Physical systems with inherent error-correcting properties

Error correction and fault-tolerant quantum computation on standard QC

- Error correction and fault-tolerant quantum computation
  - Requires tremendous number of qubits and non-local entanglement
- Example: Factorize a 130-digit (432-bit) number by Shor's algorithm
  - Total number of qubits ~ 10^6
    - including additional ancillary qubits to implement gates fault-tolerantly
  - Non-local entanglement
    - over 7^3 qubits to encode one qubit!
- Need for better design
  - with inherent error-correcting and/or fault-tolerant properties

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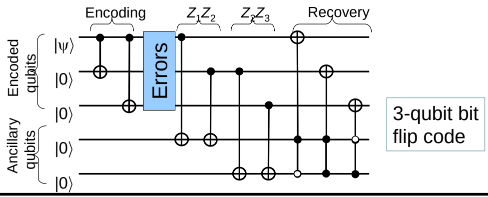
Good qubit design for syndrome measurement

- Active error correction requires syndrome measurement involving ancilla qubits
- Arrangement of qubits/ancilla qubits matters!



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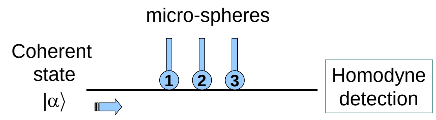
Example (1): Non-linear optics

- Error correcting storage for photonic qubit
  - 3-qubit bit-flip (phase) flip code
  - Store qubit information using three micro-spheres
  - Syndrome measurement by sending coherent states



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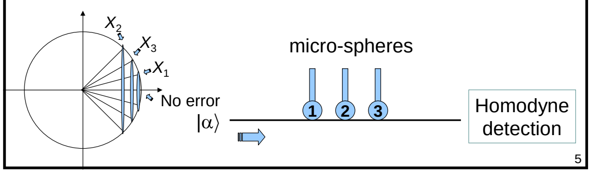
Syndrome measurement

- Syndrome measurement (3-qubit bit flip code)
  - Code subspace {|000⟩, |111⟩}
  - Phase shift
    - No errors (|000⟩, |111⟩): |α⟩
    - Bit flip X_1 (|100⟩, |011⟩): $|\alpha\, e^{\pm i\theta}\rangle$
    - Bit flip X_2 (|010⟩, |101⟩): $|\alpha\, e^{\pm 3i\theta}\rangle$
    - Bit flip X_3 (|001⟩, |110⟩): $|\alpha\, e^{\pm 2i\theta}\rangle$
  - No need for fault-tolerant ancilla preparation
    - ← Use of robust coherent state as an ancilla

	Micro-spheres		
	1	2	3
|0⟩	0	θ	–θ
|1⟩	θ	–2θ	θ



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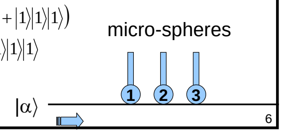
Encoding

1. Transfer qubit information from a photon to micro-sphere #1
2. Prepare micro-spheres #1 and #2 in |0⟩+|1⟩ state
3. Run error correcting procedure

$$
\begin{aligned}
&(\alpha|0\rangle + \beta|1\rangle)(|0\rangle + |1\rangle)(|0\rangle + |1\rangle) \\
&= \alpha(|0\rangle|0\rangle|0\rangle + |0\rangle|0\rangle|1\rangle + |0\rangle|1\rangle|0\rangle + |0\rangle|1\rangle|1\rangle) \\
&+ \beta(|1\rangle|0\rangle|0\rangle + |1\rangle|0\rangle|1\rangle + |1\rangle|1\rangle|0\rangle + |1\rangle|1\rangle|1\rangle) \\
&\xrightarrow{\text{Error Correction}} \alpha|0\rangle|0\rangle|0\rangle + \beta|1\rangle|1\rangle|1\rangle
\end{aligned}
$$



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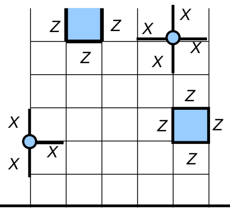
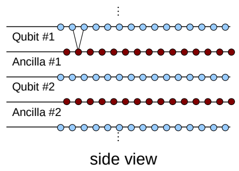
Example (2): 2D nuclear spin network

- Topological (planar) code
  - Check operators = *XXXX* or *ZZZZ*
  - Stack of 2D nuclear spin network
    - Planes of encoded qubits
    - Planes of ancilla qubits





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Naturally fault tolerant quantum computation

Natural error suppression

- Physical system with Hamiltonian
  $H = -J\,(g_1 + g_2 + g_3 + g_4 + \cdots)$
  - g_i (i = 1, 2, ⋯, *n*-*k*) : Stabilizer generators of error correcting code
  - No errors: Ground state
  - An error raises energy by 2*J*
- Natural error suppression
  - $2J \ll k_BT$ → No-error states energetically favored
  - Timescale ≫ 1/J → Probability amplitude of state with an error $e^{-iJt} \sim 0$

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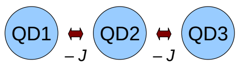
Example (1): 3-qubit bit flip code on coupled quantum dots

- 3-qubit bit flip code
  - Stabilizer generators: Z_1Z_2, Z_2Z_3
  - Hamiltonian: $H = -J(Z_1Z_2+Z_2Z_3)$
    [Ferromagnetic interaction]
  - Natural error suppression
    - No error: Ground state $E = -2J$
    - Bit flip error X_1, X_2, or X_3: $E = 0$



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Example (2): Topological code

- Consider the Hamiltonian
  $$H = -J\sum_P Z_aZ_bZ_cZ_d - J\sum_S X_aX_bX_cX_d$$
- Ground state of the system forms topological code subspace
  - Errors would increase the energy and be suppressed by $e^{-2J/kT}$
  - Physical realization
    - Intrinsically fault-tolerant
    - Much less overhead for error correction
    - Likely to be scalable!

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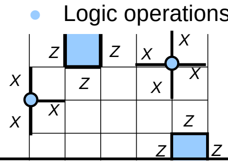
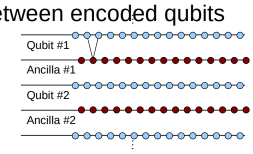
Physical system: Nuclear spin network

- Physical system
  - Stack of 2D nuclear spin network
    - Planes of encoded qubits
    - Planes of ancilla qubits
  - + constantly simulated 4-body interactions (using ancilla qubits)
  - Logic operations between encoded qubits





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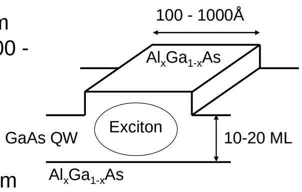
Physical system: $Al_xGa_{1-x}As$/GaAs fluctuating quantum dots

- $Al_xGa_{1-x}As$/GaAs well structures
  - 10-20 monolayer (ML) thick
  - Fluctuating quantum dots at interface (100 - 1000Å) by growth interruption
  - An exciton (Bohr radius ~ 300Å) is trapped in a quantum dot



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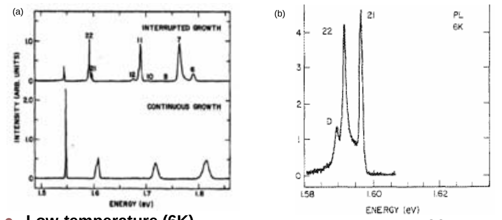
Photoluminescence spectra from QWs



- **Low-temperature (6K) photoluminescence (PL) spectra of four single quantum wells grown continuously (bottom) and with interruption at hetero-interfaces (top)**
- **6 K PL spectrum of free excitons of a single quantum well containing 21 and 22 monolayers. D denotes defect-related bound excitons.**

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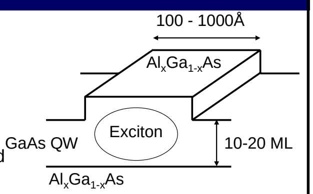
Proposed system

- GaP monolayer
  - Grown in the quantum well
  - P nuclear spins = qubits
  - Optical initialization and readout via hyperfine interaction
  - Dipole coupling among qubits
  - Akin to arrangement of qubits for topological code.



- Four body interactions simulated by applying constant NMR pulse sequence
- Code subspace = ground state
- → Inherent error suppression

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Summary

- *Error correction* and *fault-tolerance* are inevitable for quantum computation
- In existing theories
  - Large number of qubits
  - Non-local operations
  - Non-local entanglement
  - All correction by code

  are required
- Good design of qubits is required at qubit element level for scalability
  - Suitable design for syndrome measurement; OR
  - System with natural fault tolerance

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