6^{th} International School on Spintronics and Quantum Information Technology (SPINTECH6)

8/2 (2011) Matsue

Quantum Control and Quantum Simulation with Ultracold Atoms with Spin

Kyoto University, JST

Y. Takahashi

Outline of Lecture

I) State-of-the-art of ultracold atom technology

II) Quantum simulation of Hubbard model with ultracold atoms in optical lattice

- II-1) quantum magnetism and superfluidity
  - metal-Mott insulator transition, anti-ferromagnetic order
- II-2) adiabatic cooling of atoms with spins
  - Pomeranchuk cooling

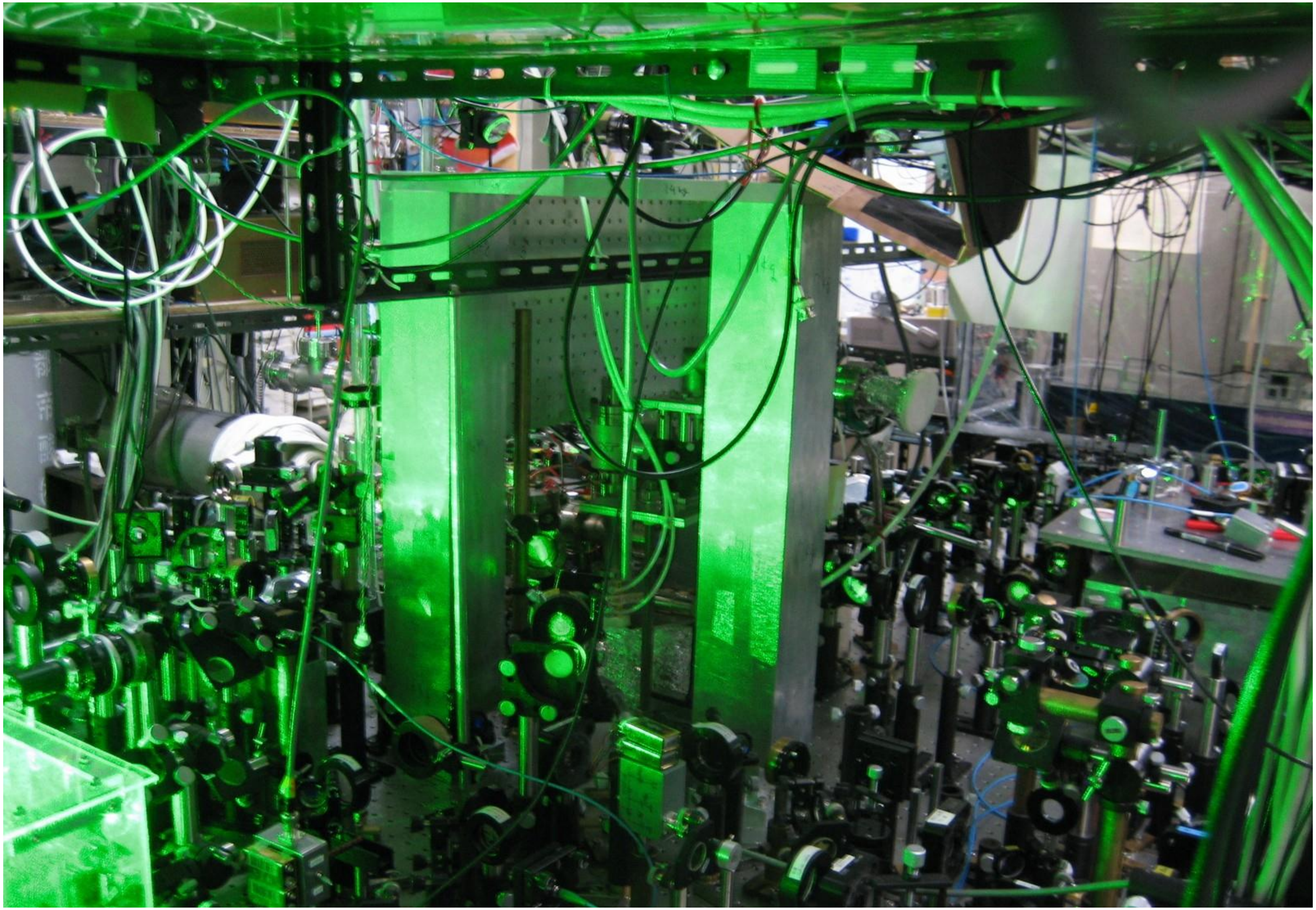
III) Quantum control of atomic spin ensemble

- III-1) quantum description of collective atomic spin
  - coherent spin state, squeezed spin state
- III-2) quantum description of polarization of light
  - quantum Stokes operators
- III-3) quantum interface and control
  - QND measurement of spin, quantum feedback control

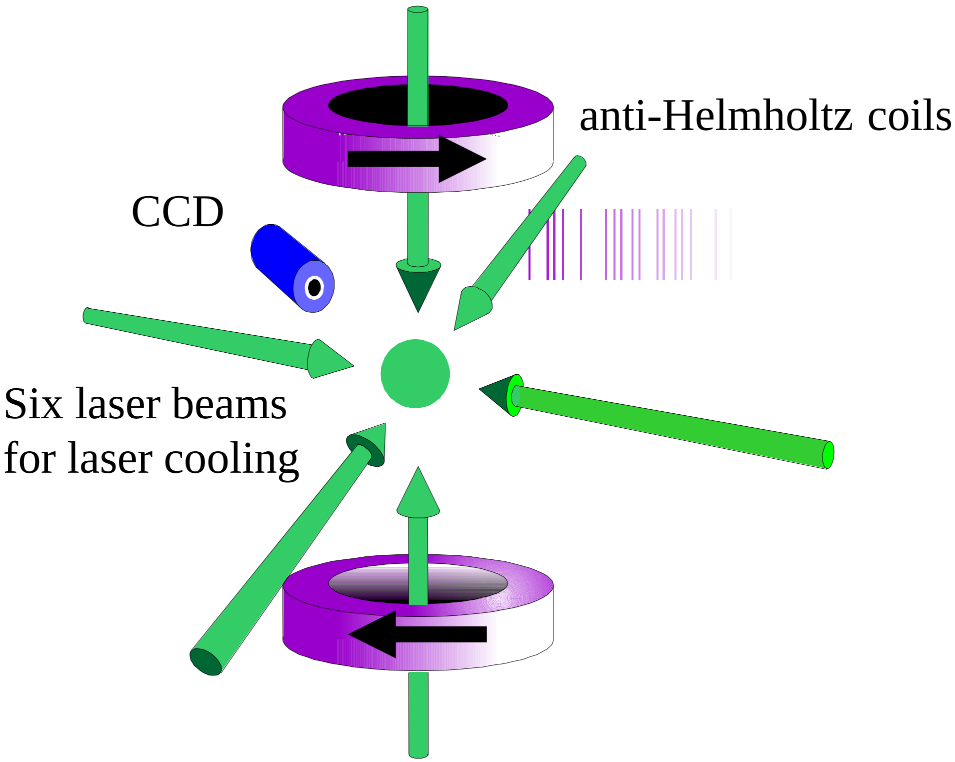
Outline of Lecture

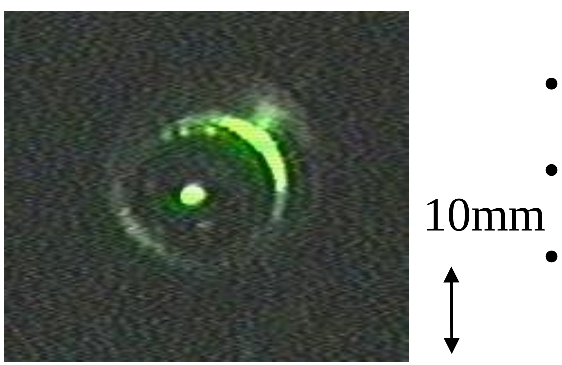
I) State-of-the-art of ultracold atom technology

typical experimental setup



Laser Cooling and Trapping



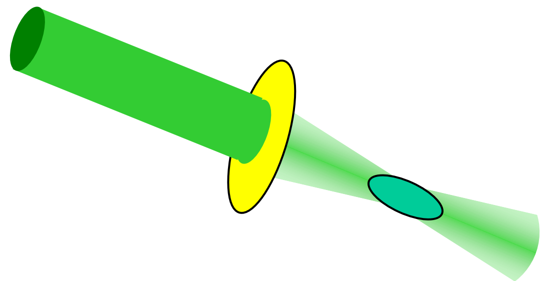


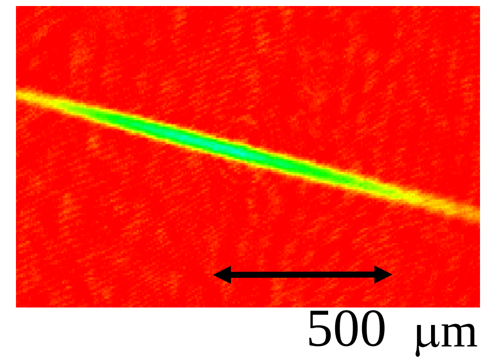
- Number: 10^7
- Density: $10^{11}/\mathrm{cm}^3$
- Temperature: 10μK

"Magneto-optical Trap"

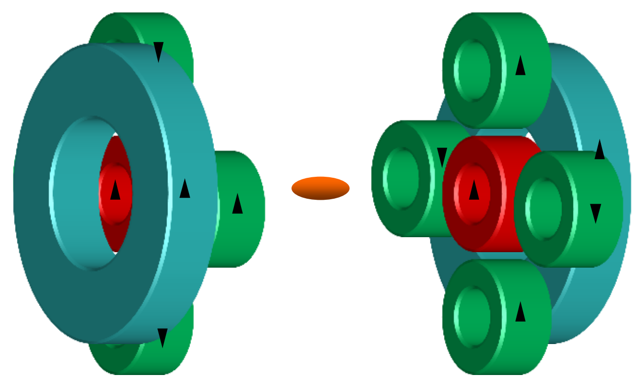
"optical trap"

$$V_{\mathrm{int}} = -p \cdot E \qquad U_{pot}(r) = -\frac{\chi E(r)^2}{2}$$





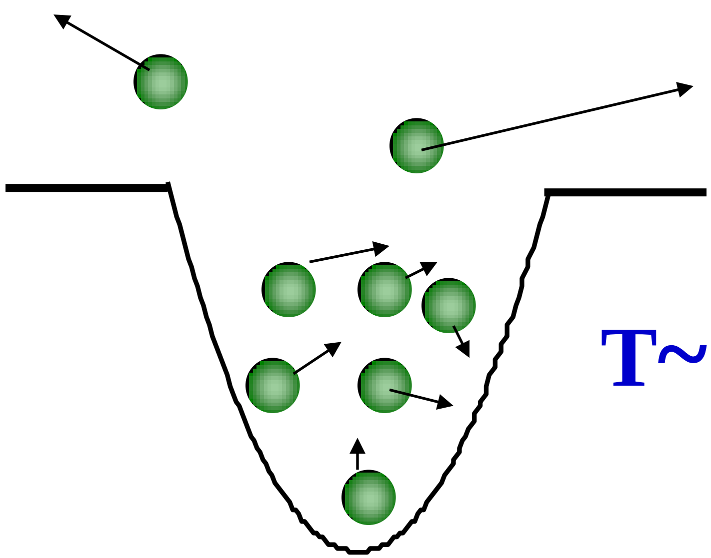
"magnetic trap" $V_{\mathrm{int}} = -\mu \cdot B$



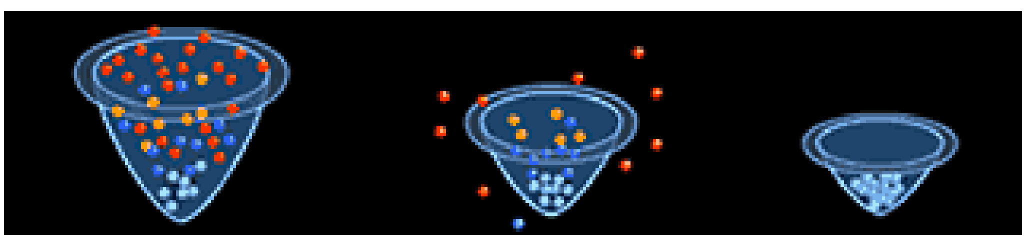
Evaporative Cooling

"remove (=evaporate) energetic atoms in a trap"

→ **"atoms remained in a trap have <u>lower average energy</u>**
(=lower temperature)"

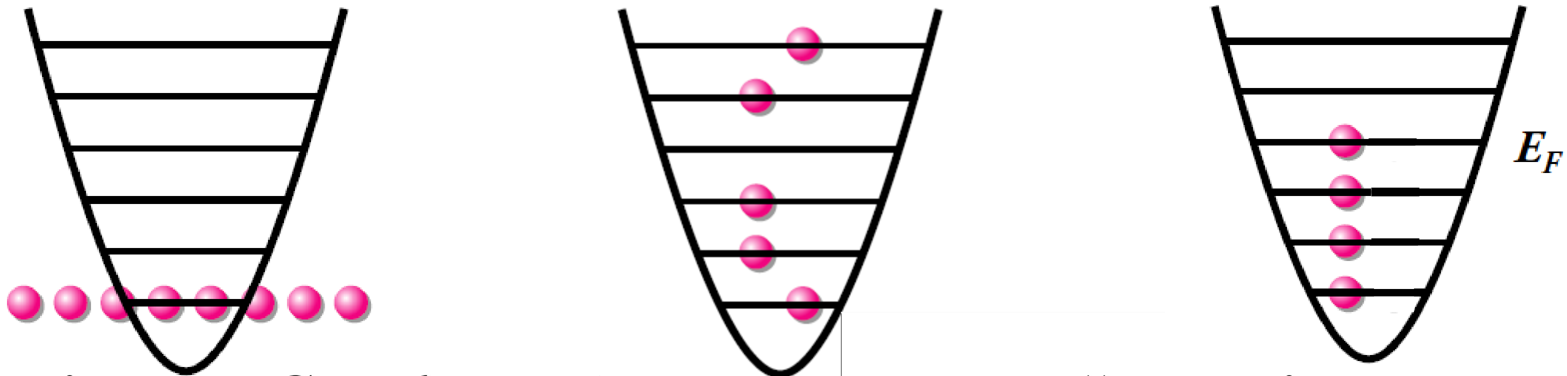


T~100 nK



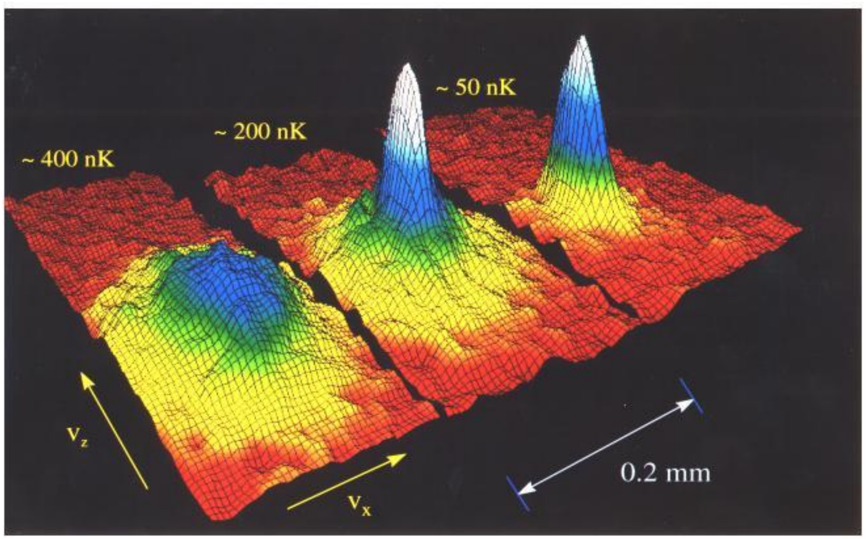
Atomic Gases Reach the Quantum Regime !

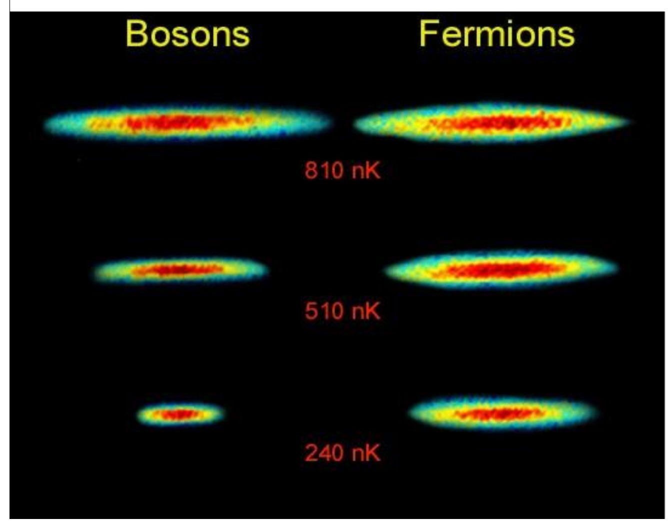
"Boson versus Fermion"



"Bose-Einsten Condensation" *"Fermi Degeneracy"*

^{87}Rb





^6Li and ^7Li

Momentum Distribution

[E. Cornell et al, (1995)]

Spatial Distribution

[R. Hulet et al, (2000)]

Feshbach Resonance:

ability to tune an inter-atomic interaction

Collision is in Quantum Regime !

It is described by s-wave scattering length a_s

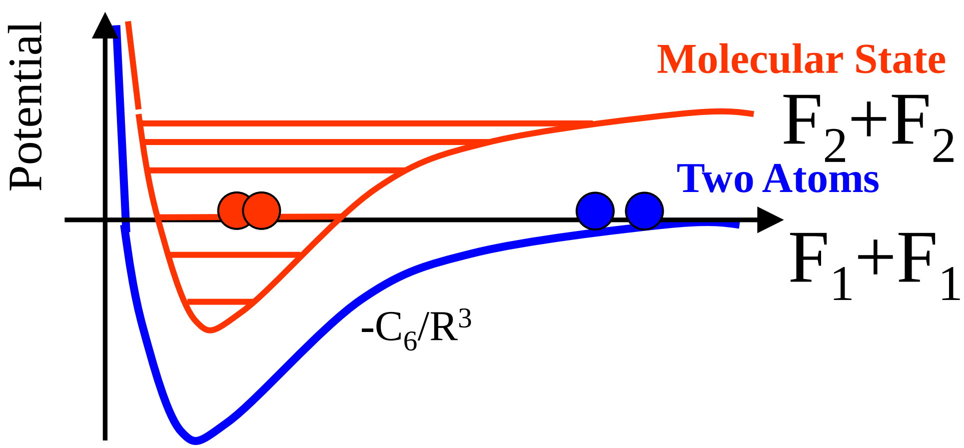
$$a_s = -\delta_l / k$$

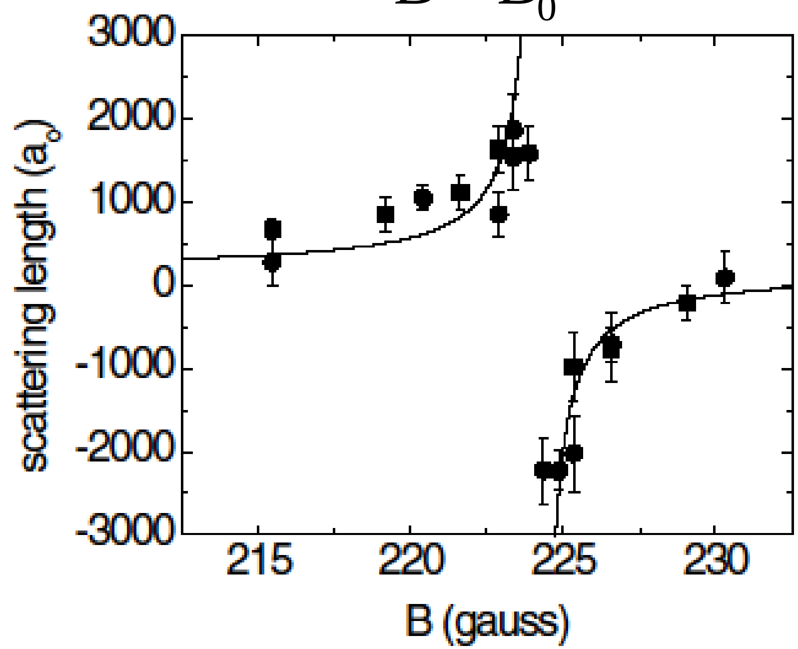
$$\sigma_0 = 4\pi |f_0|^2 = 4\pi |a_s|^2$$

Coupling between "**Open Channel**" and "**Closed Channel**"

→ Control of Interaction(a_s)

$$a_s(B) = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$



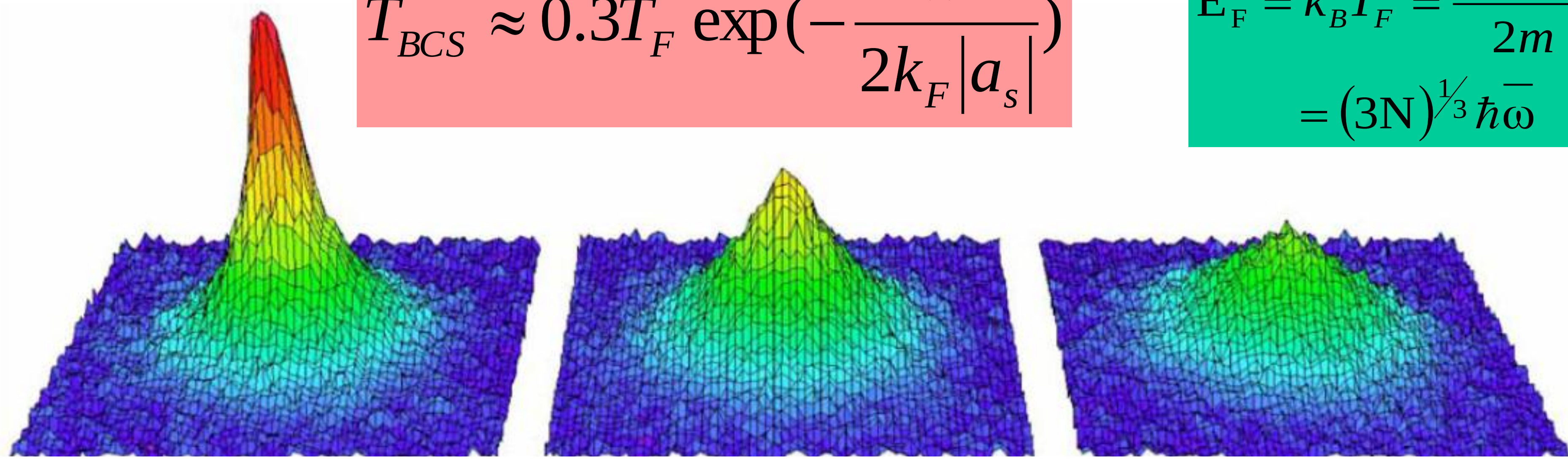


[C. Regal and D. Jin, PRL**90**, 230404(2003)]

Realization of Atomic BCS

$$T_{BCS} \approx 0.3 T_F \exp\left(-\frac{\pi}{2k_F |a_s|}\right)$$

$$E_F = k_B T_F = \frac{(\hbar k_F)^2}{2m} = (3N)^{1/3} \hbar\bar{\omega}$$



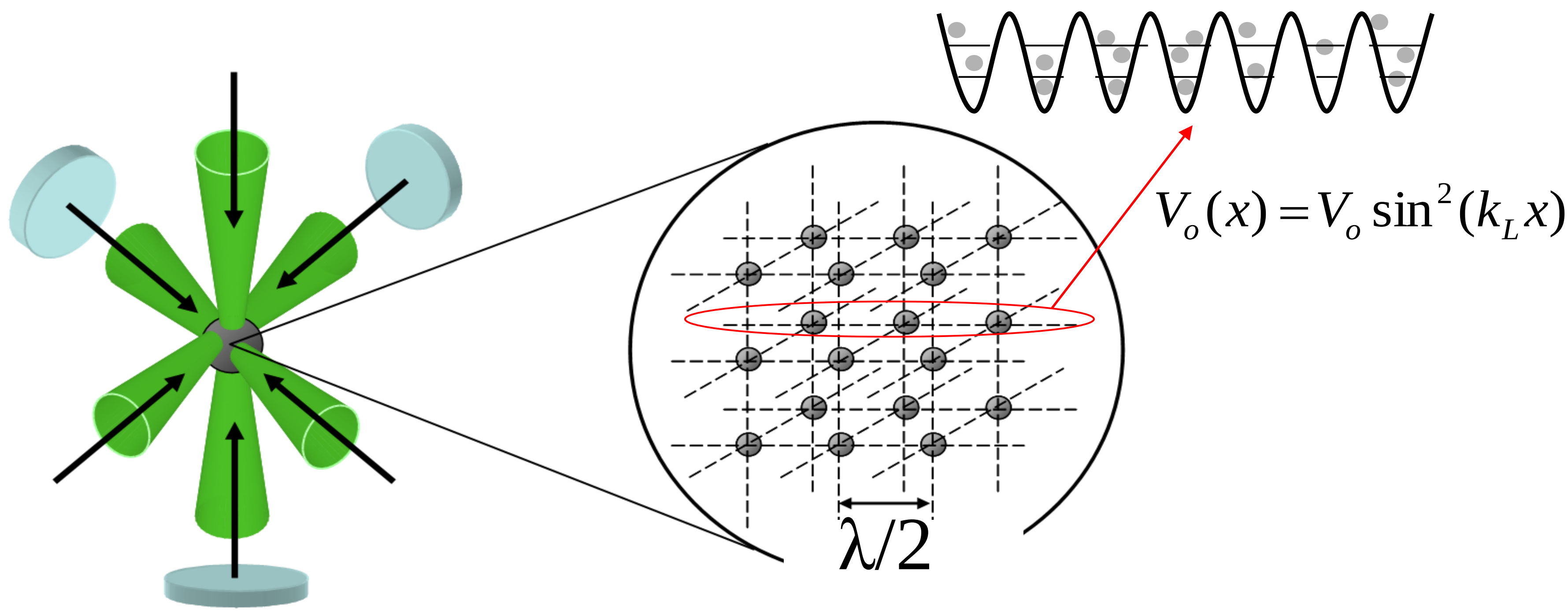
$T < T_{BCS}$ $B \sim B_{res}$

$T \sim T_{BCS}$ $B \gtrsim B_{res}$

$T > T_{BCS}$ $B \gg B_{res}$

D. Jin Group

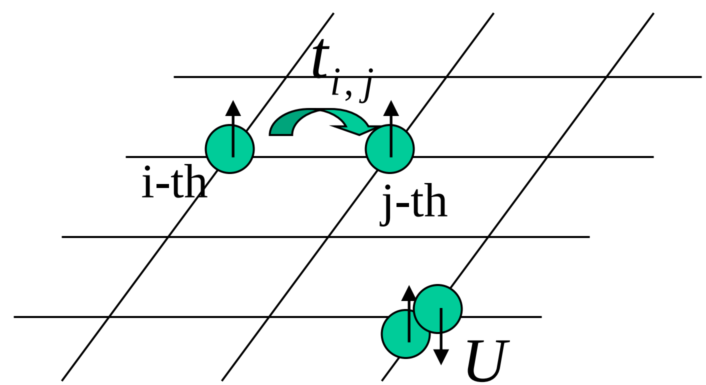
Optical Lattice



Atoms in optical lattice ⟺ **electrons in crystalline lattice**

Hubbard Model:

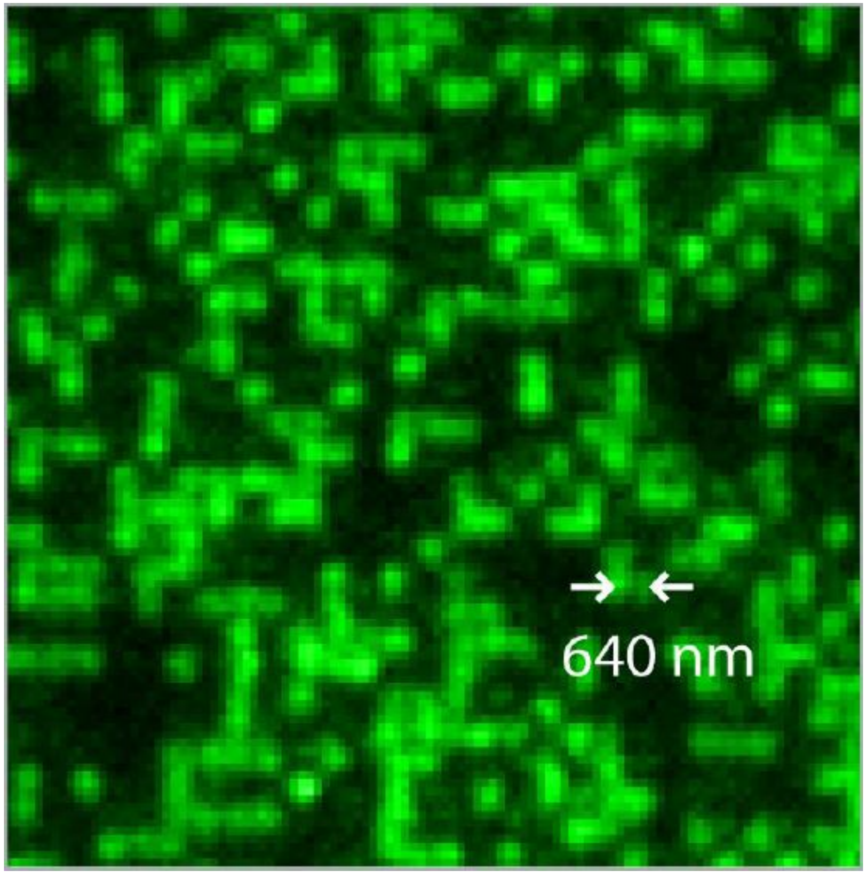
$$H = -t \sum_{<i,j>} c_i^{+} c_j + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

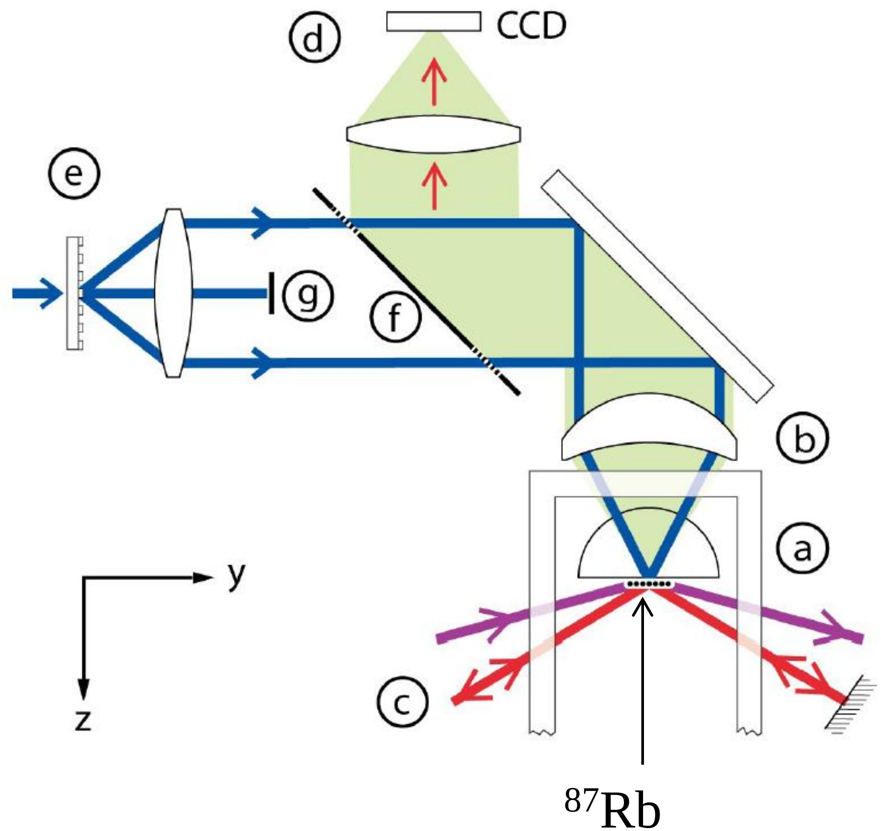


New Technique: Single Site Observation

[WS. Bakr, I. Gillen, A. Peng, S. Folling, and M. Greiner, Nature 462(426), 74-77(2009)]

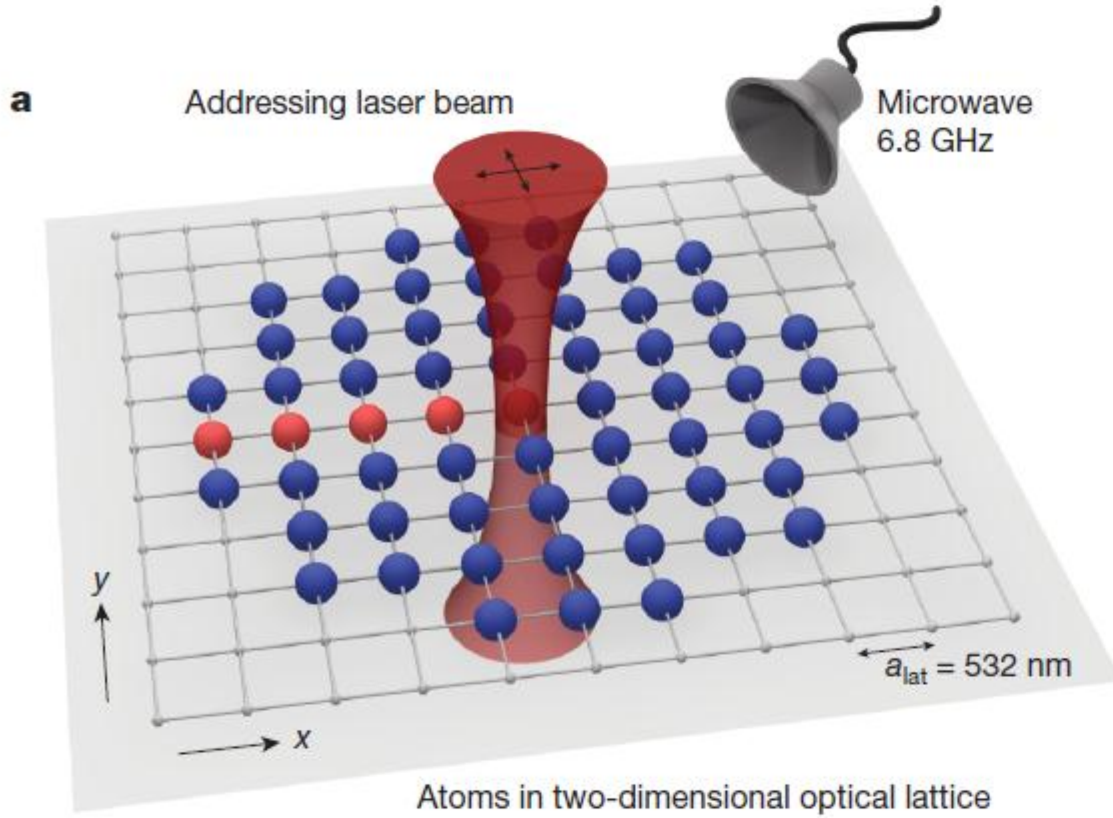
Fluorescence Imaging

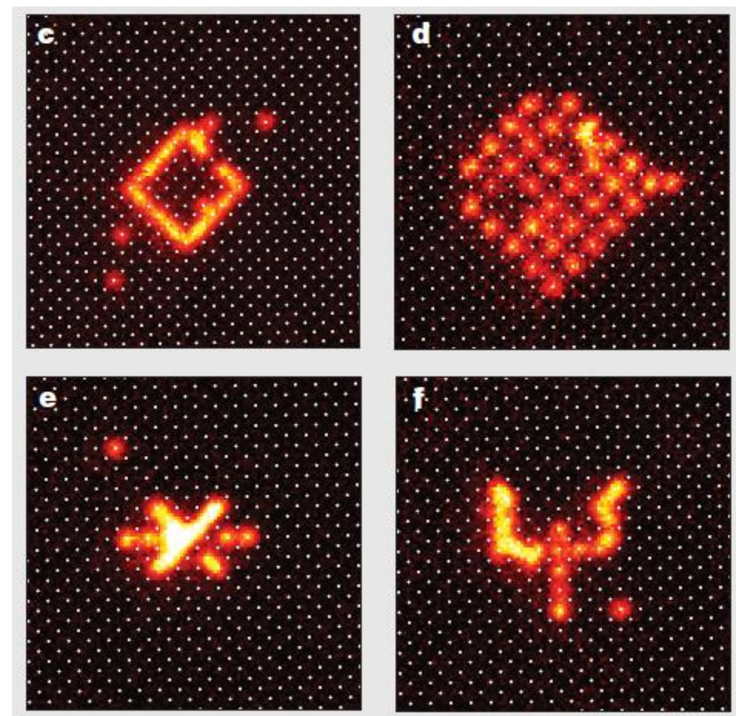




New Technique: Single Site Manipulation

[C. Ewitenberg *et al*, Nature 471, 319(2011)]





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in optical lattice

II-1) quantum magnetism and superfluidity

metal-Mott insulator transition, anti-ferromagnetic order

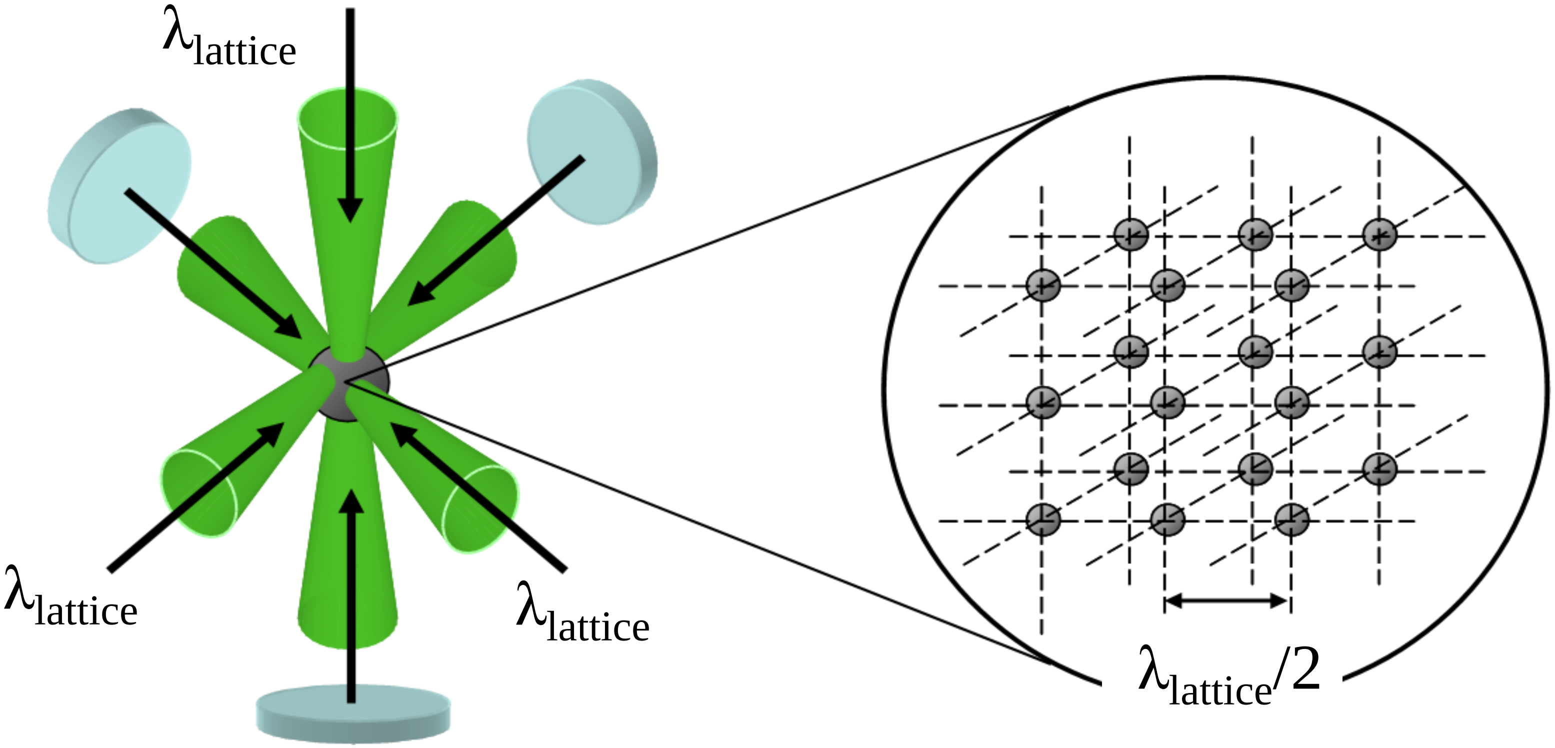
II-2) adiabatic cooling of atoms with spins

Pomeranchuk cooling

Fermions in a 3D optical lattice

$$H = -J\sum_{<i,j>} c_i^{+} c_j + U\sum_i n_{i,\uparrow} n_{i,\downarrow} + \sum_i \varepsilon_i n_i$$

"Fermi-Hubbard Model"

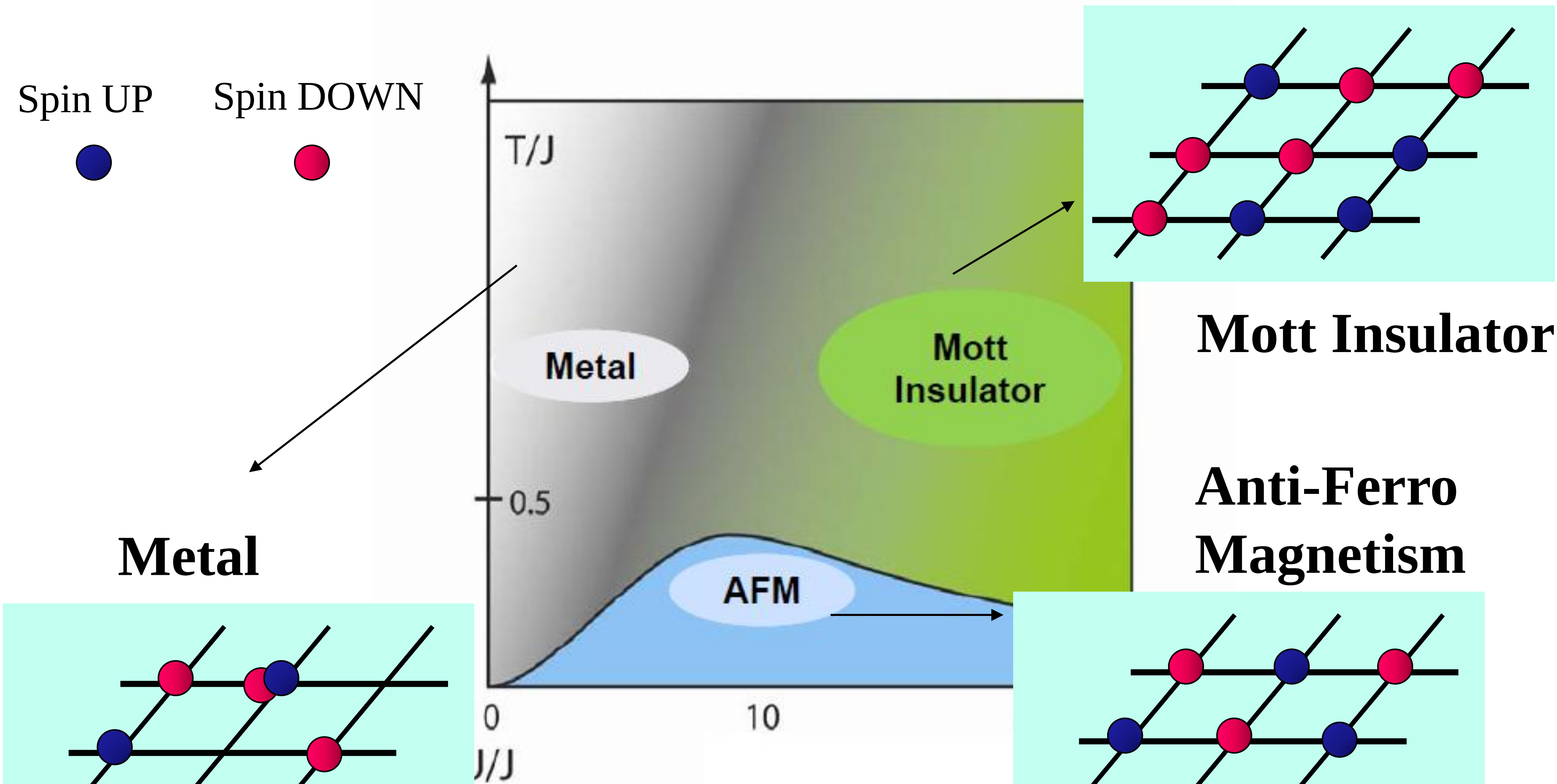


Tunability of parameters

luck of defects

luck of impurities

Phase Diagram of Repulsive Fermi-Hubbard Model

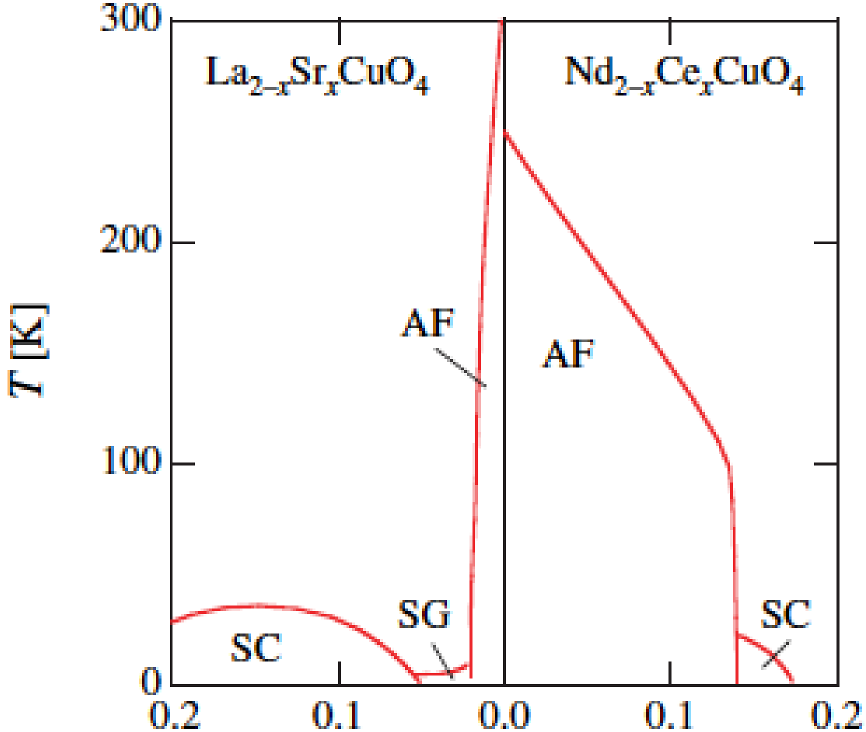


Annu. Rev. Condens. Matter Phys

Ranninger, S. Roaszkiewicw, Rev.

One of the Goals:
quantum simulation of high-T_c super-conductivity

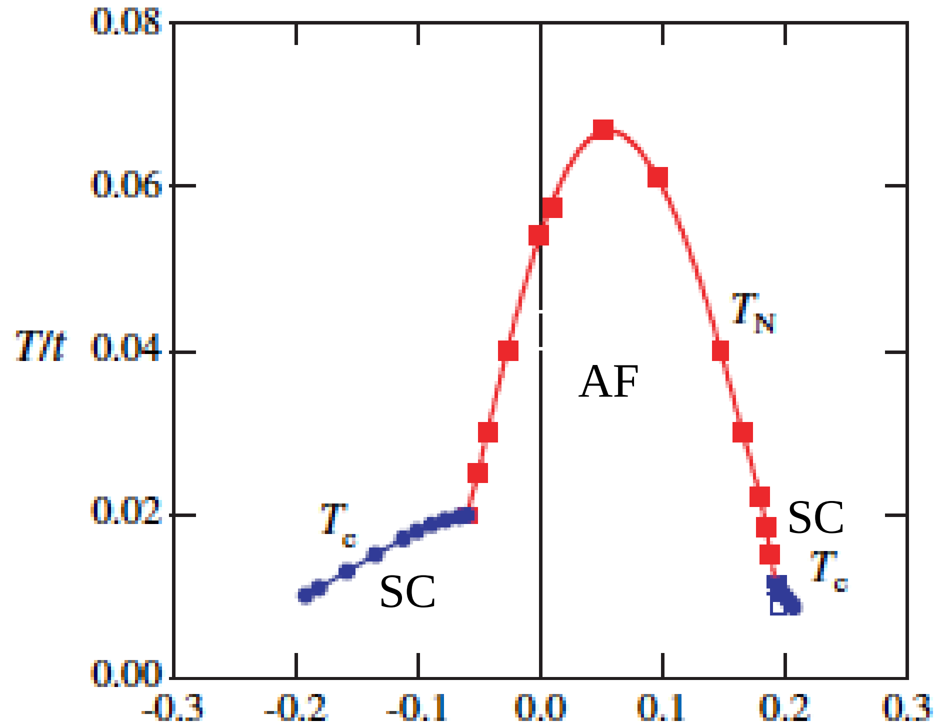
experiment



hole ⟵ x ⟶ electron

(carrier doping)

theory



hole ⟵ x ⟶ electron

(carrier doping)

[in T. Moriya and K. Ueda, Rep. Prog.Phys.66(2003)1299]

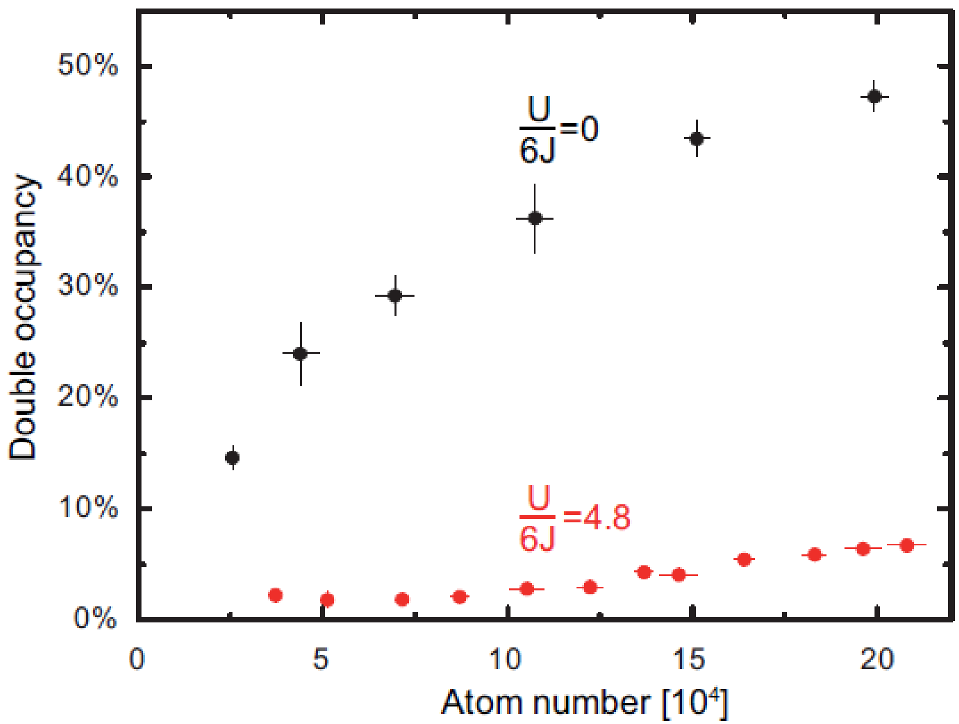
Current Status of

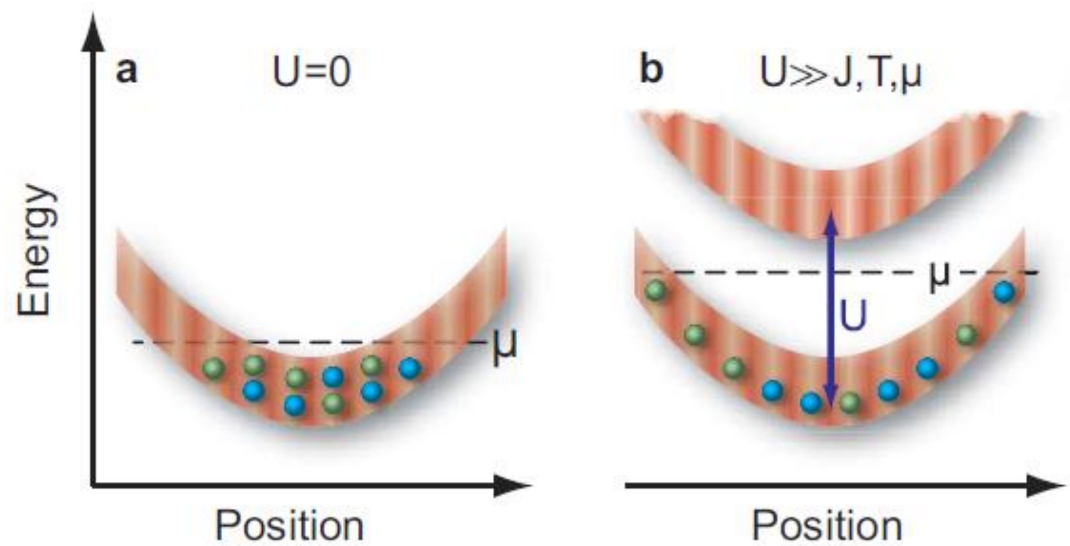
Quantum Simulation of Fermi Hubbard Model:

"Formation of (paramagnetic) Mott insulator"

[R. Jördens *et al.*, Nature **455**, 204 (2008)]   [U. Schneider, *et al.*, Science **322**,1520(2008)]

"Suppression of Doubly Occupied Sites by Strong Repulsion"





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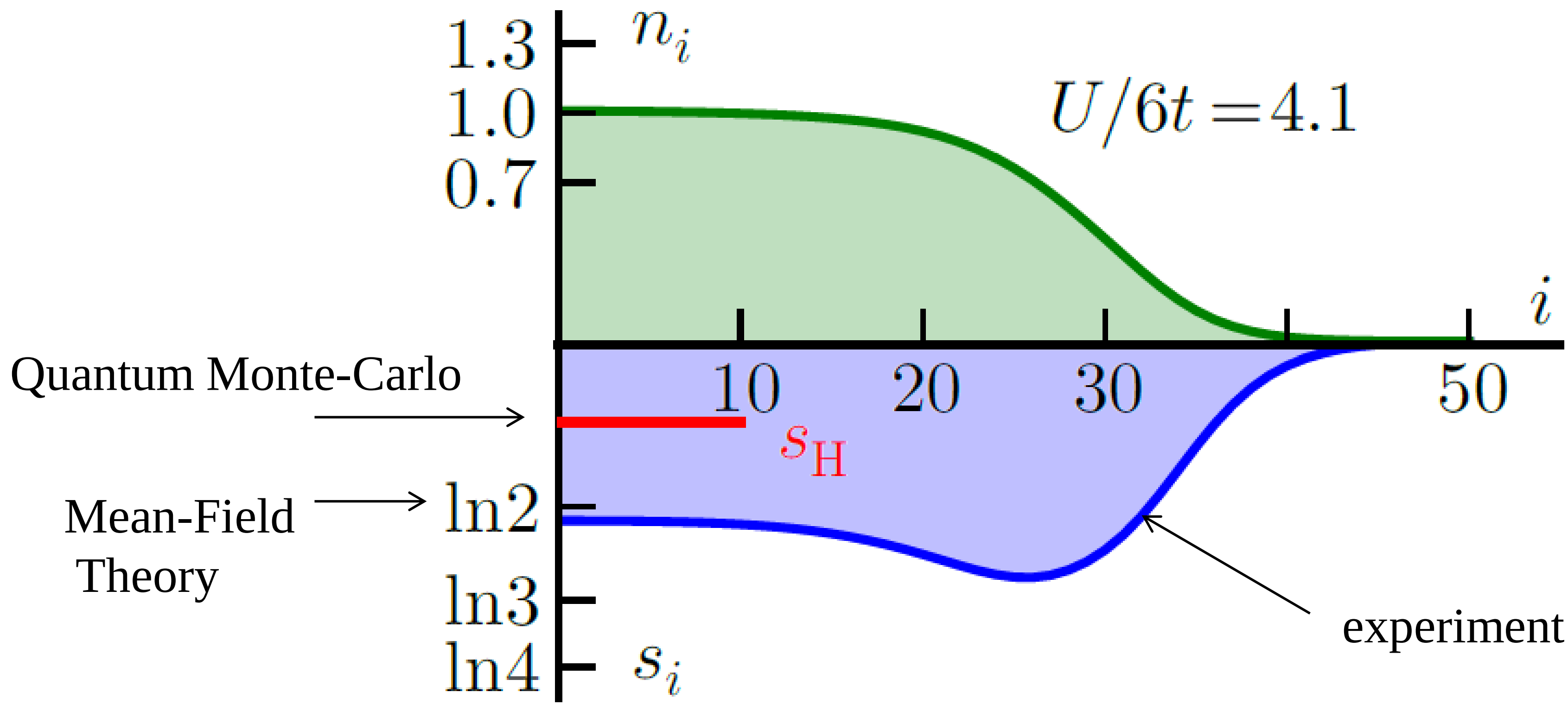
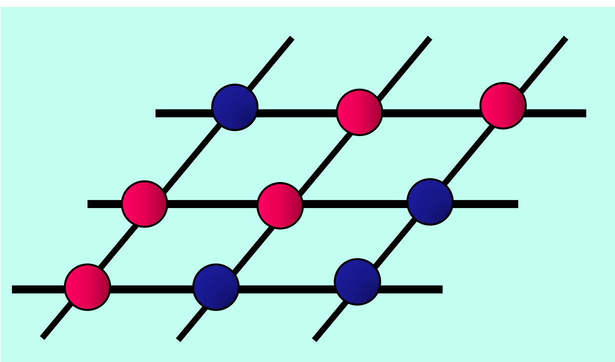


You Need Cooling for Quantum Magnetism

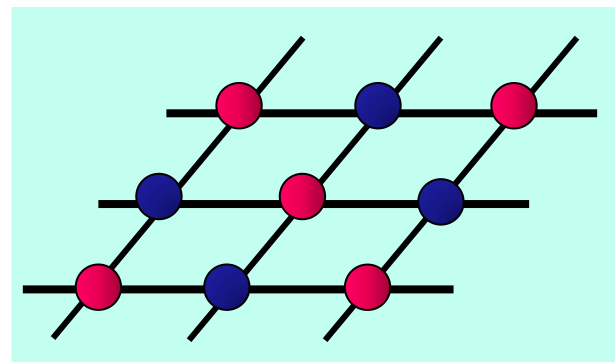
(paramagnetic) Mott Insulator

Anti-Ferro Magnetic Order



Cooling

$$s < k_B \ln(2)$$



Spin Degree of Freedom *is Cool*

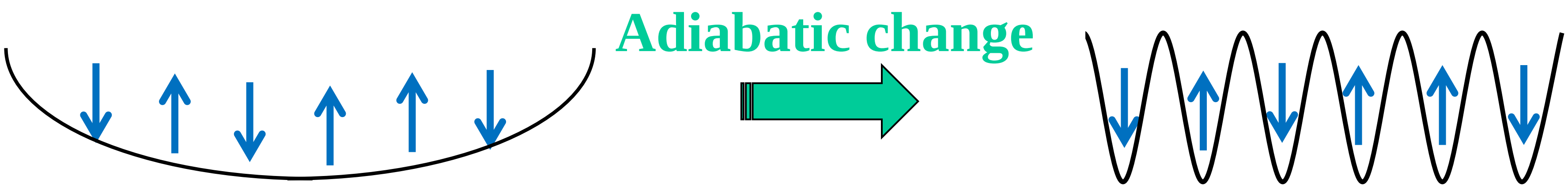
Pomeranchuk Cooling

[Pomeranchuk, (1950)]

⟶ Discovery of Superfluid ^{3}He by Osheroff, Lee, Richardson

Initial state: Spin-*depolarized* and also with *degeneracy*:

Final state: Spin depolarized:



$s \sim k_B \pi^2 T/T_F$

liquid ^{3}He atoms in trap

$s \sim k_B \ln(N)$

solid ^{3}He atoms in Mott Insulator

"entropy flows from **motional** degrees of freedom to **spin**, which results in the cooling of the system"

Outline of Lecture

III) Quantum control of atomic spin ensemble

III-1) quantum description of collective atomic spin

coherent spin state, squeezed spin state

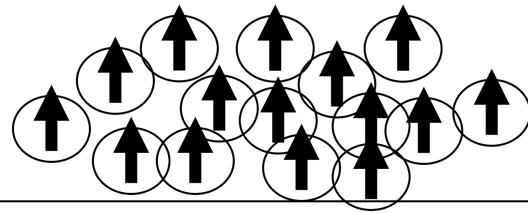
III-2) quantum description of polarization of light

quantum Stokes operators

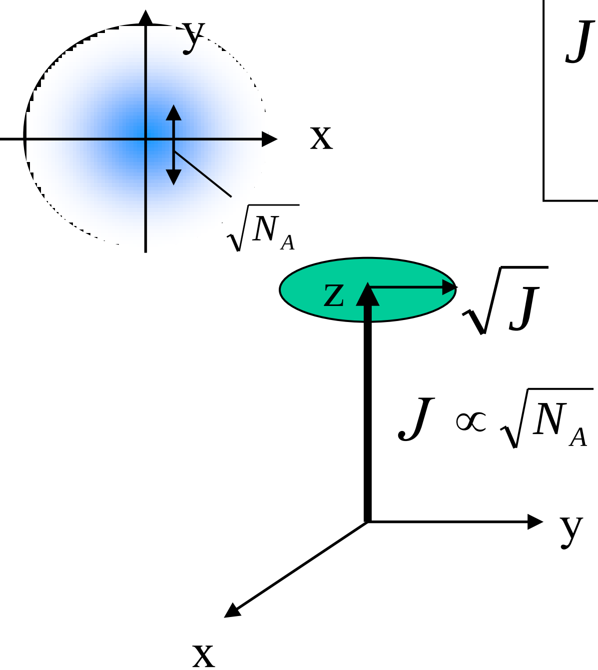
III-3) quantum interface and control

QND measurement of spin, quantum feedback control

collective atomic spin

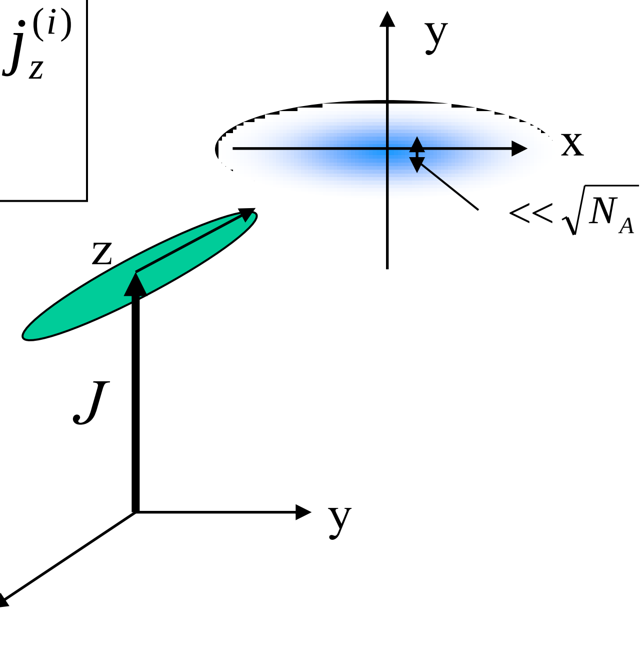


$$J_X = \sum_i j_x^{(i)}, J_Y = \sum_i j_y^{(i)}, J_Z = \sum_i j_z^{(i)}$$



$$[J_x , J_y] = i\, J_z$$

$$\longrightarrow \Delta J_X \Delta J_Y \geq \frac{\langle J_Z \rangle}{2}$$



Coherent Spin State

$$\Delta J_X = \Delta J_Y = \sqrt{J/2}$$

Squeezed Spin State

$$\Delta J_X > \sqrt{J/2}, \Delta J_Y < \sqrt{J/2}$$

Quantum Interface

"Memory"

Atomic Spins:

Collective Spin Operators

QND-interaction:

$$H_{int} = \alpha S_x J_x$$

"Communication"

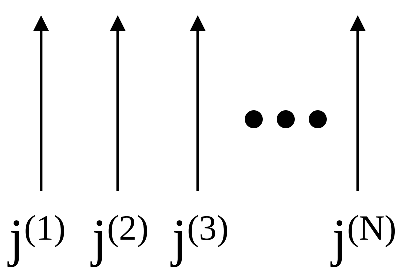
Light Polarization:

Quantum Stokes Operators

$$J_X = \sum_i j_x^{(i)}, J_Y = \sum_i j_y^{(i)}, J_Z = \sum_i j_z^{(i)}$$

$$[J_\lambda, J_\mu] = i\varepsilon_{\lambda\mu\nu} J_\nu$$

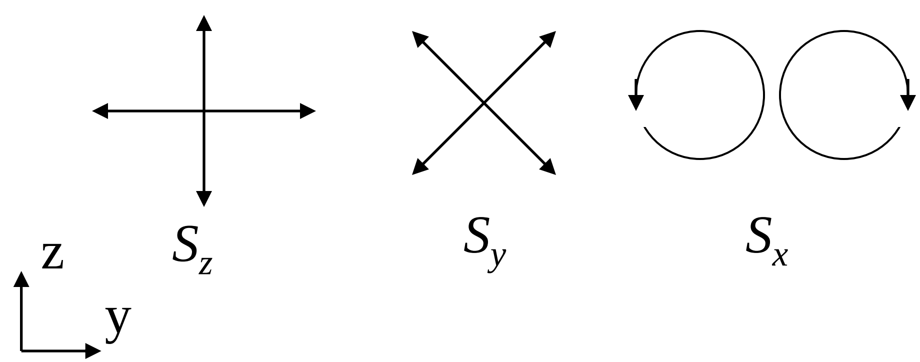
$$\lambda,\mu,\nu = x,y,z$$



$$S_x = (a_+^\dagger a_+ - a_-^\dagger a_-)/2,$$
$$S_y = (a_+^\dagger a_- - a_-^\dagger a_+)/2i,$$
$$S_z = (a_+^\dagger a_- + a_-^\dagger a_+)/2,$$

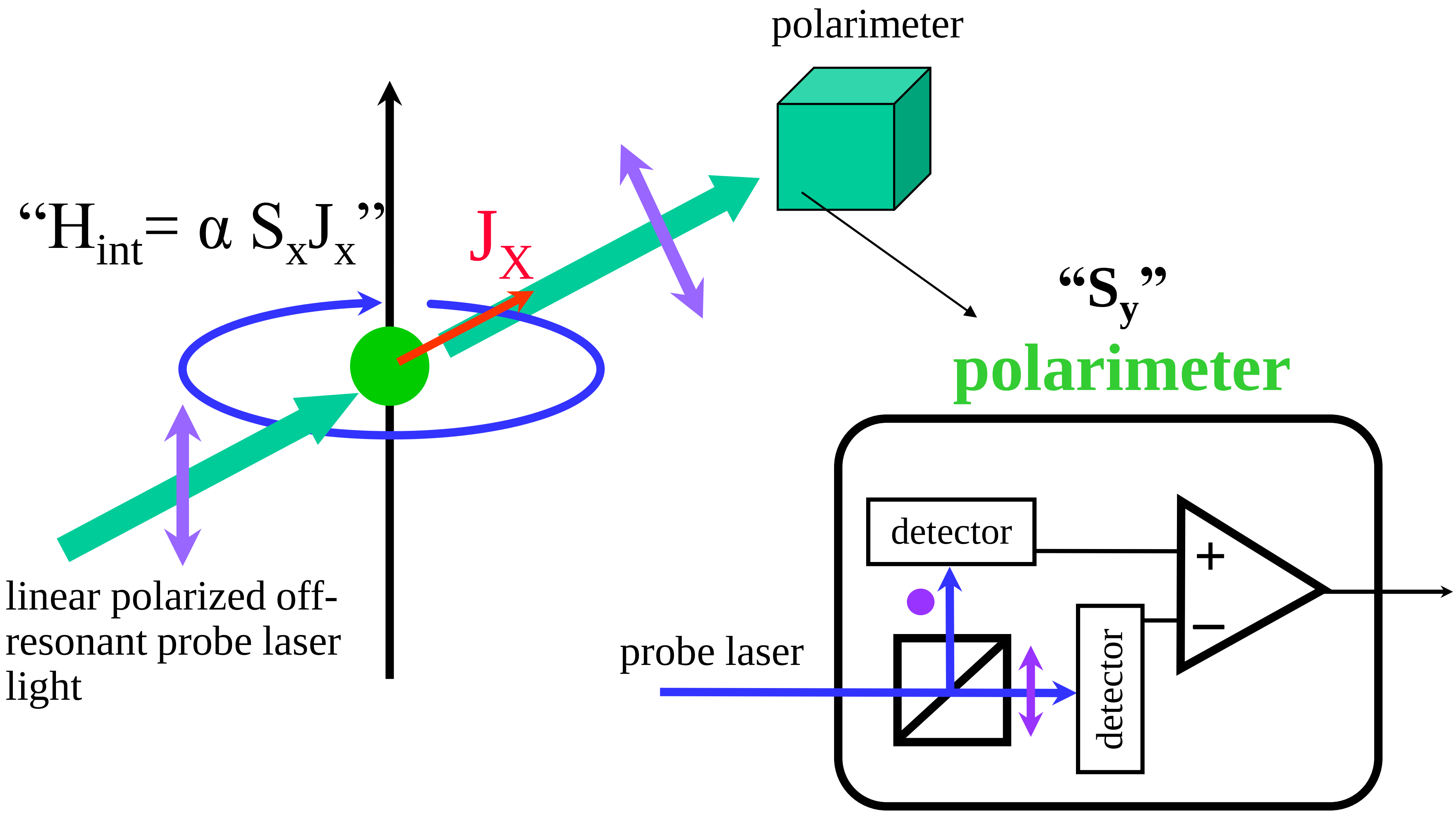
$$[S_\lambda, S_\mu] = i\varepsilon_{\lambda\mu\nu} S_\nu$$

$$\lambda,\mu,\nu = x,y,z$$



"Faraday Rotation" as QND interaction

YT *et al.*, PRA **60**, 4974, (1999);
A. Kuzmich *et al.*, Europhys. Lett. 42, 481(1998)

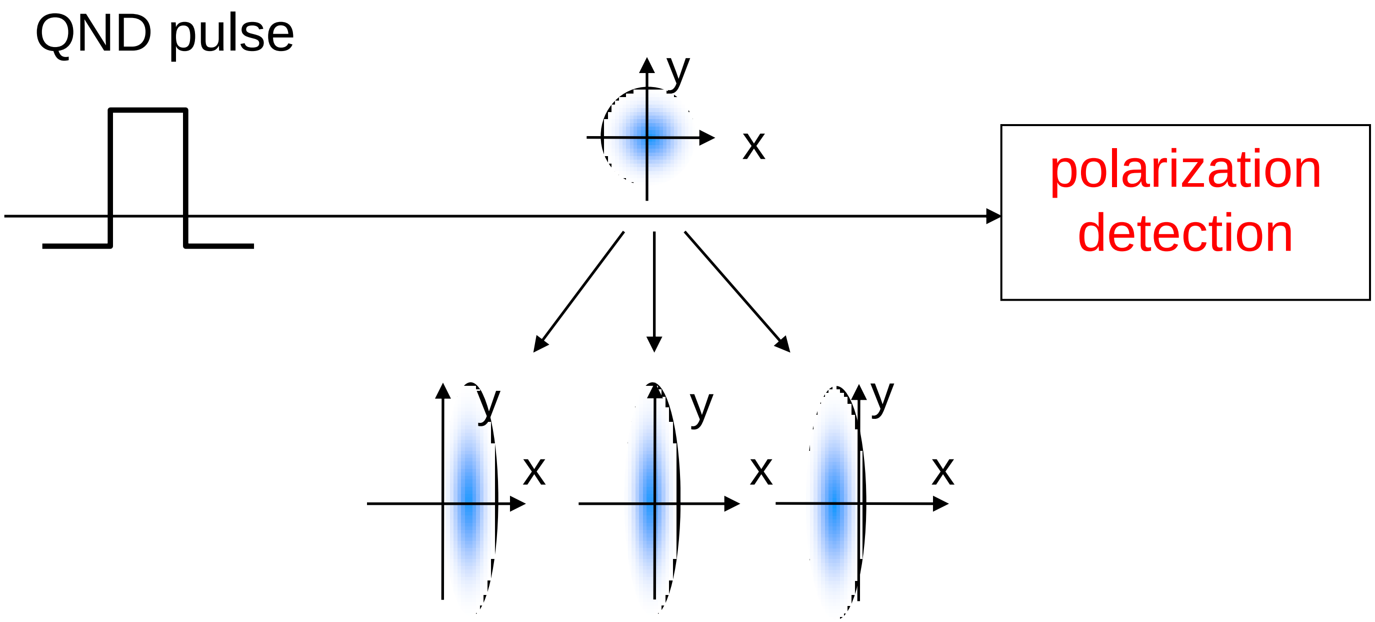


“Quantum State Preparation”

Projection of the Result of **QND** Measurement

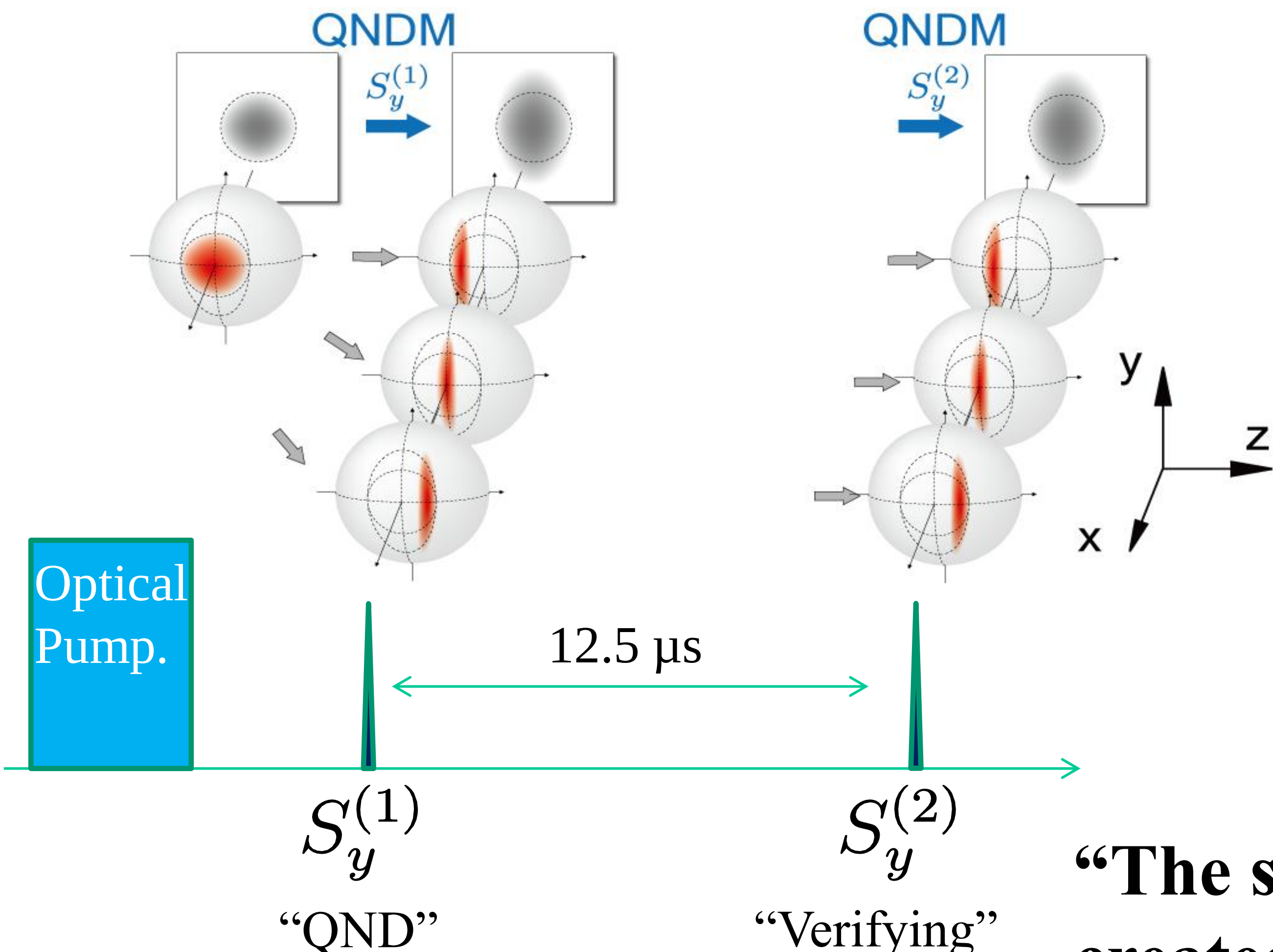
$J_x(0)$ → $H_{int} = \alpha S_x J_x$ → $J_x(t) = J_x(0)$

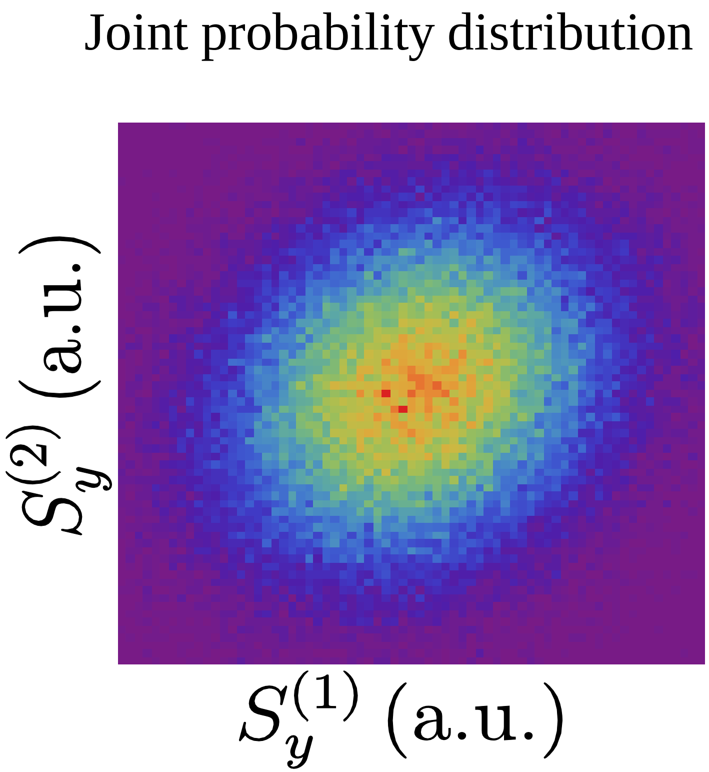
$S_y(0)$ → $H_{int} = \alpha S_x J_x$ → $S_y(t)$:**Measurement**



Now, 5 groups in the world have successfully created **Squeezed Spin States**

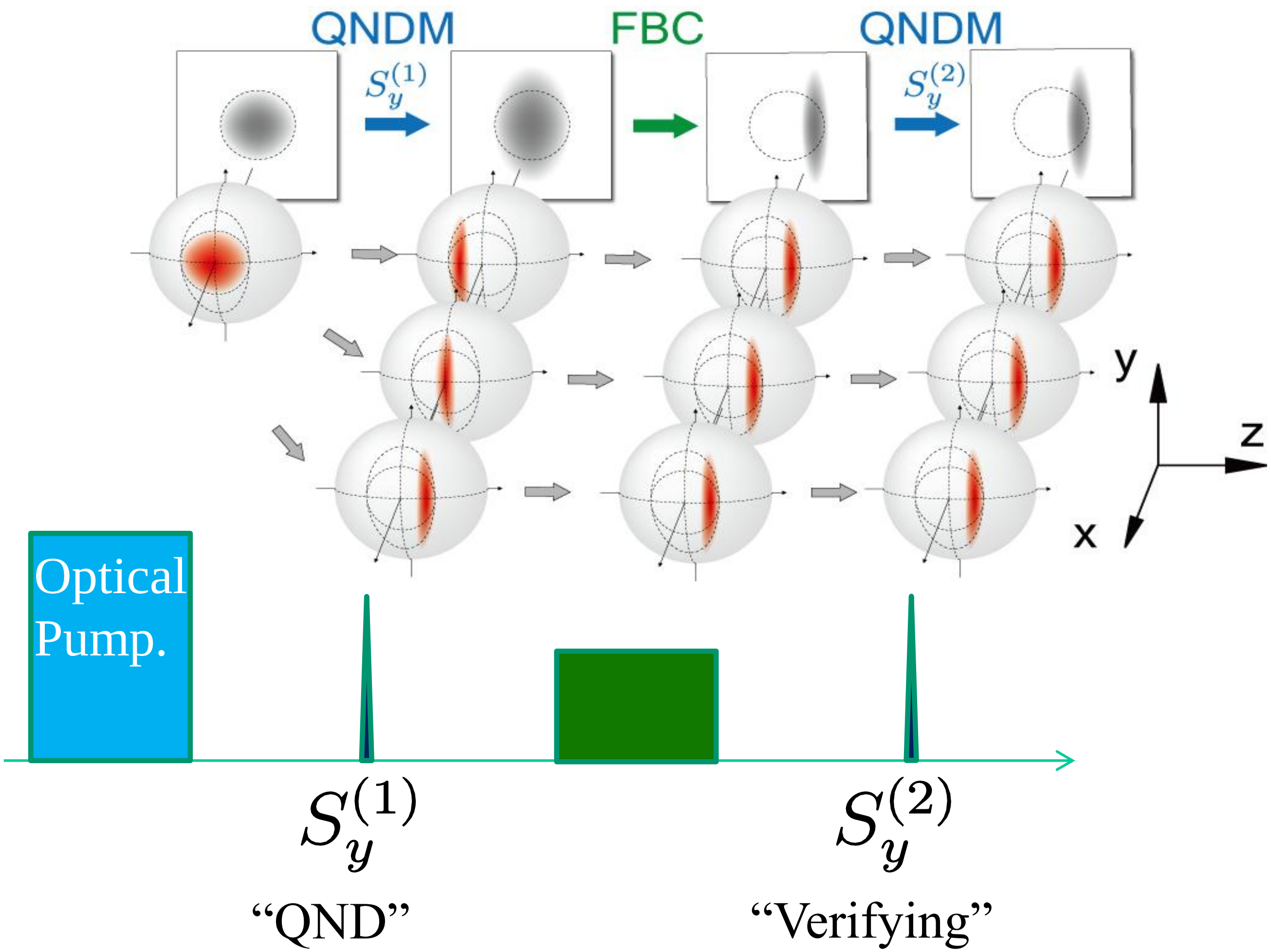
Typical Spin-QND Measurement: Scheme and Result



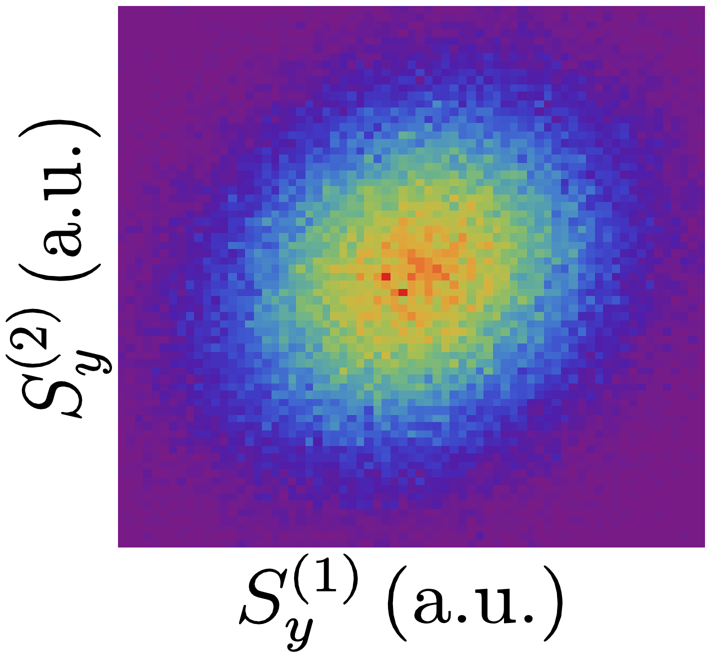


"The squeezed spin state is created CONDITIONALLY"

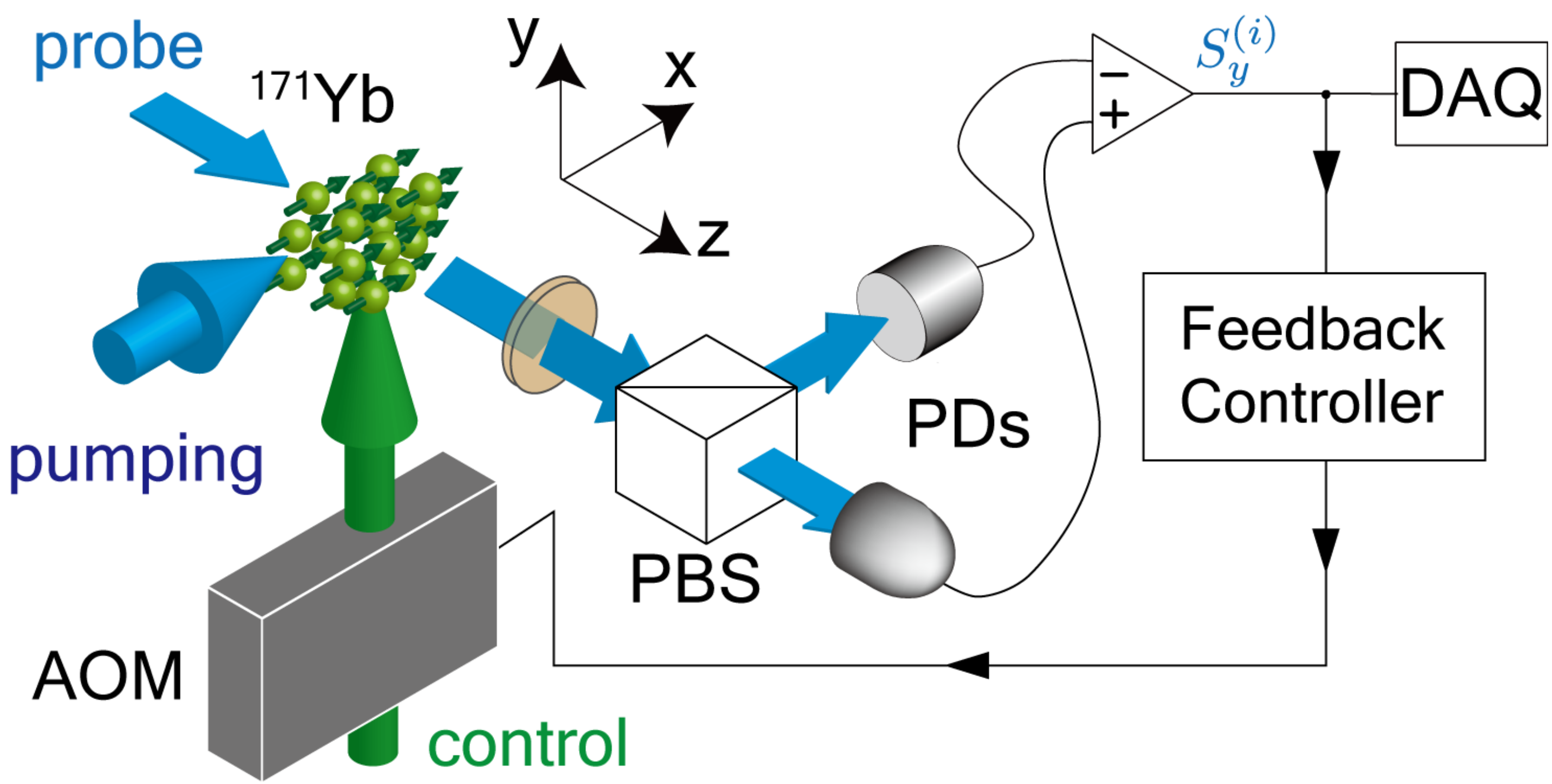
Implementing Quantum Feedback Control



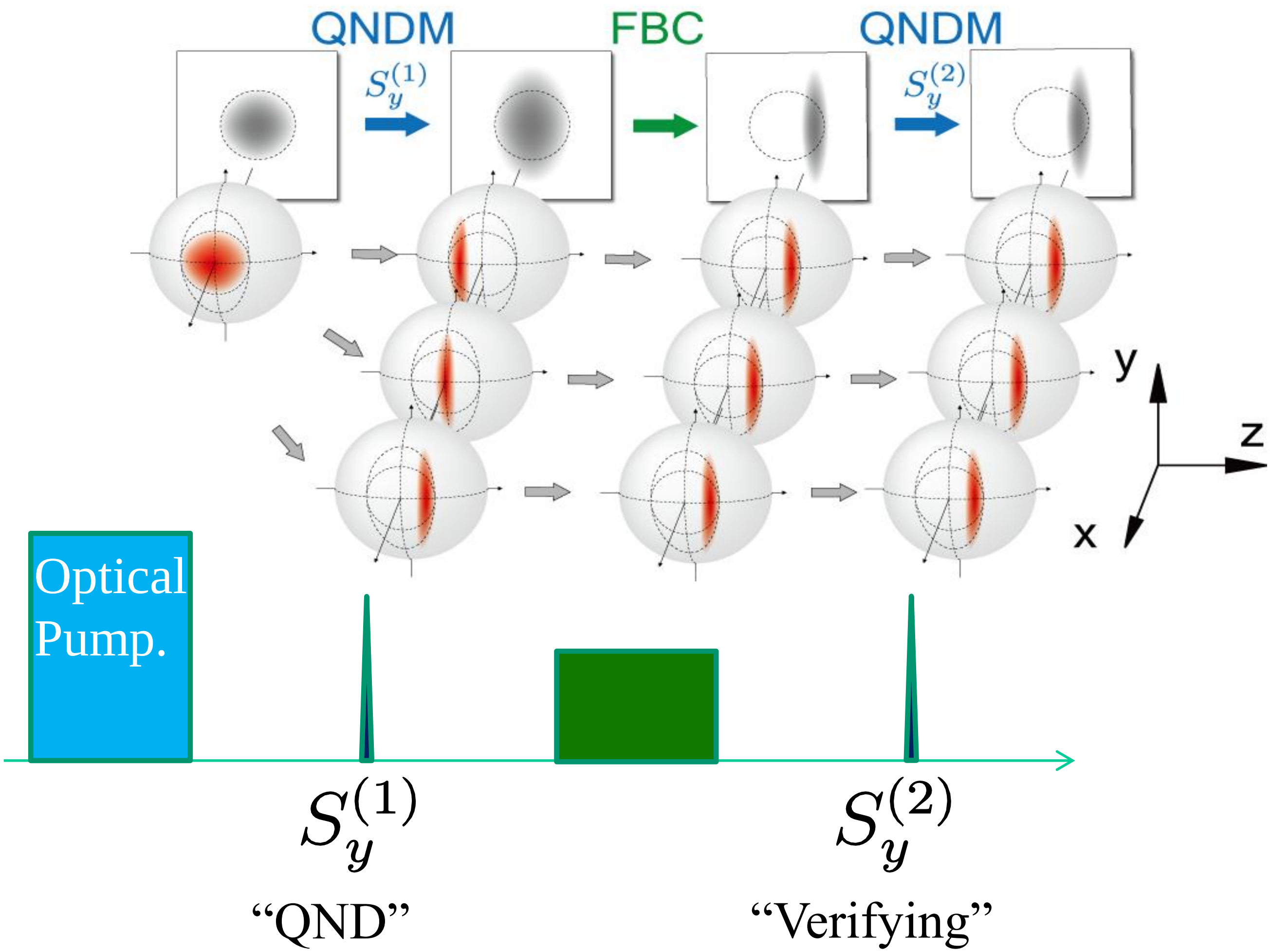
Joint probability distribution



Experimental Setup for Quantum Feedback Control



Implementing Quantum Feedback Control

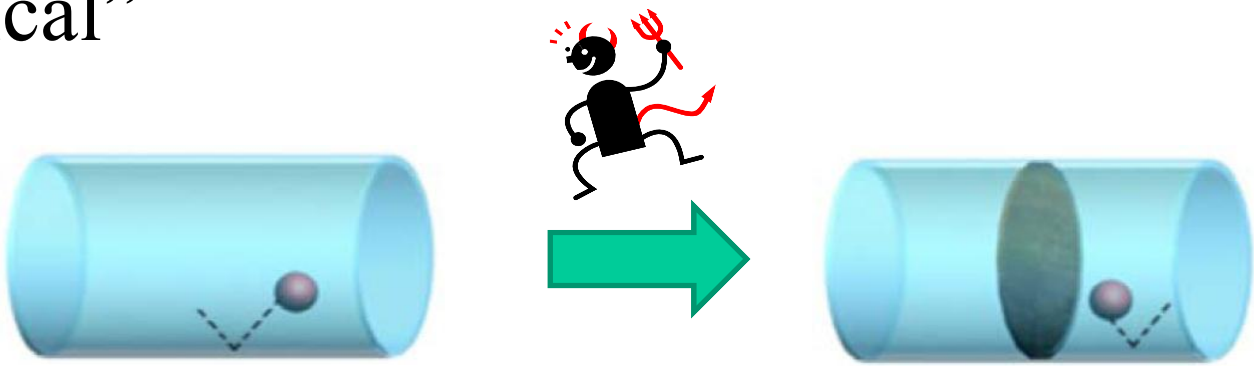


Joint probability distribution

$(\Delta y)^2 \longrightarrow$ **-1.4dB Reduction**

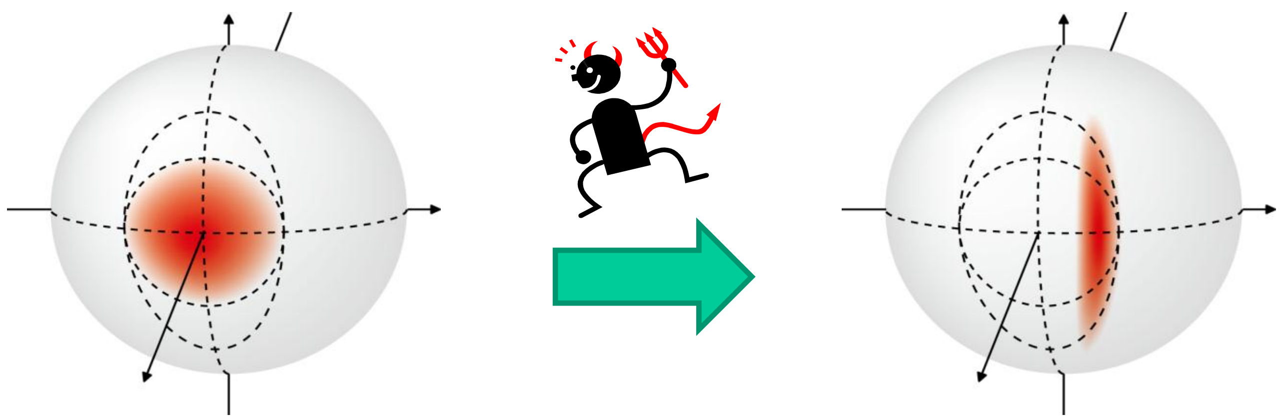
Maxwell Demon

“Classical”



Reduction of (Thermodynamic) Entropy without Work

“Quantum”



Reduction of Shannon Entropy without Work

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  - demagnetization cooling, Pomeranchuk cooling

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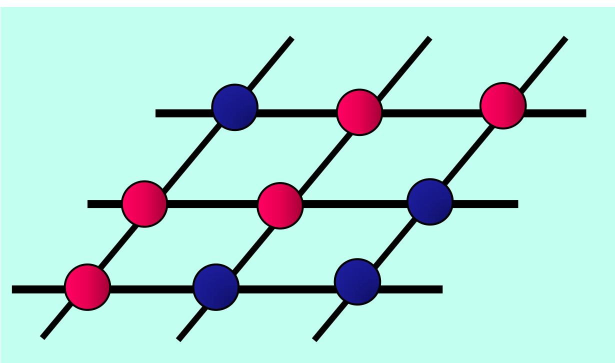
Quantum Magnetism via Quantum Feedback ?

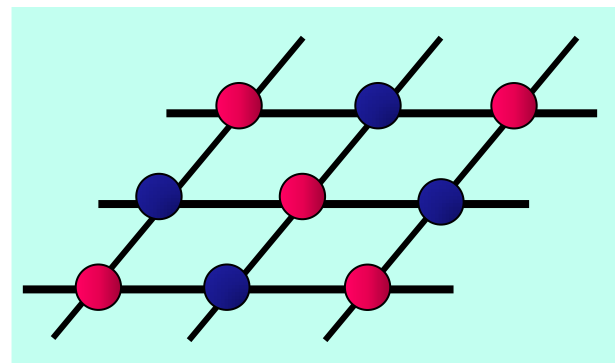
(paramagnetic)
Mott Insulator

Anti-Ferro Magnetic Order



Cooling

$$s < k_B \ln(2)$$



Measurement & Feedback Control
With Single Atom Level

Thank you very much for attention

16 August Mount Daimonji at Kyoto