm{a}}_k(\omega)]^*$$

Perturbation expansion : Attack difficult problem!

- Hamiltonian : $H = H_0 + V$ V: interaction Unsolvable
- Interaction representation Remove solvable part (H_0)

$$O_{\mathrm{H}}(t) = [\bar{\mathrm{T}} e^{\frac{i}{\hbar}\int^t dt_1 V_{H_0}(t_1)}]\; O_{H_0}(t)\; [\mathrm{T} e^{-\frac{i}{\hbar}\int^t dt_1 V_{H_0}(t_1)}]$$

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- Path ordered representation

$$G(\tau,\tau') = -i\langle\langle \mathrm{T}_C e^{-\frac{i}{\hbar}\int_C d\tau_1 V_{H_0}} c_{H_0}(\tau) c^\dagger_{H_0}(\tau')\rangle\rangle$$

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- Expansion w.r.t V_{H_0} — Perturbation expansion

$$G(\tau,\tau') = -i\langle\!\langle \mathsf{T}_C c_{H_0}(\tau) c^\dagger_{H_0}(\tau')\rangle\!\rangle$$

$$+ \frac{(-i)^2}{\hbar}\langle\!\langle \mathsf{T}_C \int_C d\tau_1 V_{H_0}(\tau_1) c_{H_0}(\tau) c^\dagger_{H_0}(\tau')\rangle\!\rangle + \cdots$$

Dyson's equation

Perturbation expansion

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- Expansion w.r.t V_{H_0} $\quad$ Perturbation expansion

$$G(\tau, \tau') = g(\tau, \tau')$$ free Green's function

$$+ \frac{(-i)^2}{\hbar} \langle\!\langle \mathsf{T}_C \int_C d\tau_1 V_{H_0}(\tau_1) c_{H_0}(\tau) c^\dagger_{H_0}(\tau') \rangle\!\rangle + \cdots$$

Dyson's equation

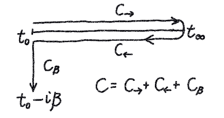
Perturbation expansion

- Example : Potential scattering : $V = \int d^3r\, v(r) c^\dagger c$

$$G(\tau, \tau') = g(\tau, \tau') + \int_C d\tau_1\, g(\tau, \tau_1) v(\tau_1) g(\tau_1, \tau')$$
$$+ \int d\tau_2 \int_C d\tau_1\, g(\tau, \tau_1) v(\tau_1) g(\tau_1, \tau_2) v(\tau_2) g(\tau_2, \tau') + \cdots$$

- Physical quantity (lesser component)

$C = C_\rightarrow + C_\leftarrow$



$$G^<(t, t') = G(\tau \in C_\rightarrow, \tau' \in C_\leftarrow)$$
$$= g^<(t, t')$$
$$+ \int_C d\tau_1\, g(\tau \in C_\rightarrow, \tau_1) v(\tau_1) g(\tau_1, \tau' \in C_\leftarrow)$$
$$+ \cdots$$

Perturbation expansion

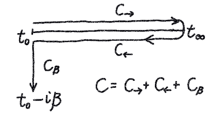
- Example : Potential scattering : $V = \int d^3r\, v(r) c^\dagger c$

$$G(\tau,\tau') = g(\tau,\tau') + \int_C d\tau_1\, g(\tau,\tau_1) v(\tau_1) g(\tau_1,\tau')$$

$$+ \int d\tau_2 \int_C d\tau_1\, g(\tau,\tau_1) v(\tau_1) g(\tau_1,\tau_2) v(\tau_2) g(\tau_2,\tau') + \cdots$$

- Physical quantity (lesser component)

$C = C_{\rightarrow} + C_{\leftarrow}$



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$$+ \cdots$$

$$= g^<(t,t') + \int_{-\infty}^{\infty} dt_1 v(t_1) [g^{\mathrm{r}}(t,t_1) g^<(t_1,t') + g^<(t,t_1) g^{\mathrm{a}}(t_1,t')] + \cdots$$

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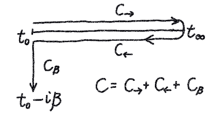

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Expressed by free Green's functions Calculable

Perturbation expansion

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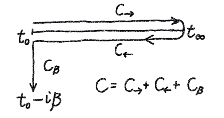

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Expressed by free Green's functions　Calculable

Fully quantum equation $\Leftrightarrow$ Boltzmann equation

Green's function and observable

- Green's function

$$G(r,t,r',t') = -i\langle\langle \mathsf{T}_C c_{\mathsf{H}}(\tau) c^\dagger_{\mathsf{H}}(\tau') \rangle\rangle$$

Space time propagation amplitude

$(r',t') \Rightarrow (r,t)$

including any interaction

- Physical Observable

$\propto \langle c^\dagger(r,t) c(r,t) \rangle$ Particule number

$=$ Lesser component $G^<(r,t,r,t)$

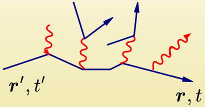
at equal time and position

$(r',t') = (r,t)$

Green's function and observable

- Green's function

$$G(r,t,r',t') = -i\langle\langle \mathrm{T}_C c_{\mathrm{H}}(\tau) c^{\dagger}_{\mathrm{H}}(\tau') \rangle\rangle$$



Space time propagation amplitude

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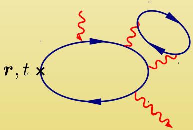

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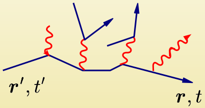

Space time propagation amplitude

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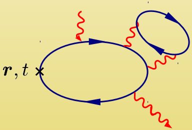

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$=$ Lesser component $G^<(r,t,r,t)$

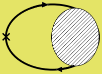
at equal time and position

$(r', t') = (r, t)$

Observable is calculated by estimating electron loop

Spin current : simpleset case

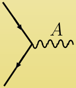
- Definition Without spin-orbit

$$j^{\alpha}_{s\mu} = -i\frac{e\hbar}{2m}\left\langle c^{\dagger}\sigma^{\alpha}\overset{\leftrightarrow}{\nabla}_{\mu}c\right\rangle =$$ 

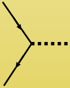
$$= -\frac{e\hbar}{2m}(\nabla_{r} - \nabla_{r'})_{\mu}G^{<}(r,t,r',t)|_{r'=r}$$

- Driving field Electric field $E = -\dot{A}$ A: Vector potential

$$H_E = \int d^3r A\cdot j = \frac{e\hbar}{2m}\int d^3r A\cdot(c^{\dagger}\overset{\leftrightarrow}{\nabla}c)$$

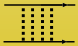


- Scattering by impurities Point-like



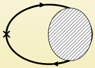
  - Elastic lifetime τ

  - Electron diffusion Multiple scattering



Application to Spin current : simpleset case

- Solution of Dyson's equation

$j^{\alpha}_{s\mu} =$ 

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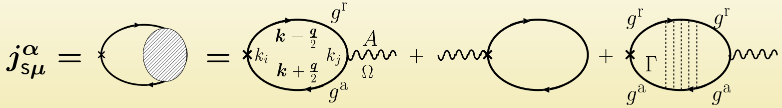
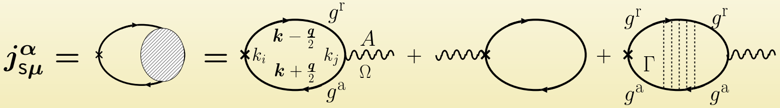


Vertex correction

Application to Spin current : simpleset case

- Solution of Dyson's equation



Vertex correction

$$j^{z}_{s,i} = -i\frac{e^2}{2\pi m^2}\int\frac{d\Omega}{2\pi}\sum_{kq} e^{-i(q\cdot x-\Omega t)}\sum_{j}\Omega A_j(q,\Omega)$$

$$\sum_{\sigma=\pm}\sigma\Big[k_i k_j g^{\mathrm{r}}_{k-\frac{q}{2},\sigma} g^{\mathrm{a}}_{k+\frac{q}{2},\sigma}$$

$$+\sum_{k'\sigma'} k_i k'_j g^{\mathrm{r}}_{k-\frac{q}{2},\sigma} g^{\mathrm{a}}_{k+\frac{q}{2},\sigma} g^{\mathrm{r}}_{k'-\frac{q}{2},\sigma'} g^{\mathrm{a}}_{k'+\frac{q}{2},\sigma'} n_{\mathrm{i}} v_{\mathrm{i}}^2 \Gamma_{\sigma\sigma'}(q,\Omega)\Big]$$

Application to Spin current : simpleset case

- Solution of Dyson's equation

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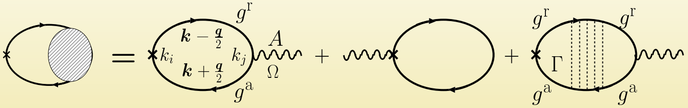

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$$= \boxed{\sigma^0_{\mathrm{s}} E_i + \nabla_i \mu^0_{\mathrm{s}}}$$

$\sigma^0_{\mathrm{s}} = e(D_+\nu_+ - D_-\nu_-) = \sigma_+ - \sigma_-$: Spin conductivity

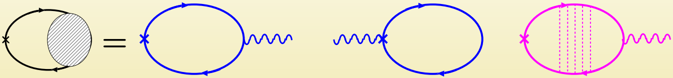
$\mu^0_{\mathrm{s}}(r) = \int d^3r \chi_0(r - r')(\nabla\cdot E)(r')$: Spin chemical potential

$\approx$ spin density $\qquad \chi_0(r) = \frac{\sigma_{\mathrm{s}}}{4\pi r}$: Diffusion

Spin current : simpleset case

- Solution of Dyson's equation

$j^{\alpha}_{s\mu} =$



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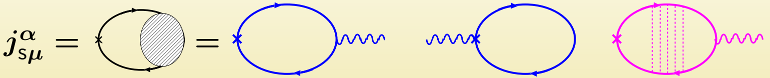

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Spin current : with spin relaxation

- Spin-orbit interaction
  - Quantum relativistic coupling spin and orbital motion
  - Spin relaxation

$$\boldsymbol{H}_{\text{so}} = -\frac{i}{2}\int d^3 r (\nabla v_{\text{so}}) \cdot [c^\dagger (\overleftrightarrow{\nabla} - 2iA) \times \sigma c]$$

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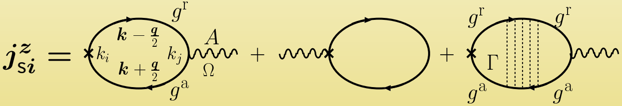
$$H_{so} = -\frac{i}{2}\int d^3r (\nabla v_{so}) \cdot [c^\dagger(\overleftrightarrow{\nabla} - 2iA) \times \sigma c]$$

- Include more diagrams



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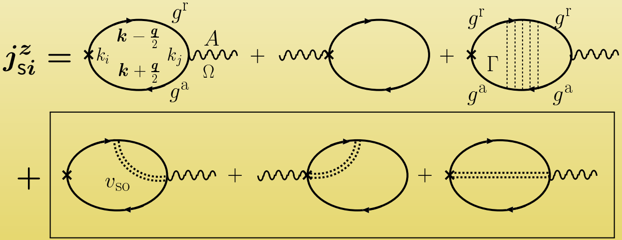

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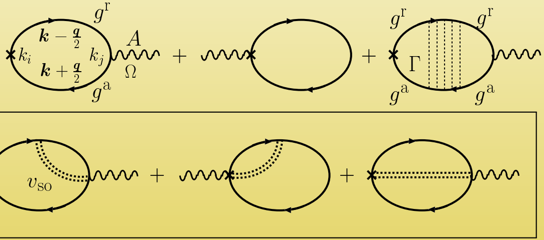

$$= \boxed{\sigma_{\mathrm{s}} E_i + \nabla_i \mu_{\mathrm{s}}}$$

$$\mu_{\mathrm{s}}(r) = \int d^3r\, \chi(r - r')(\nabla \cdot E)(r')$$ Spin chemical potential

$$\chi(r) = \frac{1}{4\pi r}\left(\sigma_+ e^{-\frac{r}{\ell_+}} - \sigma_- e^{-\frac{r}{\ell_-}}\right)$$ Spin diffusion with decay

Spin 'conservation' law

- Continuity equation of spin

field operators

$$\dot{\hat{s}}^{\alpha} = i \left\langle [\widehat{H}, c^{\dagger}] \sigma^{\alpha} c - \text{c.c.} \right\rangle$$

Spin 'conservation' law

- Continuity equation of spin

field operators

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$$\mathcal{T} = i \left\langle c^{\dagger} [\sigma \times (\nabla v_{\mathrm{so}} \times \overleftrightarrow{\nabla})] c \right\rangle$$

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    $\Leftrightarrow \dot{\rho} + \nabla \cdot j = 0$ for charge

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- Spin relaxation torque $\mathcal{T}$
  - Arises from spin-orbit (spin flips scattering)
  - Spin source and sink

Generation and absorption of spin current

Essential for spintronics

Spin relaxation

- Spin relaxation torque $\mathcal{T}$

$$\boxed{\dot{\boldsymbol{s}}^{\boldsymbol{\alpha}} + \boldsymbol{\nabla} \cdot \boldsymbol{j}_{\mathsf{s}}^{\boldsymbol{\alpha}} = \boldsymbol{\mathcal{T}}^{\boldsymbol{\alpha}}}$$

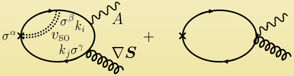
- What is $\mathcal{T}$?

Spin relaxation

- Spin relaxation torque $\mathcal{T}$

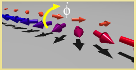
$$\boxed{\dot{s}^{\alpha} + \nabla \cdot j_{\rm s}^{\alpha} = \mathcal{T}^{\alpha}}$$

- What is $\mathcal{T}$?
  - Inhomeneous spin ∇S

$\mathcal{T} =$ 

$$= \boxed{\beta[S \times (j \cdot \nabla)S]}$$

β torque



- Current-driven domain wall motion

GT,Phys.Rep.(2008)

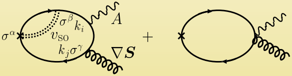
Spin relaxation

- Spin relaxation torque $\mathcal{T}$

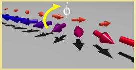
$$\boxed{\dot{s}^{\alpha} + \nabla \cdot j_{\mathrm{s}}^{\alpha} = \mathcal{T}^{\alpha}}$$

- What is $\mathcal{T}$?
  - Inhomeneous spin ∇S

$\mathcal{T} =$



$$= \boxed{\beta[S \times (j \cdot \nabla) S]} \qquad \beta \text{ torque}$$



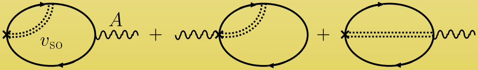
  - Current-driven domain wall motion

GT,Phys.Rep.(2008)

Nakabayashi,PRB(2010)

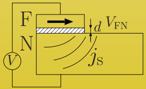
  - Inhomogeneous external field

$\mathcal{T} =$



$$= \boxed{\gamma(\nabla \cdot E)} \qquad \gamma = (D_{+}\tau_{\mathrm{s}+} - D_{-}\tau_{\mathrm{s}-}) \approx \left(\frac{1}{\ell_{\mathrm{s}+}^{2}} - \frac{1}{\ell_{\mathrm{s}-}^{2}}\right)$$

  - Spin injection

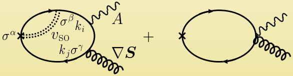


Spin relaxation

- Spin relaxation torque $\mathcal{T}$ arises from inhomogenuity

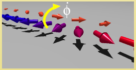
$$\boxed{\dot{s}^{\alpha} + \nabla \cdot j_{\mathrm{s}}^{\alpha} = \mathcal{T}^{\alpha}}$$

- What is $\mathcal{T}$?
  - Inhomeneous spin ∇S

$\mathcal{T} =$ 

$$= \boxed{\beta[S \times (j \cdot \nabla) S]}$$

β torque



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GT,Phys.Rep.(2008)

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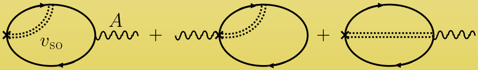
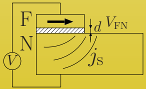
Nakabayashi,PRB(2010)

$\mathcal{T} =$ 

$$= \boxed{\gamma(\nabla \cdot E)}$$

$$\gamma = (D_+ \tau_{\mathrm{s}+} - D_- \tau_{\mathrm{s}-}) \approx \left(\frac{1}{\ell_{\mathrm{s}+}^2} - \frac{1}{\ell_{\mathrm{s}-}^2}\right)$$

  - Spin injection



Spin 'conservation' law

$$\boxed{\dot{\boldsymbol{s}} + \nabla \cdot \boldsymbol{j}_{\mathrm{s}} = \boldsymbol{\mathcal{T}}}$$

Microscopic approach

- Each term is defined and calculable   fully quantum
  No phenomenological parameter
- Continuity equation is automatically satisfied

Spin 'conservation' law

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Microscopic approach

- Each term is defined and calculable fully quantum
  No phenomenological parameter
- Continuity equation is automatically satisfied

  if calculation is correct Consistency check

Spin transport equation (semiclassical)

Valet-Fert approach Valet&Fert'93

- Spin dependent distribution function $f_{\pm}(r,p)$ $\sigma = \pm$: spin
- Transport equation Boltzmann equation

$$(v\cdot\nabla)f_\sigma - (v\cdot E)\frac{df_\sigma}{d\epsilon} = \sum_{v'\sigma'} P_{\sigma'\sigma}(f_{\sigma'}(v') - f_\sigma(v))$$

$P_{\sigma'\sigma}$: Scattering probability with & without spin flip

- Driven part $f_{\pm} = f^0 + \frac{df^0_\sigma}{d\epsilon}(\mu_0 - \mu_\sigma + g_\sigma)$

μ_σ : Spin-dependent chemical potential Spin accumulation

g_σ : Spin current contribution

- Approximation of scattering term

$$P_{-\sigma,\sigma}(f_{-\sigma}(v') - f_\sigma(v)) \Rightarrow \frac{\mu_\sigma - \mu_{-\sigma}}{\tau_{\mathrm{sf}}}$$

τ_{sf} : Spin flip time

- Spin current (due to spin accumulation) Diffusive

$$\nabla\cdot j_{\mathrm{s}} = \frac{\mu_\sigma - \mu_{-\sigma}}{\ell_{\mathrm{s}}^2} \quad j_{\mathrm{s}} = \nabla\mu_{\mathrm{s}}$$

Spin transport equation (semiclassical)

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Phenomenological parameters : μ_σ, g_σ, τ_{sf}, μ_{s},...

Spin transport equation (Semiclassical vs. quantum)

- Valet-Fert equation

  Valet&Fert'93

  Diffusion equation for $\boldsymbol{\mu}_{\mathrm{s}}$

$$\boxed{\nabla^2 \boldsymbol{\mu}_{\mathrm{s}} = \frac{\boldsymbol{\mu}_{\mathrm{s}}}{\ell_{\mathrm{s}}^2}}$$

$$j_{\mathrm{s}} = \nabla \boldsymbol{\mu}_{\mathrm{s}}$$

Semiclassical transport equation is needed to solve for unknown $\boldsymbol{\mu}_{\mathrm{s}}$

Spin transport equation (Semiclassical vs. quantum)

- Valet-Fert equation

  Valet&Fert'93

  Diffusion equation for μ_s $\boxed{\nabla^2 \mu_s = \frac{\mu_s}{\ell_s^2}}$ $\quad j_s = \nabla \mu_s$

  Semiclassical transport equation is needed to solve for unknown μ_s

- Field theory $\boxed{\dot{s} + \nabla \cdot j_s = \mathcal{T}}$

  Each term is directly calculable

  No phenomenological anzatz

$$\boxed{j_s = \sigma_s E \nabla \mu_s} \qquad \boxed{\mathcal{T} = \gamma(\nabla \cdot E)}$$

$$\mu_s(r) = \int d^3 r\, \chi(r - r')(\nabla \cdot E)(r')$$

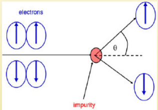
$$\chi(r) = \frac{1}{4\pi r}\left(\sigma_+ e^{-\frac{r}{\ell_+}} - \sigma_- e^{-\frac{r}{\ell_-}}\right)$$

$$\gamma = (D_+ \tau_{s+} - D_- \tau_{s-}) \approx \left(\frac{1}{\ell_{s+}^2} - \frac{1}{\ell_{s-}^2}\right)$$

$$\Rightarrow \left(\nabla^2 - \frac{1}{\ell_s^2}\right) \mu_s = -\sigma_s (\nabla \cdot E)$$

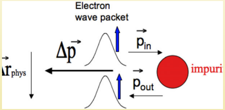
Side jump and skew scattering

Skew scattering



Scattering term $\boldsymbol{F}_{\mathrm{sc}}$

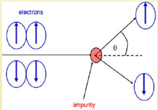
Sidejump



Energy shift $\delta\epsilon$

Side jump and skew scattering

Skew scattering



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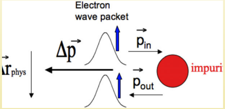
Sidejump



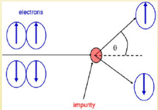
Energy shift $\delta\epsilon$

- Different meaing in Boltzmann equation

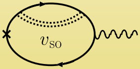
$$\dot{f} + (v \cdot \nabla) f = \boldsymbol{F}_{\text{sc}}$$

Side jump and skew scattering

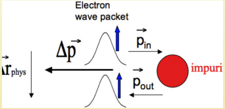
Skew scattering



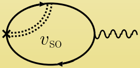
Scattering term $\boldsymbol{F}_{\text{sc}}$



Sidejump



Energy shift $\boldsymbol{\delta\epsilon}$

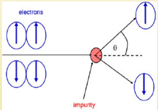


- Different meaing in Boltzmann equation

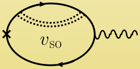
$$\dot{f} + (v \cdot \nabla) f = \boldsymbol{F}_{\text{sc}}$$

Side jump and skew scattering

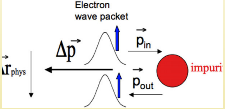
Skew scattering



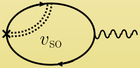
Scattering term $\boldsymbol{F}_{\rm sc}$



Sidejump



Energy shift $\boldsymbol{\delta\epsilon}$



- Different meaing in Boltzmann equation

$$\dot{\boldsymbol{f}} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{f} = \boldsymbol{F}_{\rm sc}$$

- Quantum : both need to be equally treated

Gauge invariance (Charge conservation) $\approx$ Cancellation among diagrams

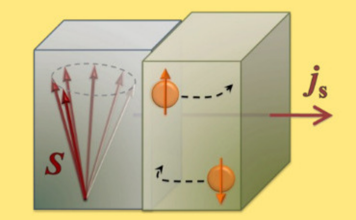
Wrong calculation $\Rightarrow$ Decay of charge !

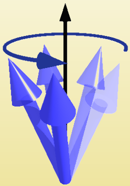
Application to recent spintronics topics

- Spin pumping
- Inverse spin Hall effect

Spin pumping

- Spin current generation from spin dynamics





- Machanism
  Spin continuity equation

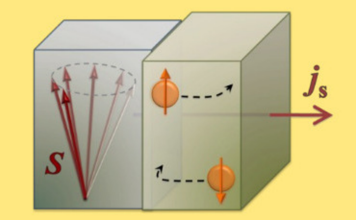
$$\dot{S} + \nabla \cdot j_{\mathrm{s}} = \mathcal{T}$$

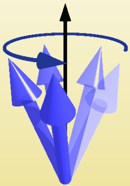
$$\mathcal{T} = \alpha(S \times \dot{S})$$

Spin damping

Spin pumping

- Spin current generation from spin dynamics





- Machanism
  Spin continuity equation

$$\dot{S} + \nabla \cdot j_{\mathrm{s}} = \mathcal{T}$$

$$\mathcal{T} = \alpha(S \times \dot{S})$$

Spin damping

$$\Rightarrow \quad \nabla \cdot j_{\mathrm{s}} = -\dot{S} + \alpha(S \times \dot{S})$$

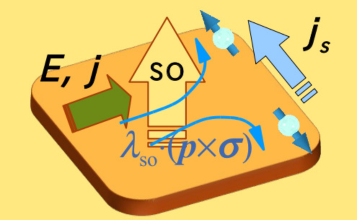
$$\Rightarrow \quad \boxed{j^{\alpha}_{\mathrm{s},i} = a_i \dot{S}^{\alpha} + b_i (S \times \dot{S})^{\alpha}}$$

Silsbee '79, Tserkovnyak '02

Spin dymanics emits spin current

Inverse spin Hall effect

- Spin Hall effect



Coupling between spin and orbital motion by spin-orbit interaction

  - Converts electric current into spin current

- Inverse spin Hall effect

  - Converts spin current into electric current

⇒ Electric detection of spin current

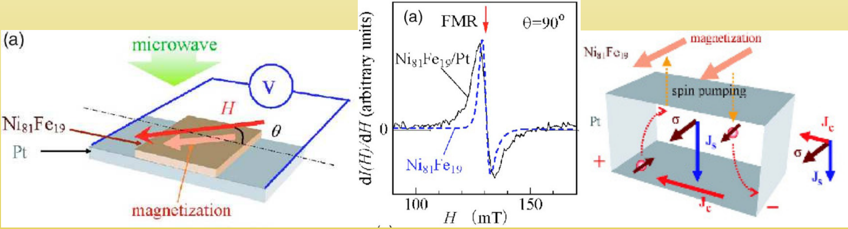
Spin pumping + Inverse spin Hall effect

$\dot{S}$ $\Rightarrow$ j_s $\Rightarrow$ j

NiFe Pt

Spin pumping Inverse spin Hall

- NiFe + Pt Saitoh '06



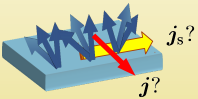
Voltage signal from magnetization precession

Spin pumping + inverse spin Hall effect

Model

- Slowly varying magnetization $\boldsymbol{S}$
- Weak exchange coupling

  $\boldsymbol{S}$ and conduction electron   Perturbation theory



$$H_{sd} = J_{sd} \int d^3 r \boldsymbol{S}(r) \cdot (c^\dagger \boldsymbol{\sigma} c)$$

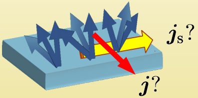
- Disordered metal   Vertex correction

Spin pumping + inverse spin Hall effect

Model

- Slowly varying magnetization $\boldsymbol{S}$
- Weak exchange coupling

$\boldsymbol{S}$ and conduction electron   Perturbation theory



$$\boldsymbol{H_{sd} = J_{sd} \int d^3 r S(r) \cdot (c^\dagger \sigma c)}$$

- Disordered metal   Vertex correction

Questions

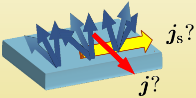
- Does j_s and j generated from magnetization dynamics?

Spin pumping + inverse spin Hall effect

Model

- Slowly varying magnetization S
- Weak exchange coupling

S and conduction electron　　Perturbation theory



$$H_{sd} = J_{sd} \int d^3 r S(r) \cdot (c^\dagger \sigma c)$$

- Disordered metal　　Vertex correction

Questions

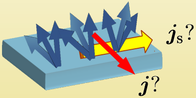
- Does j_s and j generated from magnetization dynamics?
- Is j proportional to j_s ?　　Inverse spin Hall effect?

Spin pumping + inverse spin Hall effect

Model

- Slowly varying magnetization $\boldsymbol{S}$
- Weak exchange coupling

  $\boldsymbol{S}$ and conduction electron   Perturbation theory



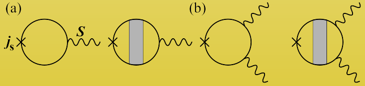
$$H_{sd} = J_{sd} \int d^3 r S(r) \cdot (c^\dagger \sigma c)$$

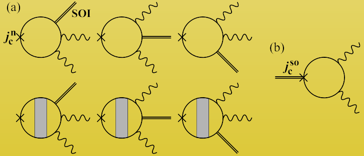
- Disordered metal   Vertex correction

Questions

- Does j_s and j generated from magnetization dynamics?
- Is j proportional to j_s ?   Inverse spin Hall effect?

Calculation

$j_s =$ 

$j =$ 

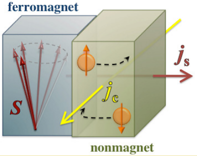
Spin pumping + inverse spin Hall effect

- Spin current (spin pumping)

$$j_s(r) = \nabla \int d^3x' \chi(r - r') \left(\dot{S} - \gamma (S \times \dot{S}) \right)_{r'}$$

$$\chi(r - r') = -J_{sd} \nu \sum_q \frac{e^{-iq\cdot(r-r')}}{q^2}$$

Diffusion



- Electric current (inverse spin Hall)

$$j = -\frac{16 e \nu \lambda J^2 \varepsilon_F \tau^2}{3\hbar^2 V} \nabla \times (S \times \dot{S}) - \frac{4 e \nu \lambda J^2 \tau^2}{\hbar^2 V} E_R \times (S \times \dot{S}) - D \nabla \rho$$

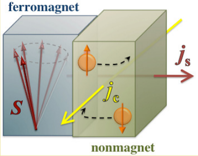
E_R: Rashba spin-orbit interaction   interface

Spin pumping + inverse spin Hall effect

- Spin current (spin pumping)

$$j_{\mathrm{s}}(r) = \nabla \int d^3x' \chi(r - r') \left(\dot{S} - \gamma (S \times \dot{S}) \right)_{r'}$$

$$\chi(r - r') = -J_{\mathrm{sd}} \nu \sum_q \frac{e^{-iq\cdot(r-r')}}{q^2}$$ Diffusion



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E_{R}: Rashba spin-orbit interaction interface

- Spin-charge conversion ?

$$j_i \neq \lambda_{\mathrm{so}} \epsilon_{ijk} j^k_{\mathrm{s},j}$$ No

Spin current picture is not good

may be o.k. at very short distance

Effective electric and magnetic fields

Spin pumping + inverse spin Hall

- Electric current generated

$$j = -\frac{16e\nu\lambda J^2\varepsilon_{\mathrm{F}}\tau^2}{3\hbar^2 V}\nabla\times(S\times\dot{S}) - \frac{4e\nu\lambda J^2\tau^2}{\hbar^2 V}E_{\mathrm{R}}\times(S\times\dot{S}) - D\nabla\rho$$

- Effective electric and magnetic fields

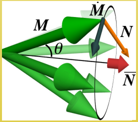
$$j = \frac{1}{\mu}\nabla\times B_{\mathrm{s}} + \sigma_{\mathrm{c}}E_{\mathrm{s}} - D\nabla\rho,$$

$$E_{\mathrm{s}} = -\alpha_{\mathrm{R}}E_{\mathrm{R}}\times N$$
$$B_{\mathrm{s}} = -\beta_{\mathrm{i}}N$$

$N = S\times\dot{S}$ spin damping

$$\alpha_{\mathrm{R}} = \frac{4e\nu\lambda J^2\tau^2}{\sigma_{\mathrm{c}}\hbar^2}$$

$$\beta_{\mathrm{i}} = \frac{16e\nu\mu\lambda J^2\varepsilon_{\mathrm{F}}\tau^2}{3\hbar^2}$$



Maxwell's equation in spin transport

Spin pumping + inverse spin Hall

- Effective electric and magnetic fields

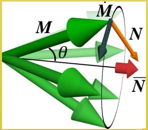
$$\boldsymbol{E}_{\mathrm{s}} = -\alpha_{\mathrm{R}} \boldsymbol{E}_{\mathrm{R}} \times \boldsymbol{N}$$
$$\boldsymbol{B}_{\mathrm{s}} = -\beta_{\mathrm{i}} \boldsymbol{N}$$

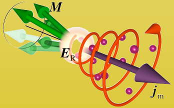
$$\alpha_{\mathrm{R}} = \frac{4e\nu\lambda J^2\tau^2}{\sigma_{\mathrm{c}}\hbar^2}$$
$$\beta_{\mathrm{i}} = \frac{16e\nu\mu\lambda J^2\varepsilon_{\mathrm{F}}\tau^2}{3\hbar^2}$$

- Maxwell's equation

$$\nabla \times \boldsymbol{E}_{\mathrm{s}} + \dot{\boldsymbol{B}}_{\mathrm{s}} = -\boldsymbol{j}_{\mathrm{m}}$$
$$\nabla \cdot \boldsymbol{B}_{\mathrm{s}} = \rho_{\mathrm{m}}$$
$$\nabla \cdot \boldsymbol{E}_{\mathrm{s}} = -\frac{\rho}{\epsilon}$$
$$\nabla \times \boldsymbol{B}_{\mathrm{s}} = \mu \boldsymbol{j} + \epsilon\mu\dot{\boldsymbol{E}}$$

$$\boldsymbol{j}_{\mathrm{m}} = \alpha_{\mathrm{R}} \nabla \times (\boldsymbol{E}_{\mathrm{R}} \times \boldsymbol{N}) + \beta_{\mathrm{i}} \dot{\boldsymbol{N}}$$
$$\rho_{\mathrm{m}} = -\beta_{\mathrm{i}} \nabla \cdot \boldsymbol{N}$$
$$\boldsymbol{N} = \boldsymbol{S} \times \dot{\boldsymbol{S}}$$ spin damping





Maxwell's equation in spin transport

Spin pumping + inverse spin Hall

- Effective electric and magnetic fields

$$\boldsymbol{E}_{\mathrm{s}} = -\alpha_{\mathrm{R}} \boldsymbol{E}_{\mathrm{R}} \times \boldsymbol{N}$$
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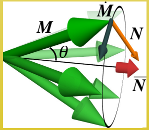
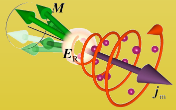
$$\alpha_{\mathrm{R}} = \frac{4e\nu\lambda J^2\tau^2}{\sigma_{\mathrm{c}}\hbar^2}$$
$$\beta_{\mathrm{i}} = \frac{16e\nu\mu\lambda J^2\varepsilon_{\mathrm{F}}\tau^2}{3\hbar^2}$$

- Maxwell's equation

$$\nabla \times \boldsymbol{E}_{\mathrm{s}} + \dot{\boldsymbol{B}}_{\mathrm{s}} = -\boldsymbol{j}_{\mathrm{m}}$$
$$\nabla \cdot \boldsymbol{B}_{\mathrm{s}} = \rho_{\mathrm{m}}$$
$$\nabla \cdot \boldsymbol{E}_{\mathrm{s}} = -\frac{\rho}{\epsilon}$$
$$\nabla \times \boldsymbol{B}_{\mathrm{s}} = \mu \boldsymbol{j} + \epsilon\mu\dot{\boldsymbol{E}}$$

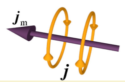
$$\boldsymbol{j}_{\mathrm{m}} = \alpha_{\mathrm{R}} \nabla \times (\boldsymbol{E}_{\mathrm{R}} \times \boldsymbol{N}) + \beta_{\mathrm{i}} \dot{\boldsymbol{N}}$$
$$\rho_{\mathrm{m}} = -\beta_{\mathrm{i}} \nabla \cdot \boldsymbol{N}$$
$$\boldsymbol{N} = \boldsymbol{S} \times \dot{\boldsymbol{S}} \qquad \text{spin damping}$$





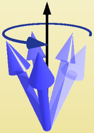
Spin damping monopole

Takeuchi '11



Angular momentum transfer Spin⇒orbital

- Spin damping $N = S \times \dot{S}$



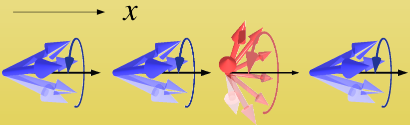
Decay of spin angular momentum

⇓ Spin-orbit interaction

Generation of orbital angular momentum

- Inhomogeneous damping

Spin angular momentum



Spin damping monopole

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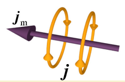
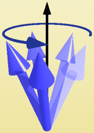


Angular momentum transfer Spin⇒orbital

- Spin damping $N = S \times \dot{S}$



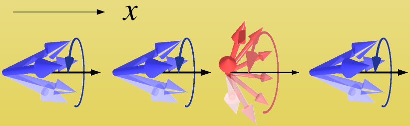
Decay of spin angular momentum

⇓ Spin-orbit interaction

Generation of orbital angular momentum

- Inhomogeneous damping

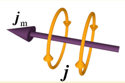
Spin angular momentum



Orbital angular momentum

Spin damping monopole

Takeuchi '11



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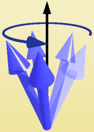

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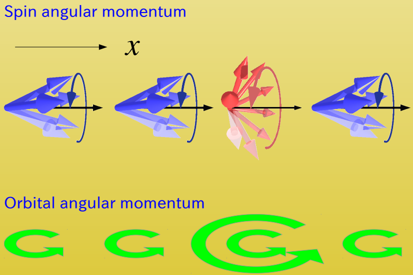
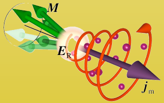


Rotational motion of electron

≃ effective magnetic flux

⇒ Monopole

$$\rho_m = \nabla \cdot N$$



Spin damping monopole

Takeuchi '11

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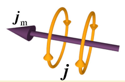
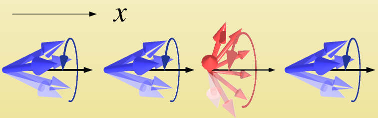


- Inhomogeneous damping

Spin angular momentum



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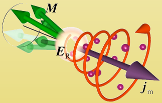

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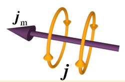

Angular momentum transfer Spin⇒orbital

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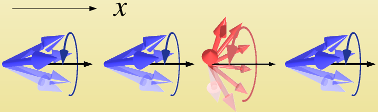
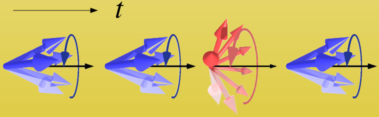
Spin angular momentum



Orbital angular momentum

- Time-dependent damping

Spin angular momentum

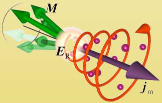


Rotational motion of electron

≃ effective magnetic flux

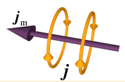
⇒ Monopole

$$\rho_{\rm m} = \nabla \cdot \boldsymbol{N}$$



Spin damping monopole

Takeuchi '11

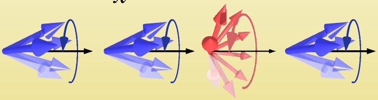


Angular momentum transfer Spin⇒orbital

- Inhomogeneous damping

Spin angular momentum

$\longrightarrow x$

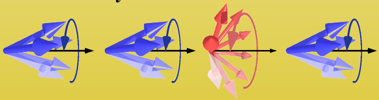


Orbital angular momentum

- Time-dependent damping

Spin angular momentum

$\longrightarrow t$



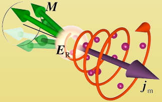
Voltage

Rotational motion of electron

$\simeq$ effective magnetic flux

$\Rightarrow$ Monopole

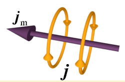
$$\rho_{\rm m} = \nabla \cdot \boldsymbol{N}$$



Spin damping monopole

Takeuchi '11

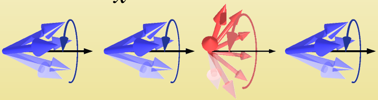
Angular momentum transfer Spin⇒orbital



- Inhomogeneous damping

Spin angular momentum

$\longrightarrow x$



Orbital angular momentum

Rotational motion of electron

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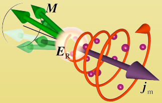
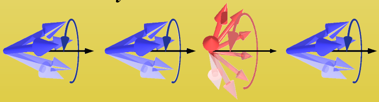


- Time-dependent damping

Spin angular momentum

$\longrightarrow t$



Voltage

Change of magnetic flux

⇒ Voltage, current

≃ Monopole current

$$j_{\mathrm{m}} = \dot{\boldsymbol{N}}$$

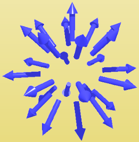
$$\nabla \times \boldsymbol{E} = -\dot{\boldsymbol{B}} + j_{\mathrm{m}}$$

Monopoles

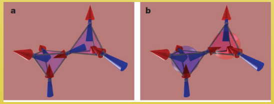
- Dirac's monopole — Dirac'31
  - String singularity
- Grand unified theory monopole — 't Hooft, Polyakov, '74
  - Symmetry breaking — SU(5) $\rightarrow$ U(1)
  - $E \gtrsim 10^{17}$ Gev
- Hedgehog monopole — Volovik '87
  - Ferromagnetic metal — SU(2) $\rightarrow$ U(1)

$$\rho_{\mathrm{H}} = -\epsilon_{ijk} \nabla_i S \cdot (\nabla_j S \times \nabla_k S)$$

$$j_{\mathrm{H},i} = \epsilon_{ijk} \dot{S} \cdot (\nabla_j S \times \nabla_k S)$$



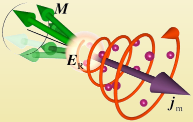
- Spin ice monopole — Castelnovo'08
  - Frustrated spin
  - Fictitious magnetic charge (?)
  - Not coupled to electromagnetism (?)



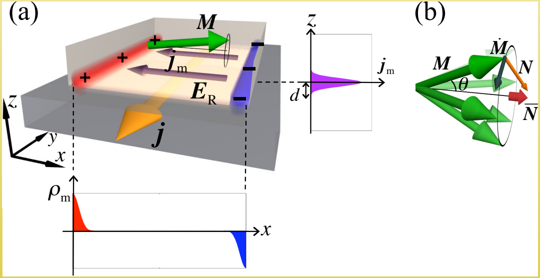
- Momentum space monopole — Nagaosa
  - Anomalous Hall effect

Monopoles

- Spin damping monopole Takeuchi ' 2011
  - Ferromagnetic metal
  - Emerges from spin dynamics + spin-orbit
  - Skewed projection SU(2) → U(1)



- Monopole pumping



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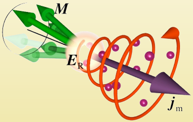

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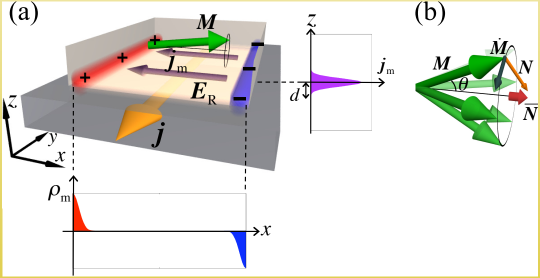

Same system as spin pumping + inverse spin Hall !!

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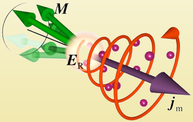

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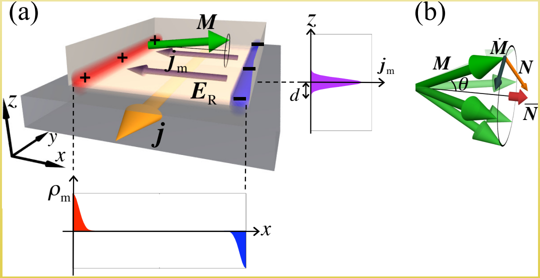

Same system as spin pumping + inverse spin Hall !!

Different (better) explanation

Summary

Microscopic formalism for spin transport

- Fully quantum calculation   Perturbation theory
- Each contribution is directly calculable
  No need for semiclassical transport equation
- Spin relaxation torque β torque, spin injection, spin chemical potential
- Spin pumping + inverse spin Hall
  Rigorous description of spin-charge conversion

- References

  G. Tatara, H. Kohno and J. Shibata, Physics Reports 468, 213 (2008).
  多々良源、スピントロニクス理論の基礎(培風館、新物理学シリーズ、2009)
  A.Takeuchi and G.Tatara, cond-mat arXiv:1104.4215 (2011).
  K.Hosono et al., J. Phys. Soc. Jpn. 79 014708(2010).
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Monopole...? → A.Takeuchi (O-3)

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