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# Electron spins in nonmagnetic semiconductors



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Physics of non-interacting spins

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Optical spin injection and detection

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Spin manipulation in nonmagnetic semiconductors

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In non-magnetic semiconductors such as GaAs and Si, spin interactions are weak; To first order approximation, they behave as non-interacting, independent spins.

- Zeeman Hamiltonian
- Bloch sphere
- Larmor precession
- $T_1$ ,  $T_2$ , and  $T_2^*$
- Bloch equation



# The Zeeman Hamiltonian

Hamiltonian for an electron spin in a magnetic field

$$H = -\vec{M} \cdot \vec{B} = g\mu_B \vec{s} \cdot \vec{B}$$

$\vec{M}$ : magnetic moment

$\vec{B}$ : magnetic field

magnetic moment of  
an electron spin

$$\vec{M} = -g\mu_B \vec{s}$$

$g$ : Landé g-factor  
(=2 for free electrons)

$$\mu_B = \frac{e\hbar}{2m} : \text{Bohr magneton} \\ = 58 \mu\text{eV/T}$$

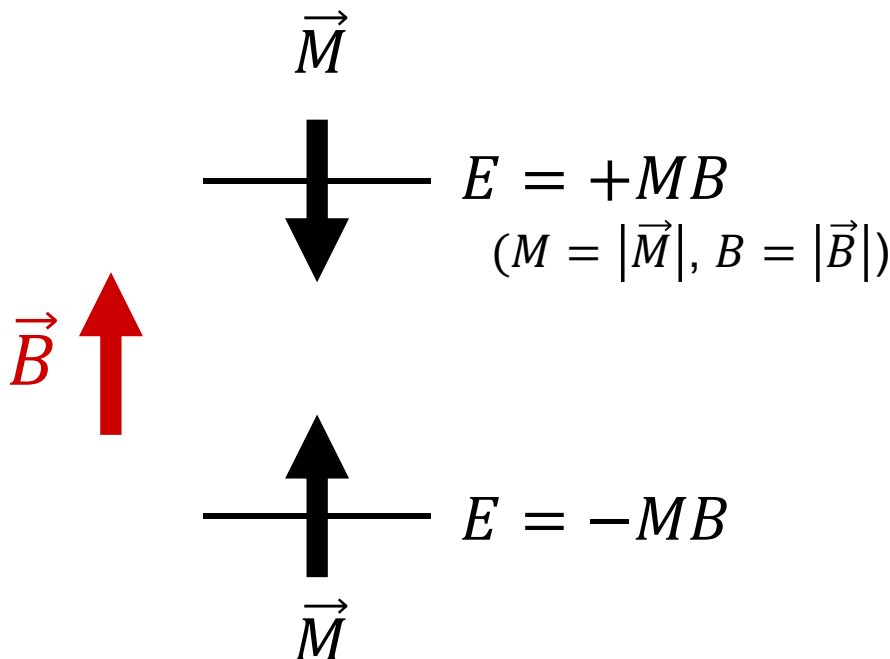
$e$ : electronic charge

$\hbar$ : Planck constant

$m$ : free electron mass

$$\vec{s} = \frac{\vec{\sigma}}{2} : \text{spin operator}$$

$\vec{\sigma}$ : Pauli operator



# The Zeeman Hamiltonian

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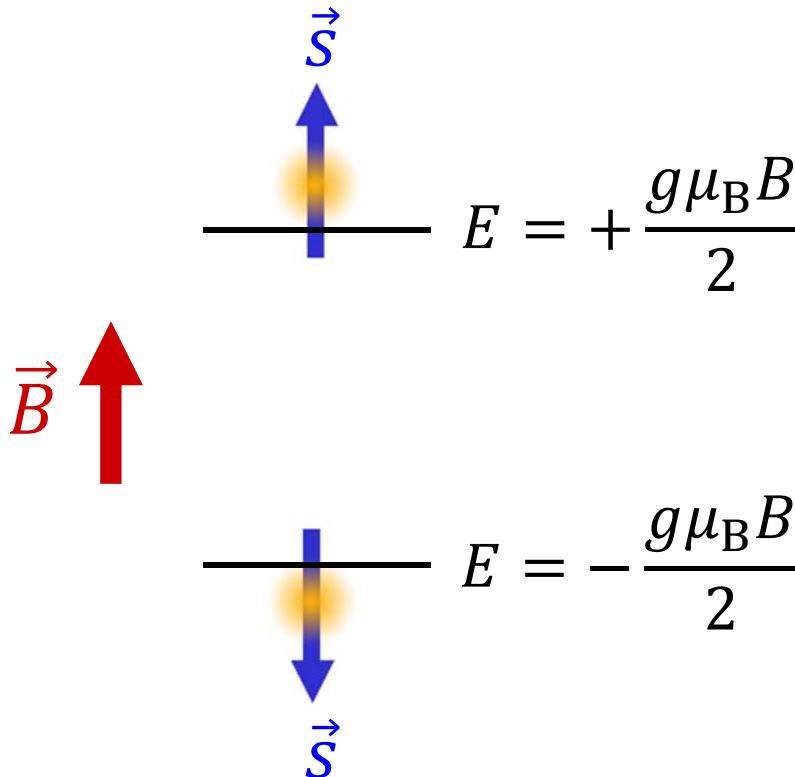
$e$ : electronic charge

$\hbar$ : Planck constant

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$$\vec{S} = \frac{\vec{\sigma}}{2} : \text{spin operator}$$

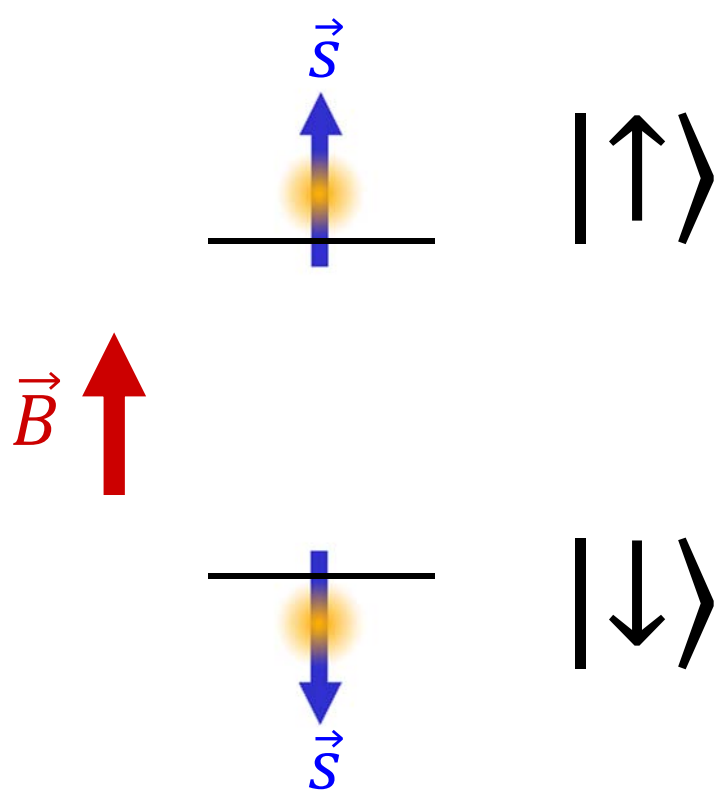
$\vec{\sigma}$ : Pauli operator



# The energy eigenstates

Without loss of generality, we can set the  $z$ -axis to be the direction of  $\vec{B}$ , i.e.,  $\vec{B} = (0,0,B)$

$$H = g\mu_B \vec{S} \cdot \vec{B} = g\mu_B B s_z$$



Spin "up"  $s_z |\uparrow\rangle = +\frac{1}{2} |\uparrow\rangle$   
 $s_z = +\frac{1}{2}$   $E = +\frac{g\mu_B B}{2}$

Spin "down"  $s_z |\downarrow\rangle = -\frac{1}{2} |\downarrow\rangle$   
 $s_z = -\frac{1}{2}$   $E = -\frac{g\mu_B B}{2}$

# Spinor notation and the Pauli operators

Quantum states are represented by a normalized vector in a Hilbert space

➔ Spin states are represented by 2D vectors

$$|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Spin operators are represented by 2x2 matrices. For example, the Pauli operators are

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

ex.  $s_z |\uparrow\rangle = +\frac{1}{2} |\uparrow\rangle$  is equivalent to

$$s_z |\uparrow\rangle = \frac{\sigma_z}{2} |\uparrow\rangle = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = +\frac{1}{2} |\uparrow\rangle$$

# Expectation values of the Pauli vector

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Expectation values for the spin up state

$$\langle \uparrow | \sigma_x | \uparrow \rangle = (1 \ 0) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = (1 \ 0) \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0$$

$$\langle \uparrow | \sigma_y | \uparrow \rangle = (1 \ 0) \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = (1 \ 0) \begin{pmatrix} 0 \\ i \end{pmatrix} = 0$$

$$\langle \uparrow | \sigma_z | \uparrow \rangle = (1 \ 0) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = (1 \ 0) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1$$

$\langle \vec{\sigma} \rangle = (0,0,1)$   spin is pointing in the +z direction



# Expectation values of the Pauli vector

One more example: Expectation values for a coherent superposition

$$|\varphi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\langle\varphi|\sigma_x|\varphi\rangle = \frac{1}{2} \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1$$

$$\langle\varphi|\sigma_y|\varphi\rangle = \frac{1}{2} \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} -i \\ i \end{pmatrix} = 0$$

$$\langle\varphi|\sigma_z|\varphi\rangle = \frac{1}{2} \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = 0$$

$\langle\vec{\sigma}\rangle = (1,0,0)$   spin is pointing in the +x direction 

spin can point in any direction, but when measured along a particular axis, it can only be “up” or “down”



# The Bloch sphere

In general,  $|\varphi\rangle = \lambda|\uparrow\rangle + \mu|\downarrow\rangle$  where  $\lambda$  and  $\mu$  are complex numbers.

Any spin state (and any qubit) can be represented by a point on a sphere

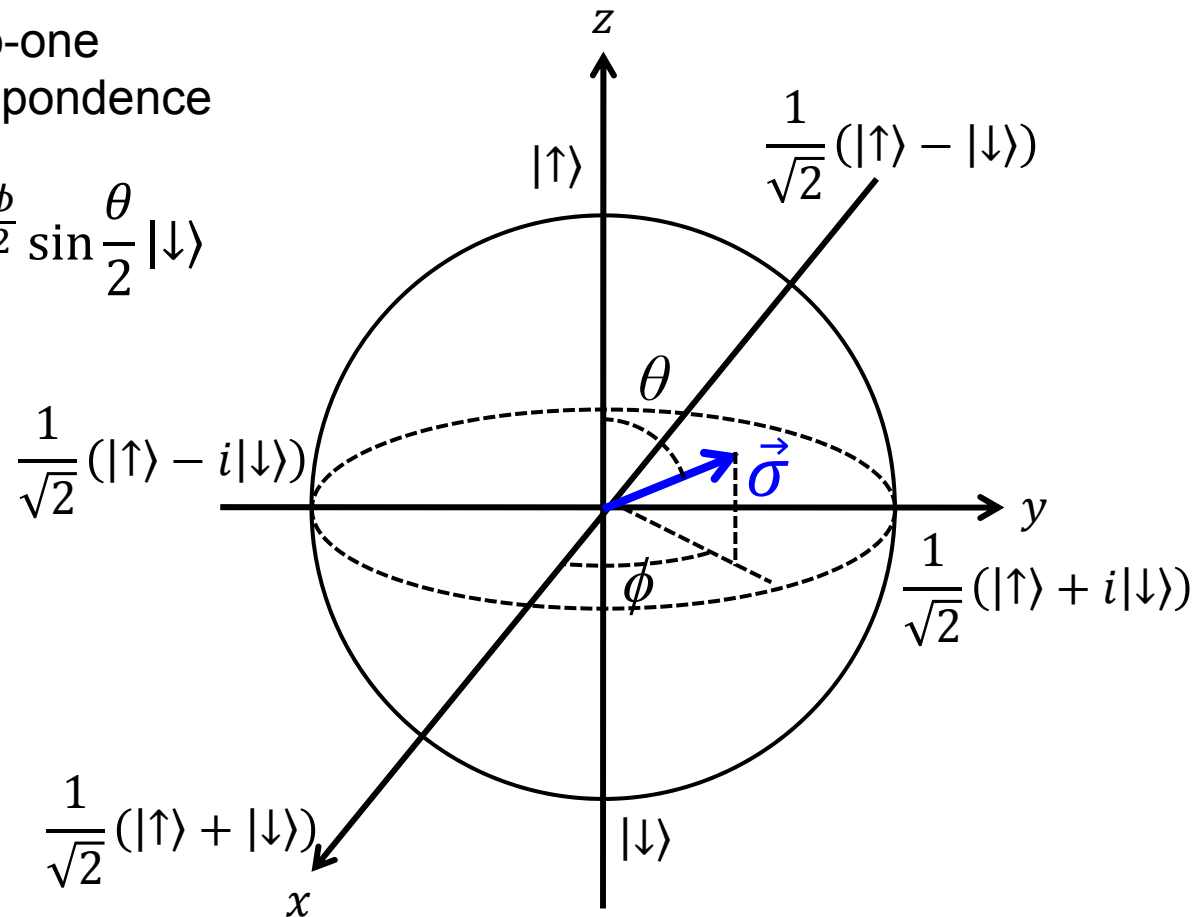
$$\langle \vec{\sigma} \rangle = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$$

↑ one-to-one  
↓ correspondence

$$|\varphi\rangle = e^{-i\frac{\phi}{2}} \cos \frac{\theta}{2} |\uparrow\rangle + e^{+i\frac{\phi}{2}} \sin \frac{\theta}{2} |\downarrow\rangle$$



This is the most general state. 2 complex coefficients have 4 degrees of freedom, minus normalization condition and arbitrariness of the global phase.



# Time evolution of spin states

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How does a spin state that is perpendicular to magnetic field evolve with time?

$$\text{Initial state: } |\varphi(t = 0)\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle + |\downarrow\rangle)$$

1. Obtain the time-evolution operator

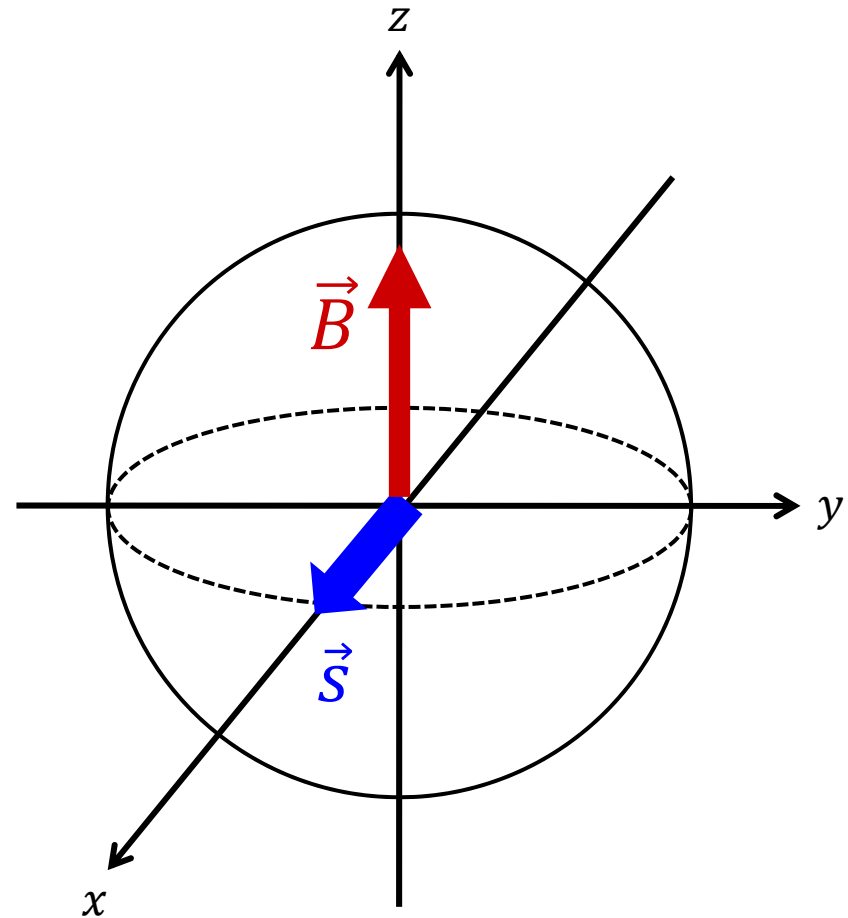
$$U(t) = \exp\left(\frac{Ht}{i\hbar}\right)$$

2. Compute the state at time  $t$

$$|\varphi(t)\rangle = U(t)|\varphi(0)\rangle$$

3. Calculate the expectation value of  $\vec{\sigma}$

$$\langle\varphi(t)|\vec{\sigma}|\varphi(t)\rangle$$



# Time-evolution operator

1. Time-evolution operator is given by

$$U(t) = \exp\left(\frac{Ht}{i\hbar}\right) \quad \text{where} \quad H = g\mu_B B s_z = \frac{g\mu_B B}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

so

$$U(t) = \begin{pmatrix} \exp\left(-i\frac{g\mu_B B t}{2\hbar}\right) & 0 \\ 0 & \exp\left(+i\frac{g\mu_B B t}{2\hbar}\right) \end{pmatrix} = \begin{pmatrix} \exp\left(-i\frac{\Omega t}{2}\right) & 0 \\ 0 & \exp\left(+i\frac{\Omega t}{2}\right) \end{pmatrix}$$

where  $\Omega = \frac{g\mu_B B}{\hbar}$  is the Larmor frequency  $\left(\frac{\mu_B}{2\pi\hbar} = 14 \text{ GHz/T}\right)$

2. The state at time  $t$  is obtained by applying  $U(t)$  on the initial state

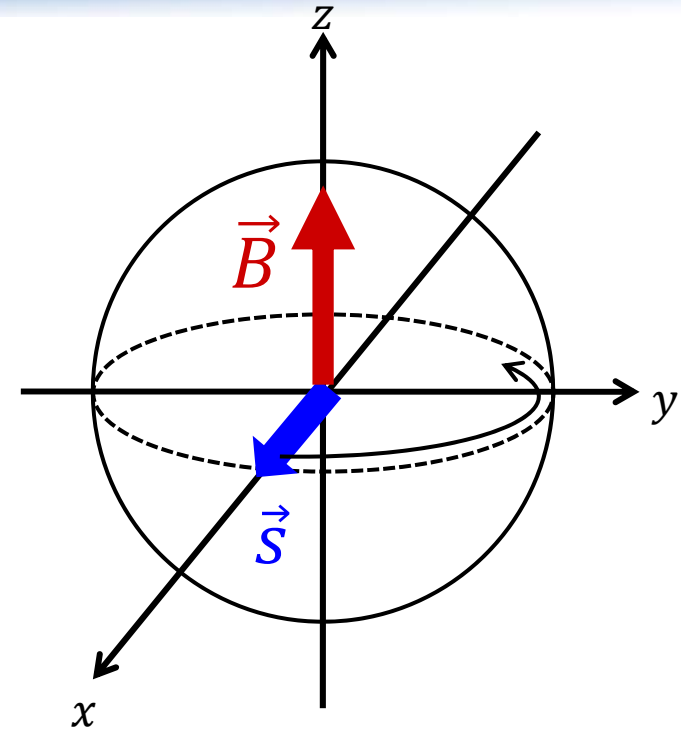
$$|\varphi(0)\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle + |\downarrow\rangle) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$|\varphi(t)\rangle = U(t)|\varphi(0)\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \exp(-i\Omega t/2) & 0 \\ 0 & \exp(+i\Omega t/2) \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \exp(-i\Omega t/2) \\ \exp(+i\Omega t/2) \end{pmatrix}$$

3. Expectation value of  $\vec{\sigma}$

$$\begin{aligned} \langle \varphi(t) | \sigma_x | \varphi(t) \rangle &= \frac{1}{2} \begin{pmatrix} e^{+i\Omega t/2} & e^{-i\Omega t/2} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} e^{-i\Omega t/2} \\ e^{+i\Omega t/2} \end{pmatrix} \\ &= \frac{1}{2} (e^{+i\Omega t} + e^{-i\Omega t}) \\ &= \cos \Omega t \end{aligned}$$

$$\begin{aligned} \langle \varphi(t) | \sigma_y | \varphi(t) \rangle &= \frac{1}{2} \begin{pmatrix} e^{+i\Omega t/2} & e^{-i\Omega t/2} \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} e^{-i\Omega t/2} \\ e^{+i\Omega t/2} \end{pmatrix} \\ &= -\frac{i}{2} (e^{+i\Omega t} - e^{-i\Omega t}) \\ &= \sin \Omega t \end{aligned}$$



spin precession in the  $xy$  plane at an angular frequency  $\Omega = g\mu_B B/\hbar$

$$\langle \varphi(t) | \sigma_z | \varphi(t) \rangle = \frac{1}{2} \begin{pmatrix} e^{+i\Omega t/2} & e^{-i\Omega t/2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} e^{-i\Omega t/2} \\ e^{+i\Omega t/2} \end{pmatrix} = 1 - 1 = 0$$

# Heisenberg picture of Larmor precession

More intuitive semi-classical picture can be obtained

$$H = g\mu_B \vec{S} \cdot \vec{B} = g\mu_B (B_x S_x + B_y S_y + B_z S_z)$$

$$\frac{dS_x}{dt} = \frac{1}{i\hbar} [S_x, H] = \frac{g\mu_B}{i\hbar} \{ [S_x, S_y] B_y + [S_x, S_z] B_z \}$$

using commutation relations:  $[S_x, S_y] = iS_z$ ,  $[S_y, S_z] = iS_x$ ,  $[S_z, S_x] = iS_y$

$$\frac{dS_x}{dt} = \frac{g\mu_B}{\hbar} \{ S_z B_y - S_y B_z \}$$

$$\frac{dS_y}{dt} = \frac{g\mu_B}{\hbar} \{ S_x B_z - S_z B_x \}$$

$$\frac{dS_z}{dt} = \frac{g\mu_B}{\hbar} \{ S_y B_x - S_x B_y \}$$

combined into one vector equation

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S}$$

where  $\vec{\Omega} = \frac{g\mu_B}{\hbar} \vec{B}$

# Semi-classical equation of motion

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S}$$

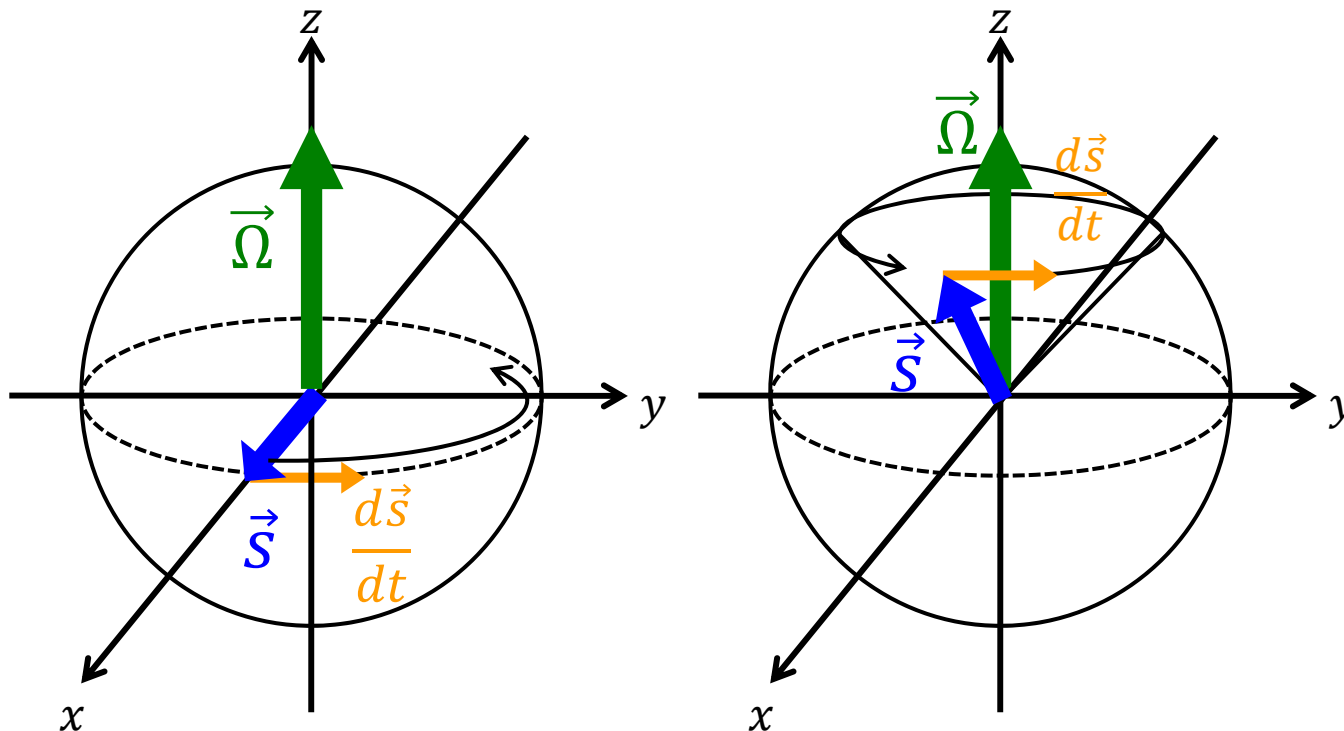
$$\vec{\Omega} = \frac{g\mu_B}{\hbar} \vec{B}$$

rate of change for angular momentum

in terms of magnetization  $\vec{M} = -g\mu_B\vec{S}$   
and spin angular momentum  $\vec{S} = \hbar\vec{s}$

$$\frac{d\vec{S}}{dt} = \vec{M} \times \vec{B} \quad (\text{agrees with classical result!})$$

torque exerted by magnetic field



- Torque is perpendicular to both magnetic field and spin
- Parallel component of spin is conserved
- Perpendicular component precesses

Spin precesses in a cone at frequency  $\Omega$

# Spin relaxation and the Bloch equation

In a real world, interaction with the environment will eventually randomize spin

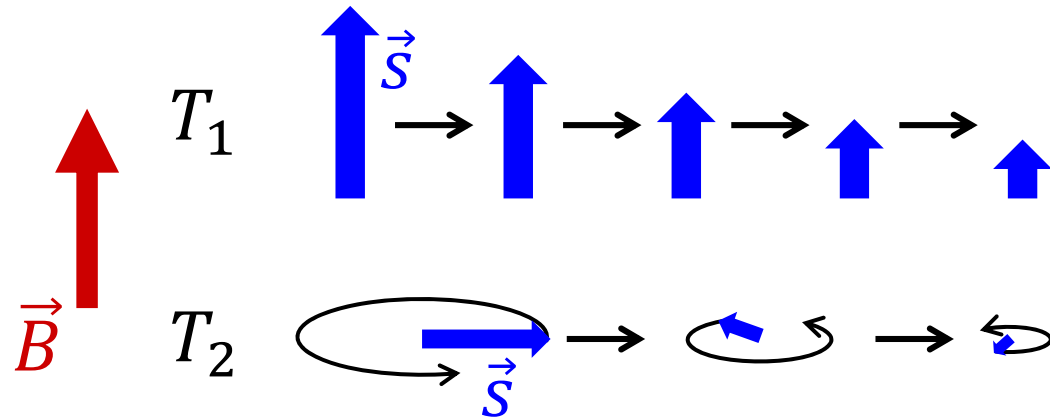
1. Longitudinal spin relaxation ( $T_1$ )

Requires energy relaxation

2. Transverse spin relaxation ( $T_2$ )

Phase relaxation is enough

For an ensemble, Inhomogeneous dephasing ( $T_2^*$ )



$$\frac{d\vec{s}}{dt} = \vec{\Omega} \times \vec{s} - \frac{(\vec{s} \cdot \hat{\Omega})\hat{\Omega}}{T_1} - \frac{\vec{s} - (\vec{s} \cdot \hat{\Omega})\hat{\Omega}}{T_2^*}$$

$$\hat{\Omega} = \frac{\vec{\Omega}}{|\vec{\Omega}|}$$

Bloch equation

$$\frac{d\vec{S}}{dt} = \vec{M} \times \vec{B} - \frac{(\vec{S} \cdot \hat{B})\hat{B}}{T_1} - \frac{\vec{S} - (\vec{S} \cdot \hat{B})\hat{B}}{T_2^*}$$

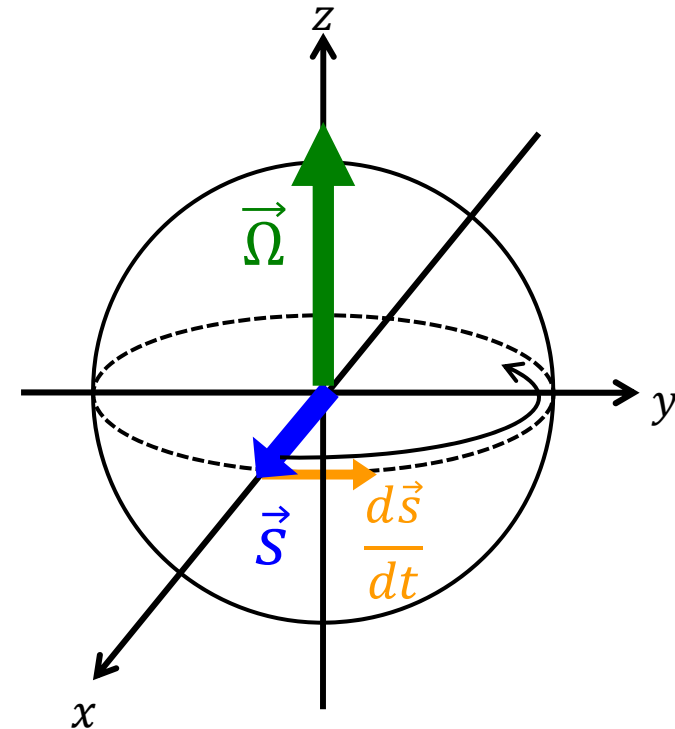
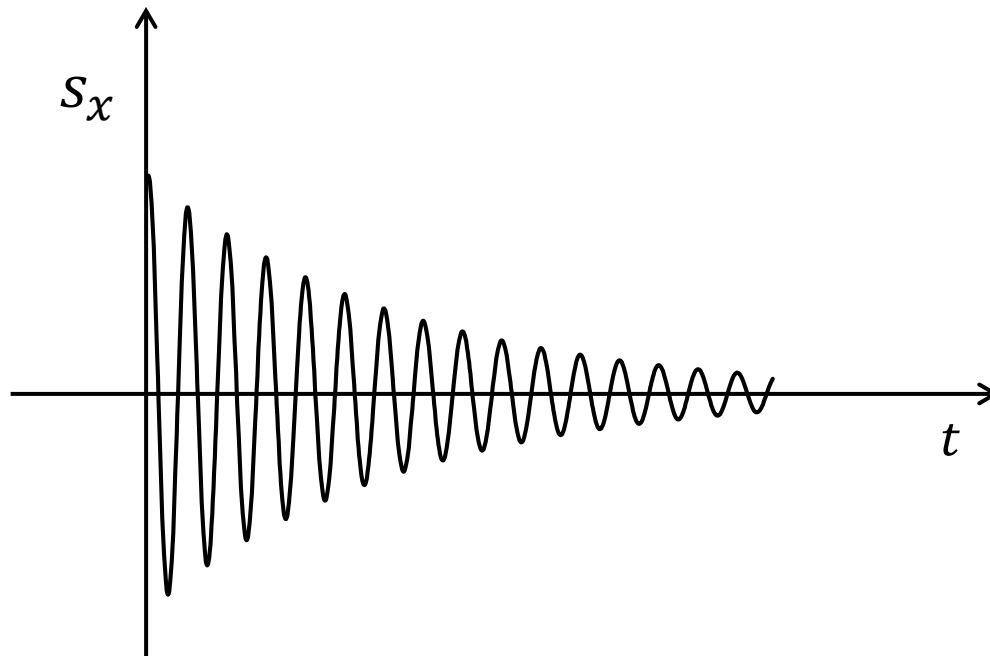
$$\hat{B} = \frac{\vec{B}}{|\vec{B}|}$$

# Larmor precession revisited

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For spins initially pointing along  $x$  in a magnetic field along  $z$ , the solution for spin component along  $x$  is given by

$$s_x(t) = \frac{1}{2} \exp\left(-\frac{t}{T_2^*}\right) \cos \Omega t$$



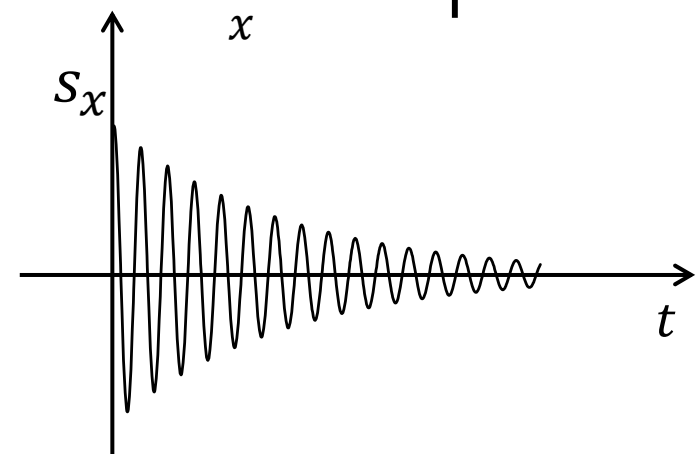
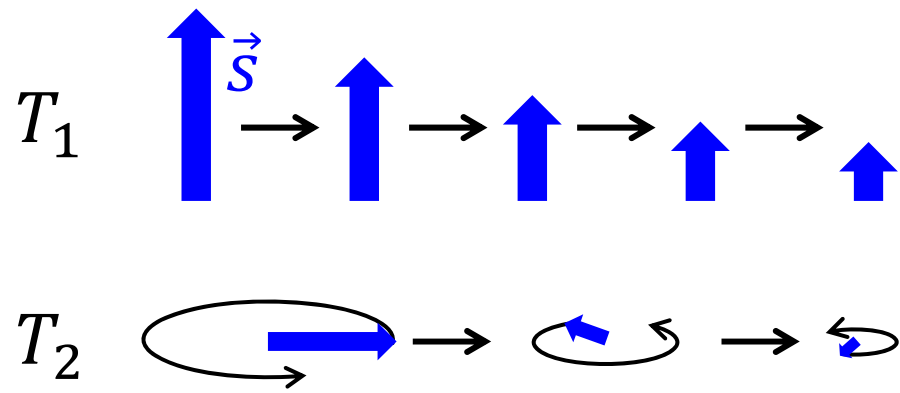
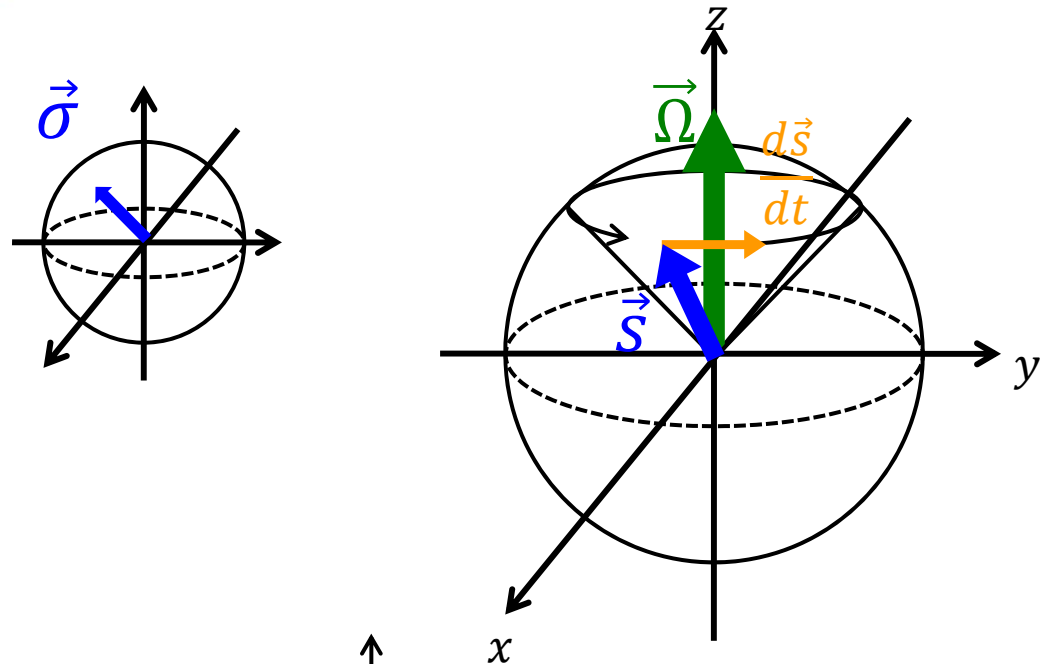


- Zeeman Hamiltonian

$$H = g\mu_B \vec{s} \cdot \vec{B}$$

- Bloch sphere

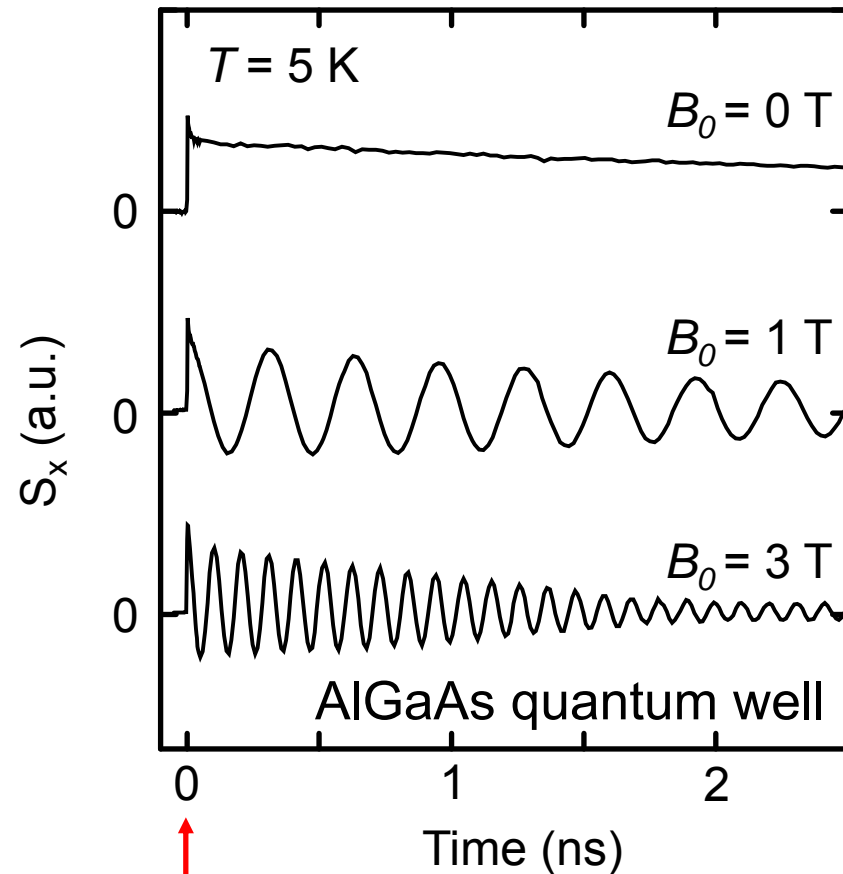
- Larmor precession
- $T_1$ ,  $T_2$ , and  $T_2^*$
- Bloch equation



In direct gap nonmagnetic semiconductors, Larmor precession can be directly observed in the time domain

- Optical selection rules
- Time-resolved Kerr/Faraday rotation
- Hanle effect

Time-resolved Kerr rotation data



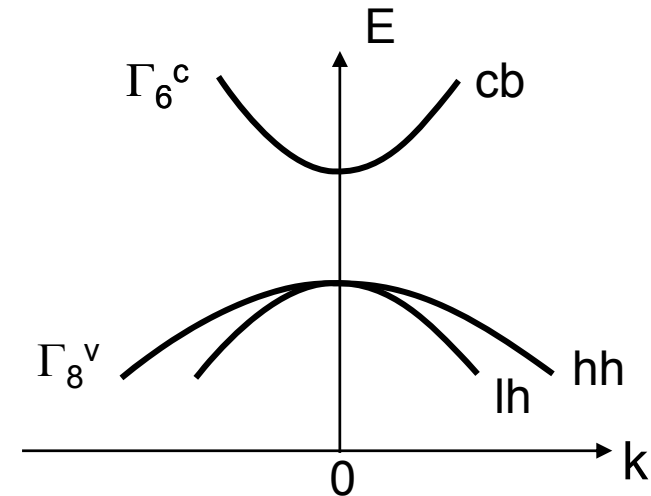
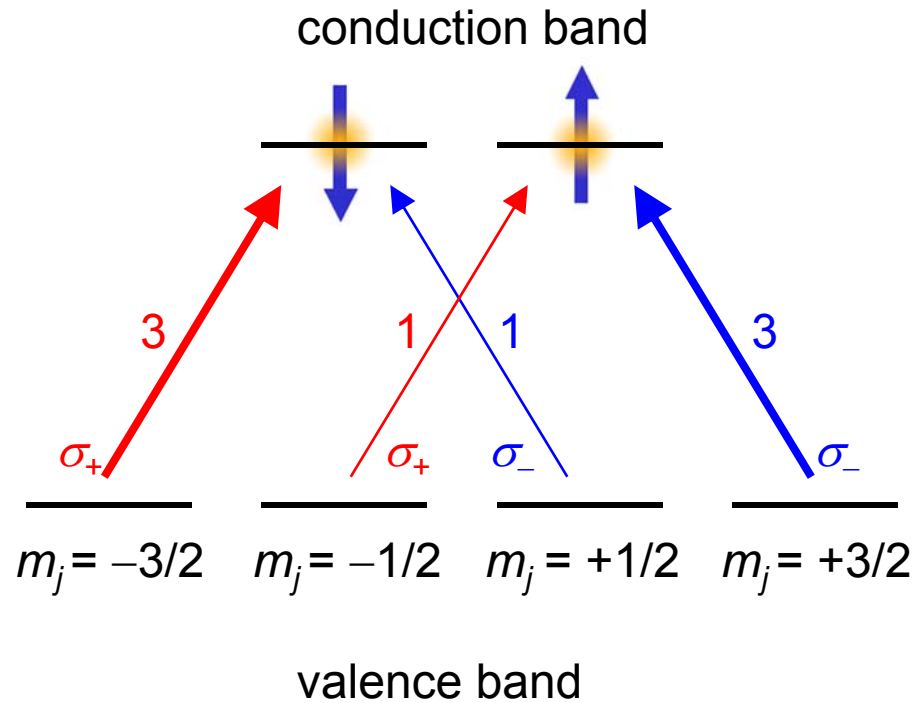
Electron spins are optically injected at  $t=0$

# Optical selection rules

left circularly polarized photons carry angular momentum of +1  
 right circularly polarized photons carry angular momentum of -1  
 (in units of  $\hbar$ )



selection rules from angular momentum conservation

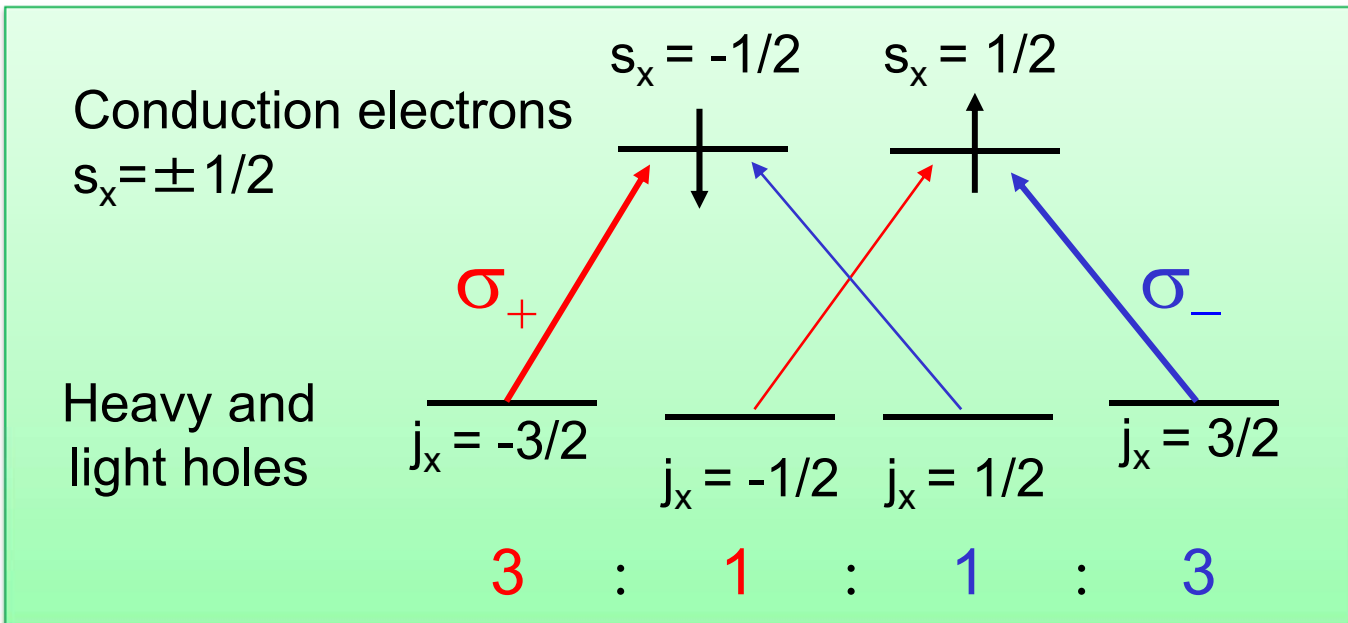
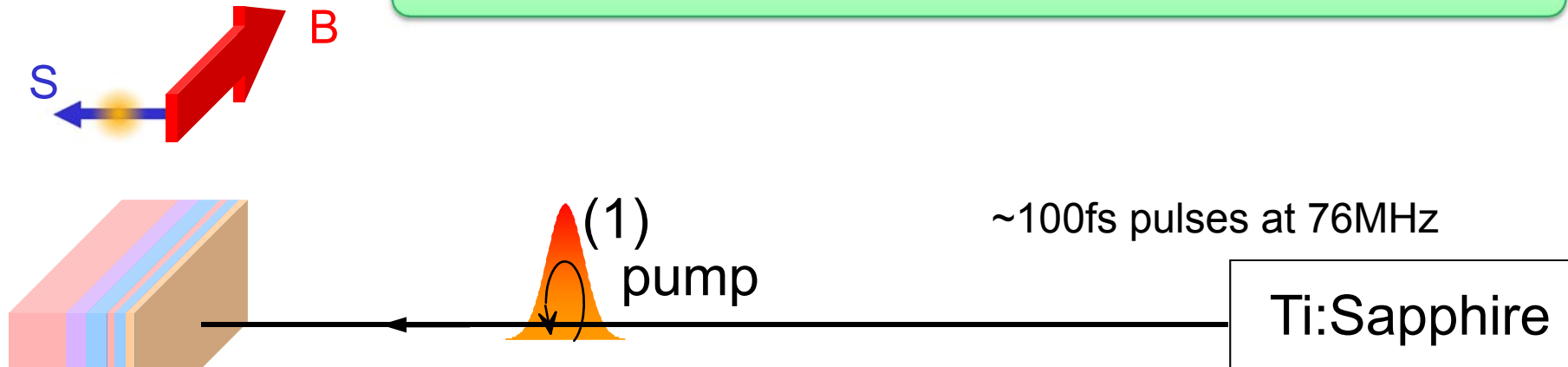


Zinc-blende: 50% polarization

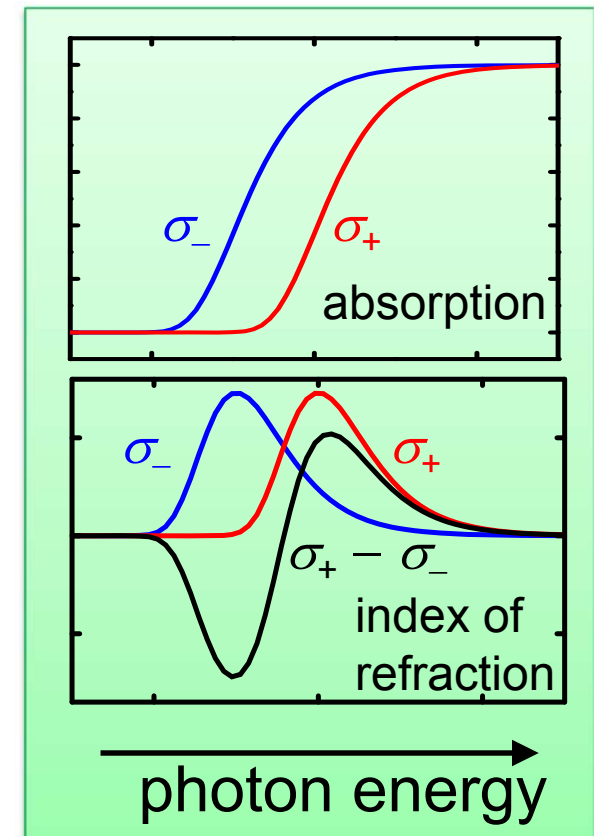
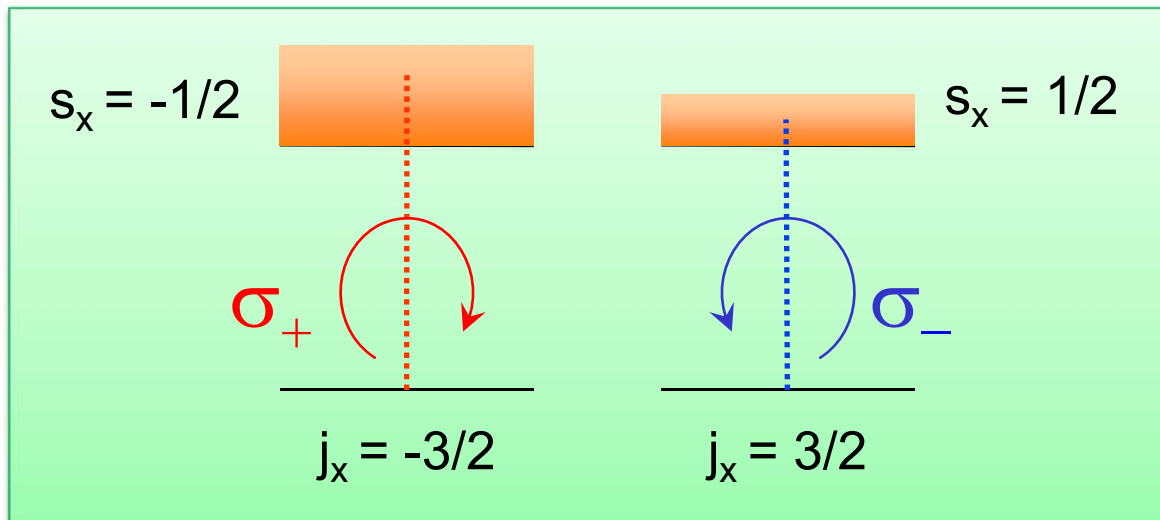
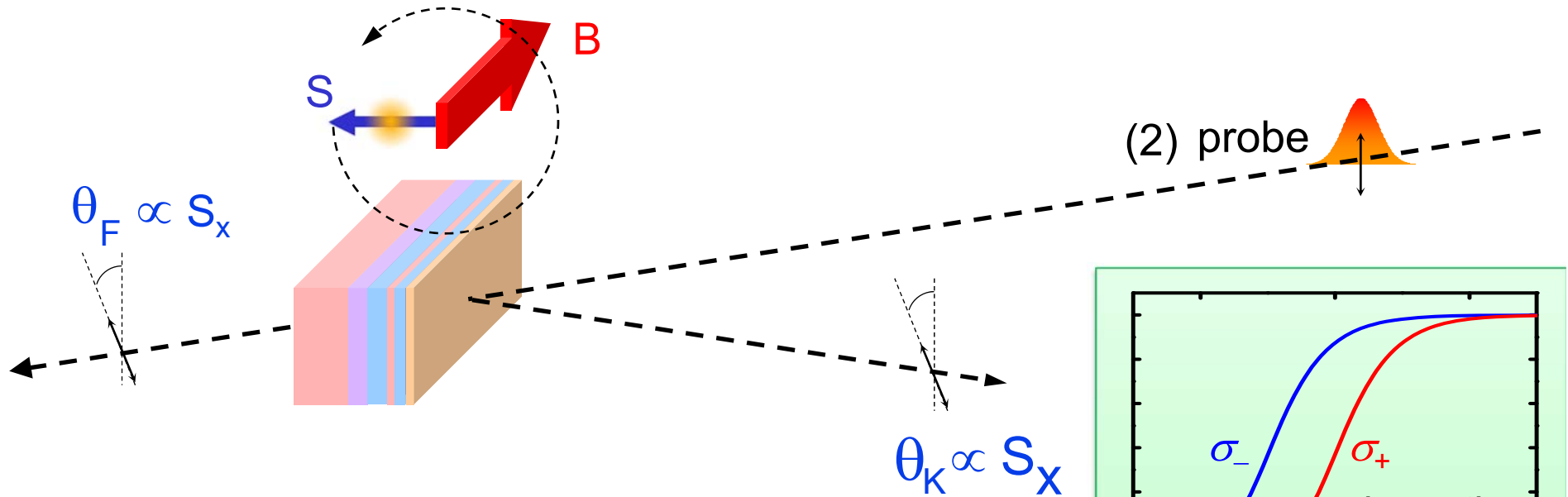
Wurtzite and quantum wells: 100% polarization

# Time-resolved Faraday (Kerr) rotation

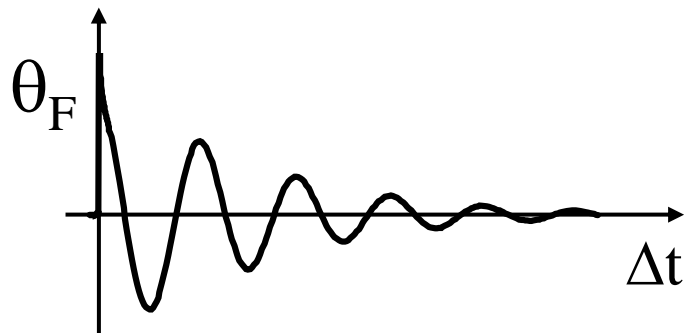
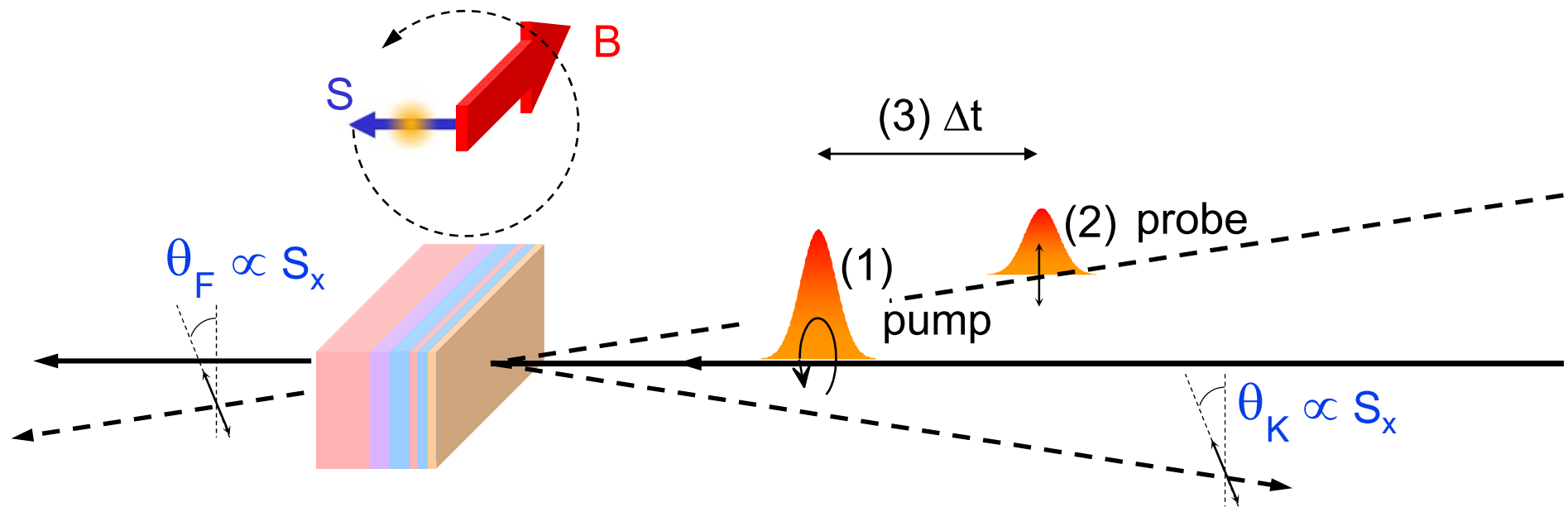
1. Circularly polarized pump creates spin population



1. Circularly polarized pump creates spin population
2. Linearly polarized probe measures  $S_x$



1. Circularly polarized pump creates spin population
2. Linearly polarized probe measures  $S_x$
3.  $\Delta t$  is scanned and spin precession is mapped out

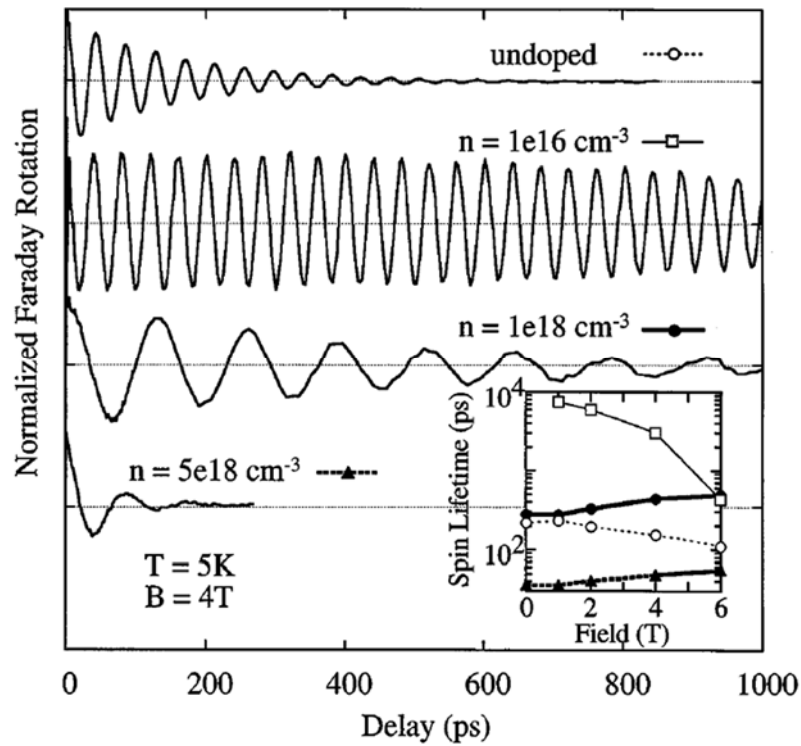


$$\theta_F = A e^{-\Delta t/T_2^*} \cos(\Omega \Delta t)$$

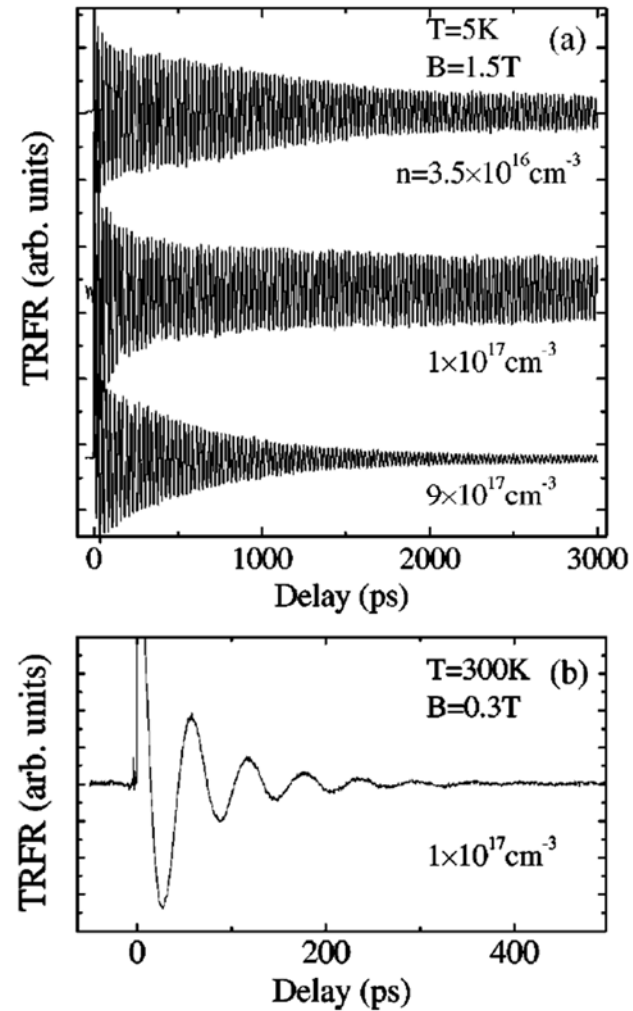
$$\Omega = g\mu_B B / \hbar$$

S. Crooker et al., *PRB* **56**, 7574 (1997)

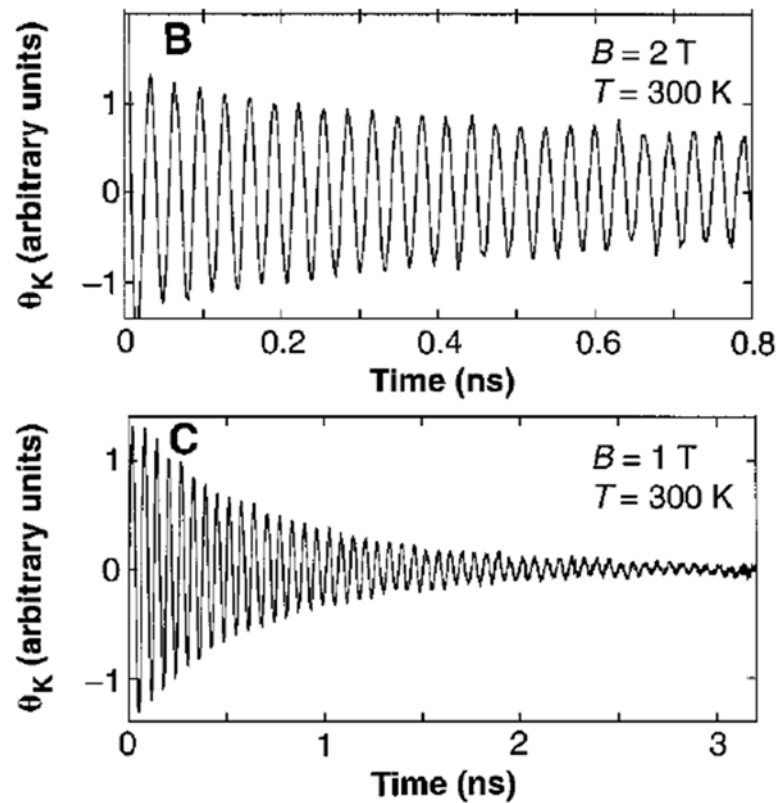
GaAs  
PRL 80, 4313 (1998)



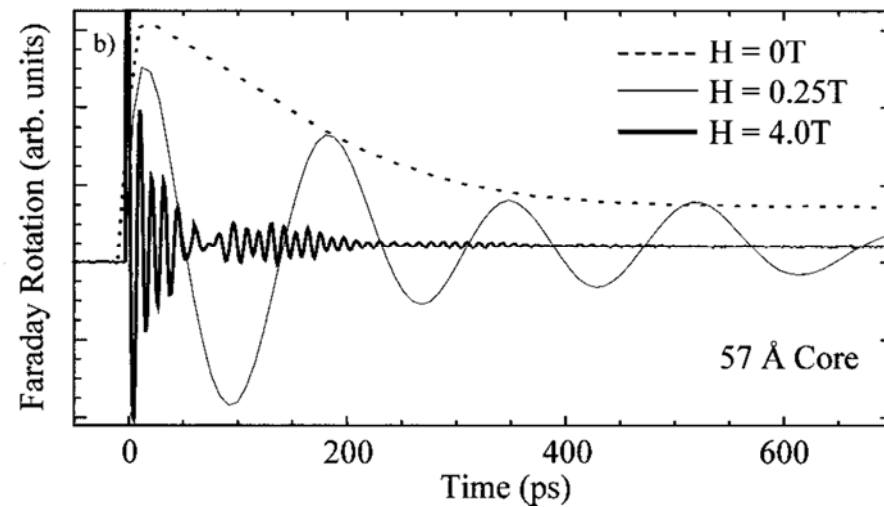
GaN  
PRB 63, 121202 (2001)



ZnCdSe quantum well  
 Science 277, 1284 (1997)



CdSe quantum dots  
 PRB 59, 10421 (1999)





In DC measurements, magnetic field dependence of the spin polarization becomes Lorentzian.

$$\int_0^{\infty} dt \left[ S_0 \exp\left(-\frac{t}{T_2^*}\right) \cos(\Omega t) \right] = \frac{\rho_{el}}{(\Omega T_2^*)^2 + 1}$$

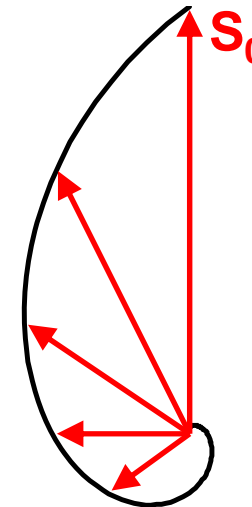
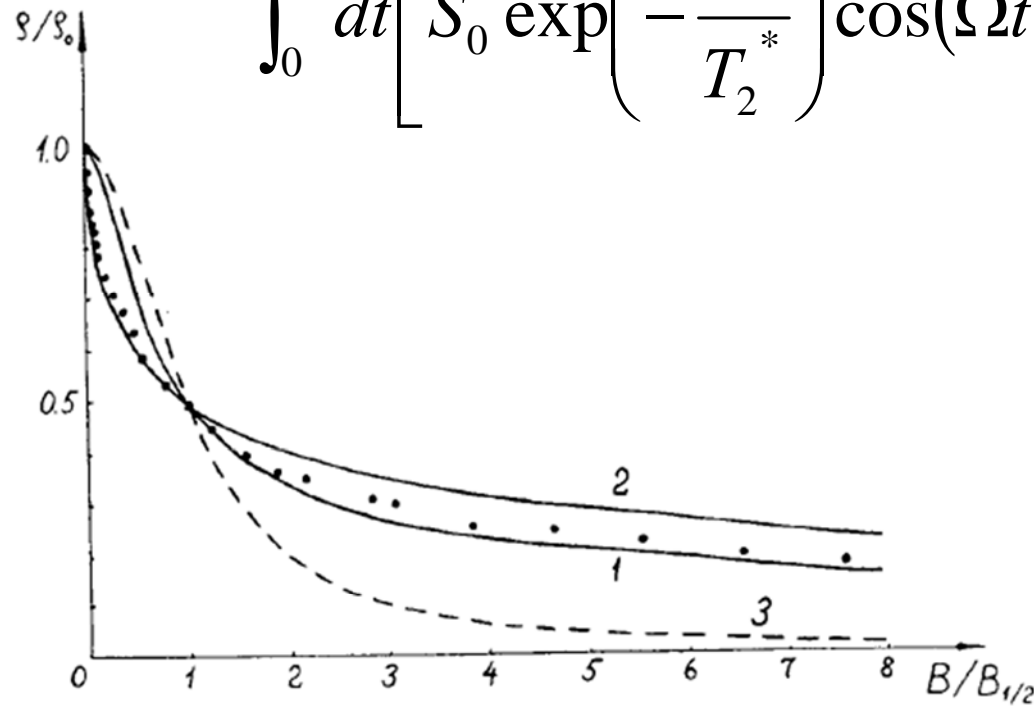
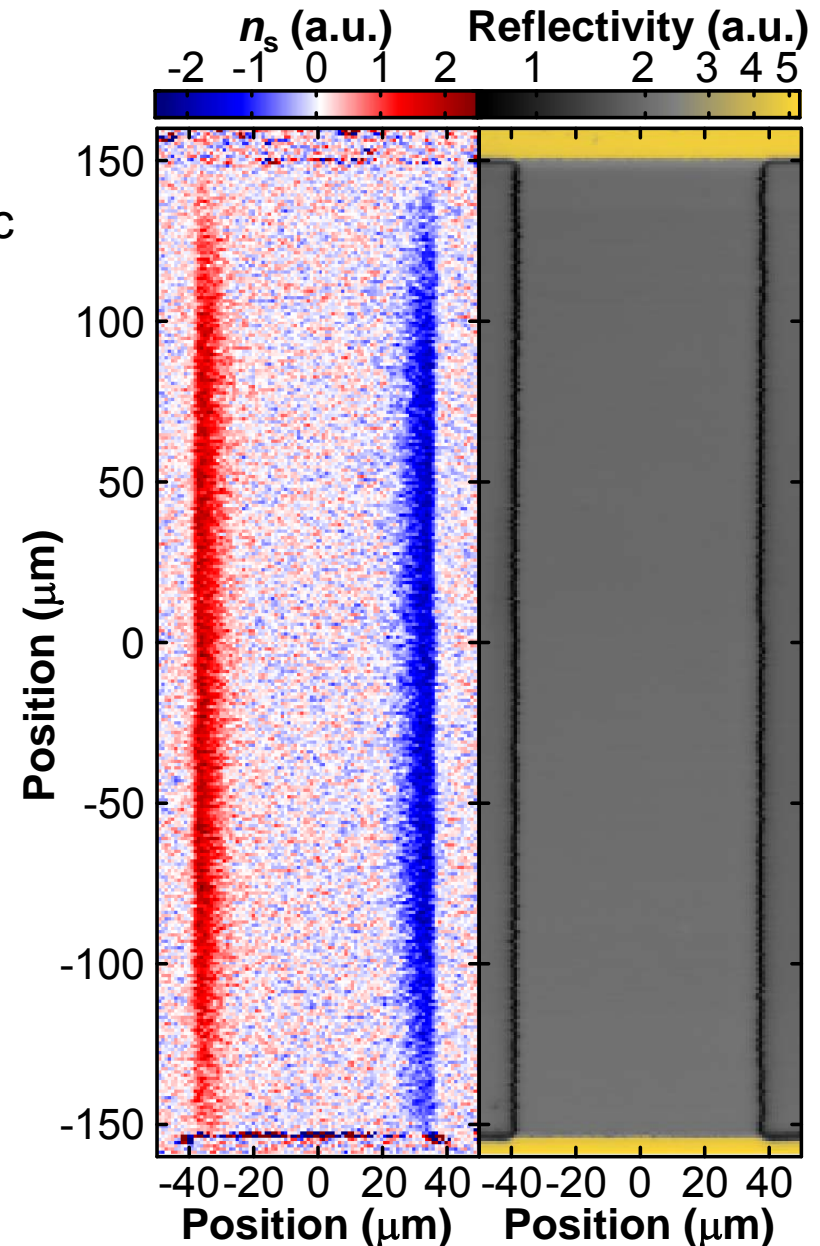
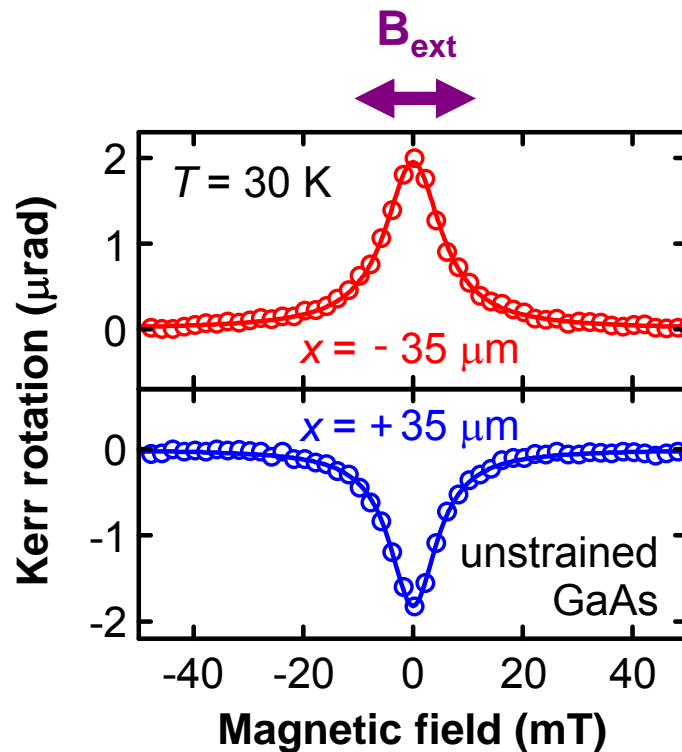


Fig. 14. The Hanle effect in an n-Ga<sub>0.8</sub>Al<sub>0.2</sub>As crystal at 4.2 K for the A-band presented in fig. 10 (Vekua et al. 1976). Curves 1 and 2 were calculated according to eqs. (63) and (64) respectively, curve 3 is the Lorentz contour with halfwidth  $B_{1/2} = 6$  G equal to the halfwidth of the experimental Hanle curve.

“Optical Orientation” (Elsevier, 1984)

Science 306, 1910 (2004)

Spin accumulation at the edges are imaged by modulating the externally applied magnetic field and measuring the signal at the second harmonic frequency



There are many interactions in nonmagnetic semiconductors that allow spin manipulation

- g-factor engineering
- spin-orbit interaction
- hyperfine interaction
- AC Stark effect

$$\mathbf{H} = g\mu_B \mathbf{B} \cdot \mathbf{S}$$

$g$ : material dependent effective g-factor  
 $\mathbf{B}$ : magnetic field

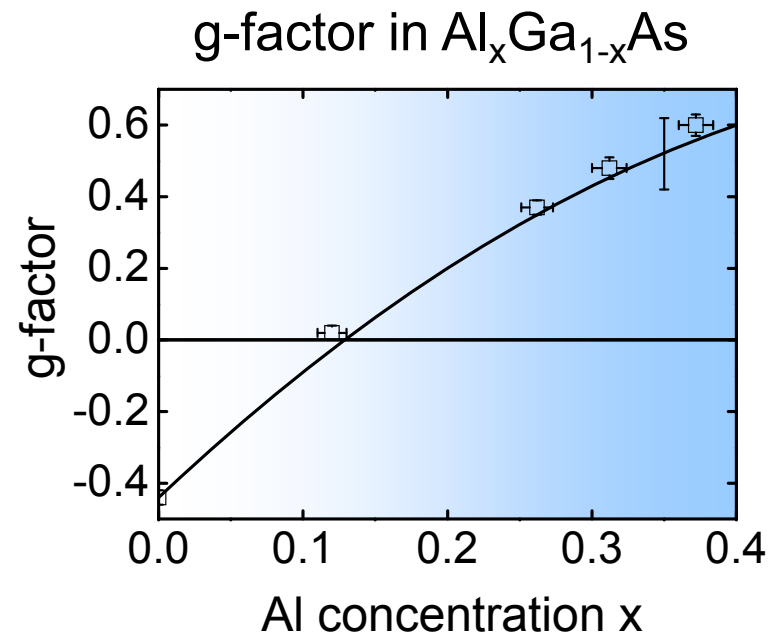
$g = -0.44$  in GaAs  
 $g = -14.8$  in InAs  
 $g = 1.94$  in GaN

Change  $g$  in a static, global  $\mathbf{B}$ !



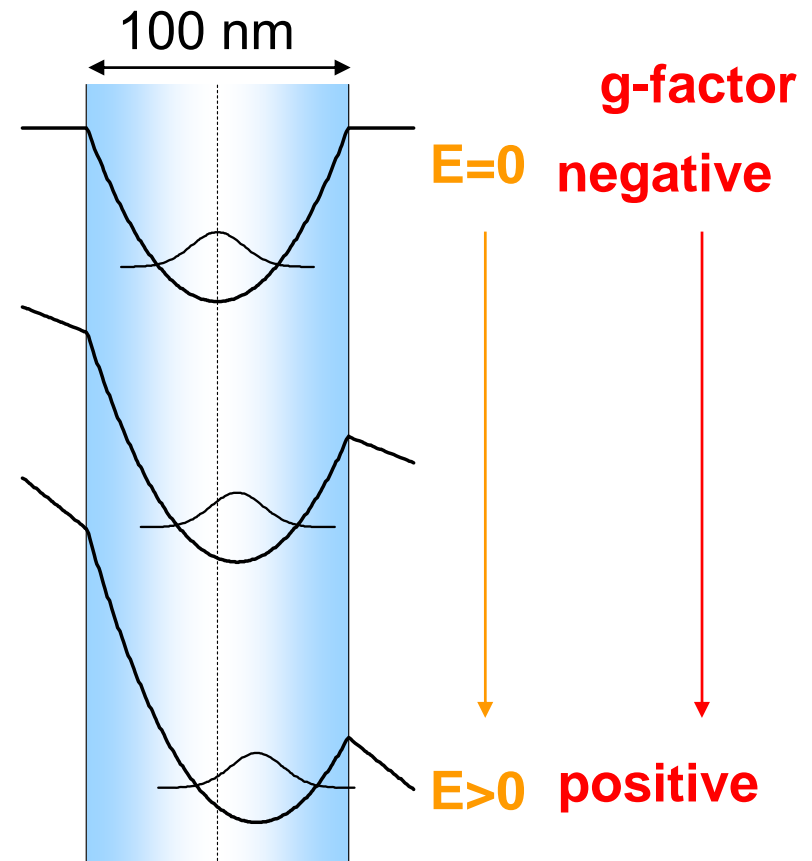
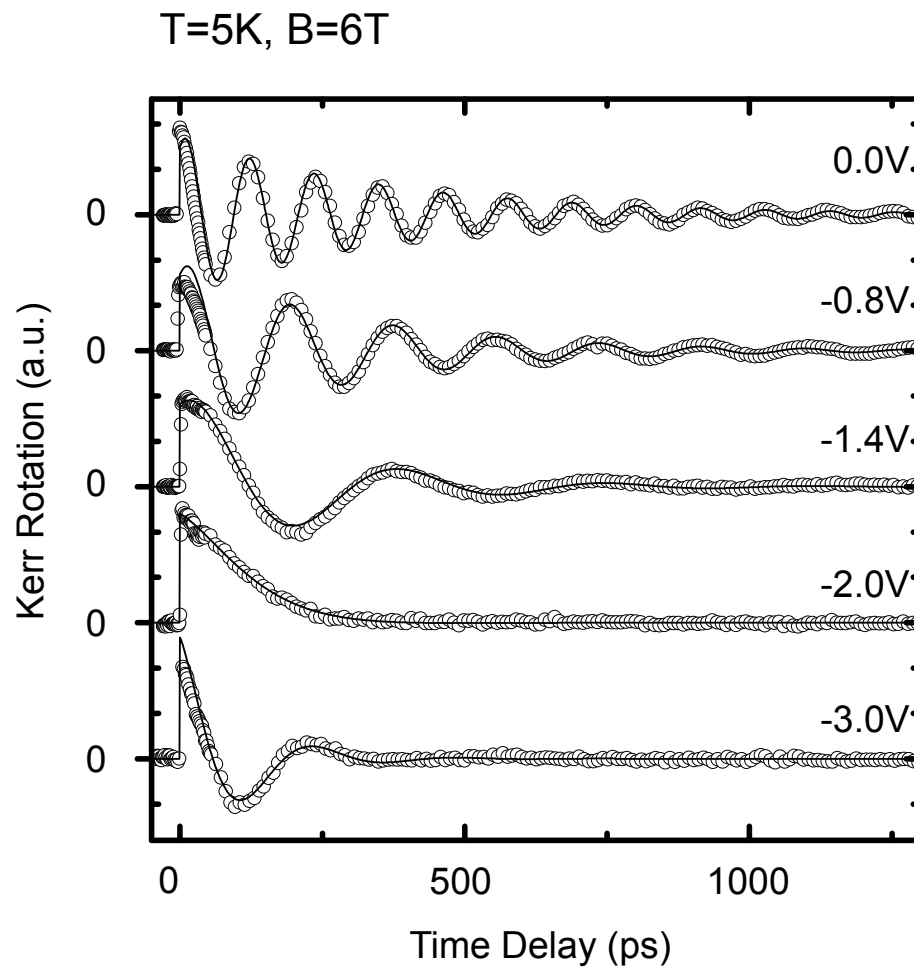
move electrons into different materials  
using electric fields

Control the g-factor through  
material composition in  
semiconductor heterostructures



# Quasi-static electrical tuning of g-factor

*Nature* **414**, 619 (2001)

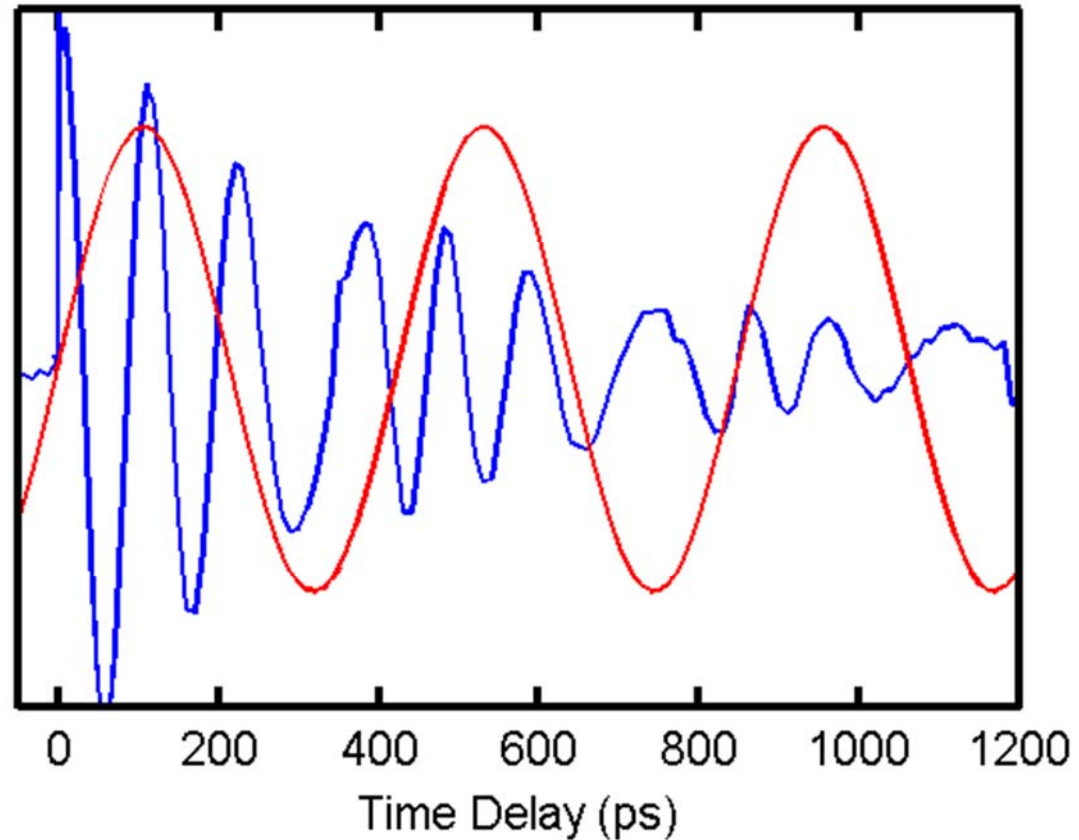
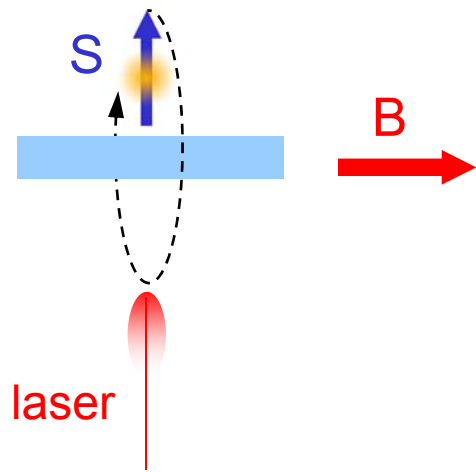


g-factor is electrically tuned

# Frequency modulated spin precession

Time dependent voltage

$$V(t) = V_0 + V_1 \sin(2\pi\nu t)$$



Blue: Kerr Rotation

Red: microwave voltage

Fits well to equation :

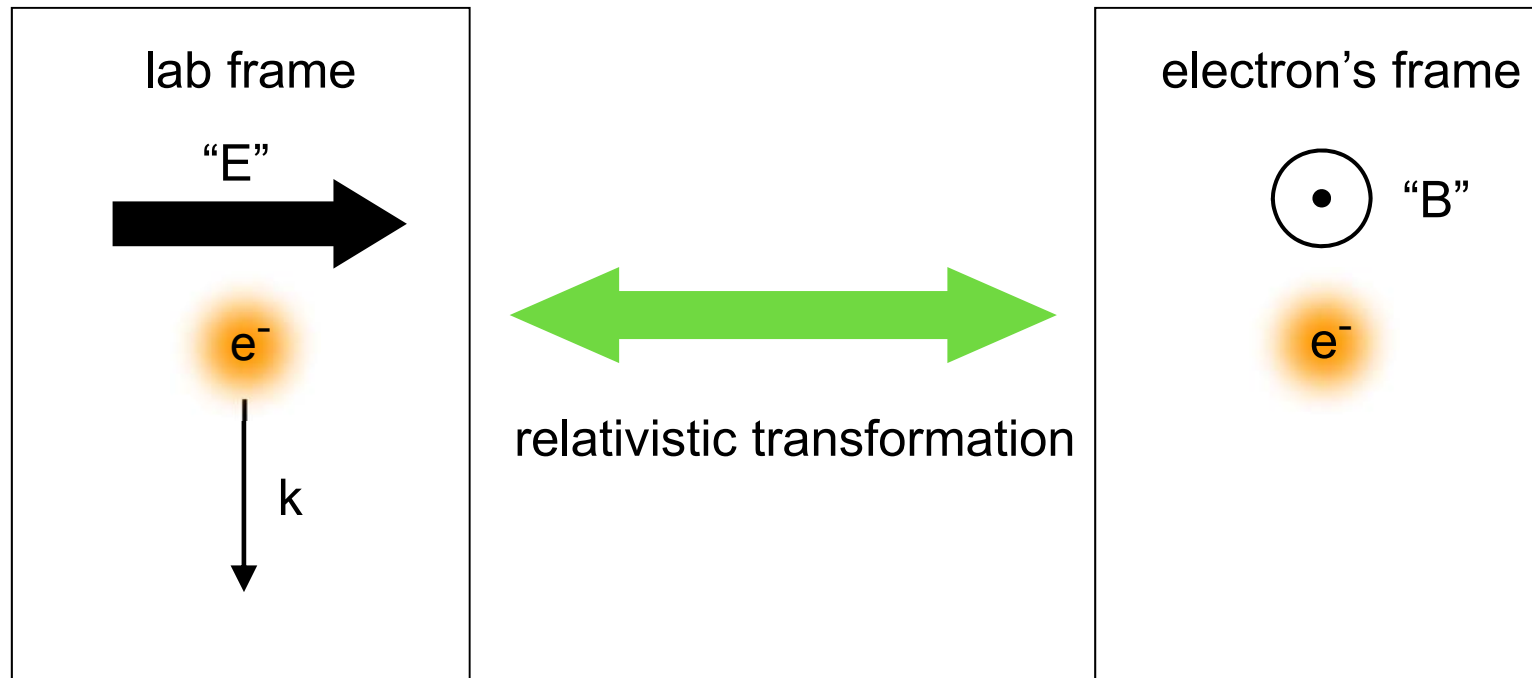
$$Ae^{-\frac{t}{\tau_1}} \cos\left(\omega_0 t + \frac{\omega_1}{\omega_\mu} \cos(\omega_\mu t + \phi_\mu)\right)$$

*Science* **299**, 1201 (2003)

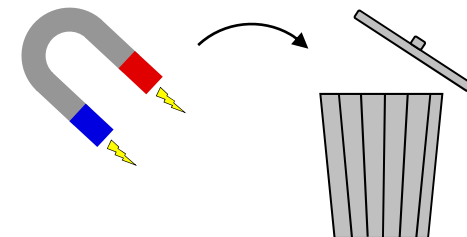
➡ Spin precession is frequency modulated

g-factor tuning at GHz frequency range


$$H = \hbar/(4m^2c^2) (\nabla V(\mathbf{r}) \times \mathbf{p}) \cdot \boldsymbol{\sigma}$$



Allows **zero-magnetic field spin manipulation** by electric field through control of  $k$



time-reversal symmetry  $E(k, \uparrow) = E(-k, \downarrow)$  Kramers degeneracy  
(i.e., at zero magnetic field)

inversion symmetry  $E(k, \uparrow) = E(-k, \uparrow)$   Zero-magnetic-field spin splitting requires asymmetry

Rashba effect: structural inversion asymmetry (SIA) of quantum wells  
Sov. Phys. Solid State 2, 1109 (1960)

$$\vec{\Omega}(\vec{k}) = \alpha(k_y, -k_x)$$

Dresselhaus effect: bulk inversion asymmetry (BIA) of zinc-blende crystal  
Phys. Rev. 100, 580 (1955)

$$\vec{\Omega}(\vec{k}) = 2\gamma\{k_x(k_y^2 - k_z^2), k_y(k_z^2 - k_x^2), k_z(k_x^2 - k_y^2)\}$$

In a (001) quantum well

$$\vec{\Omega}(\vec{k}) = 2\gamma\langle k_z^2 \rangle(k_x, -k_y)$$

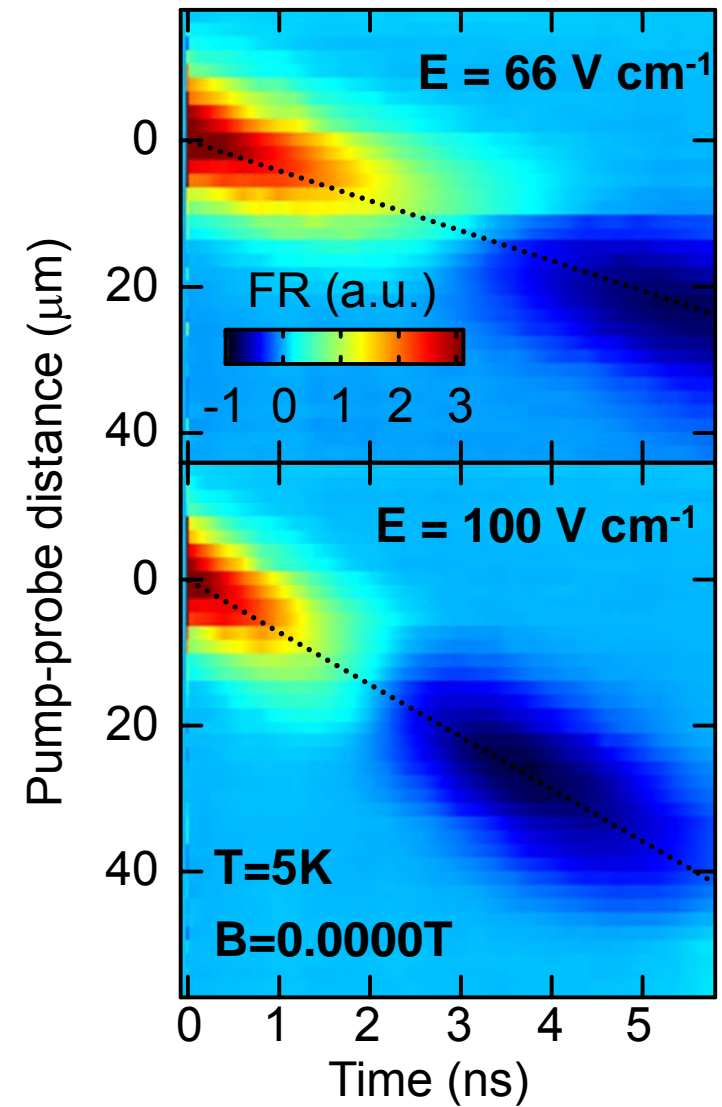
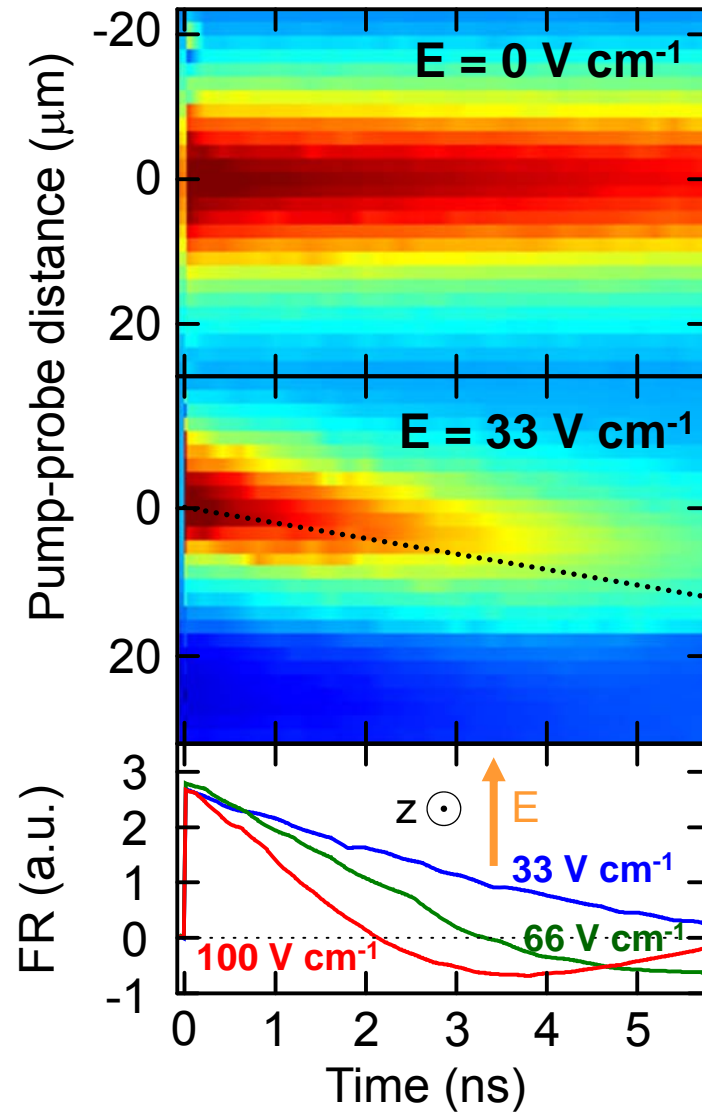
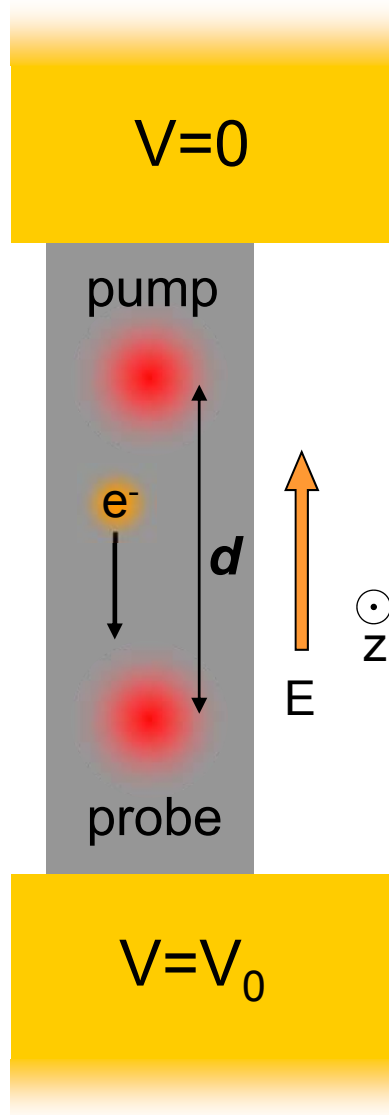
Strain-induced terms

$$\vec{\Omega}_3(\vec{k}) \propto \{(\varepsilon_{yy} - \varepsilon_{zz})k_x, (\varepsilon_{zz} - \varepsilon_{xx})k_y, (\varepsilon_{xx} - \varepsilon_{yy})k_z\}$$

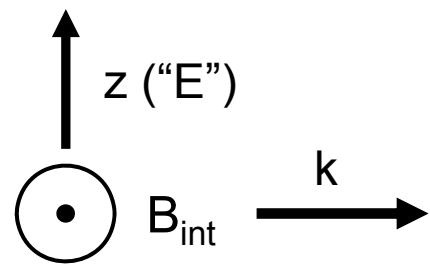
$$\vec{\Omega}_4(\vec{k}) \propto (\varepsilon_{zx}k_x - \varepsilon_{xy}k_y, \varepsilon_{xy}k_y - \varepsilon_{yz}k_z, \varepsilon_{yz}k_z - \varepsilon_{zx}k_x)$$



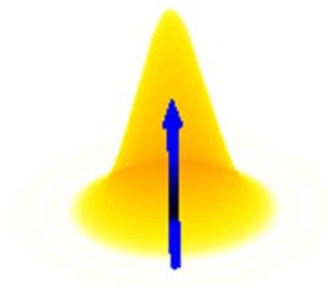
## Spatiotemporal evolution of spin packet in strained GaAs

*Nature* **427**, 50 (2004)

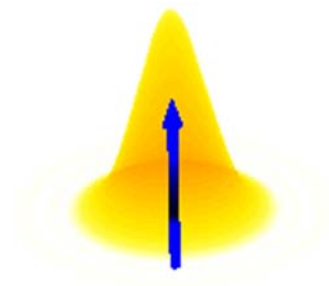
Spin precession at zero magnetic field!



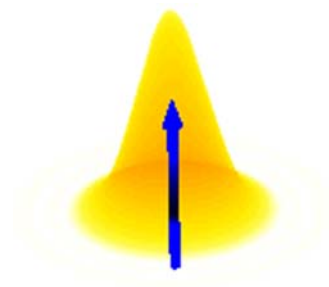
0 V cm<sup>-1</sup>



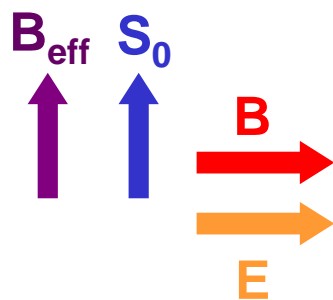
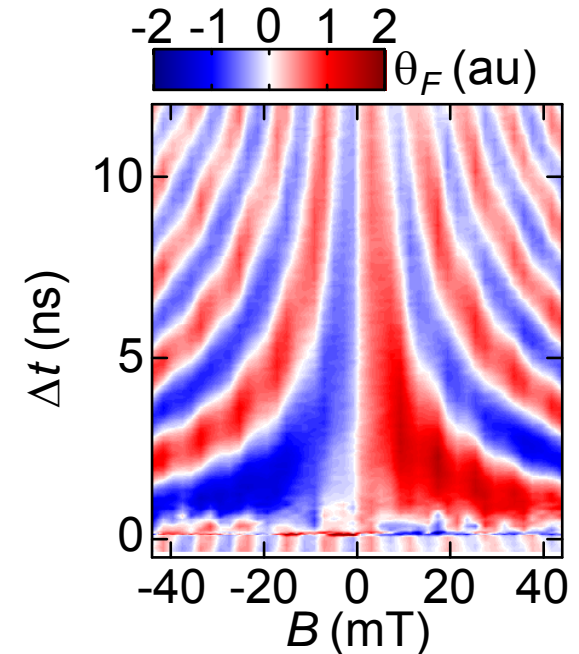
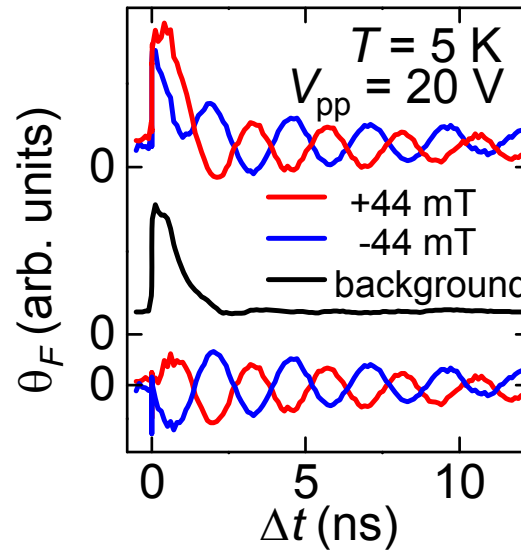
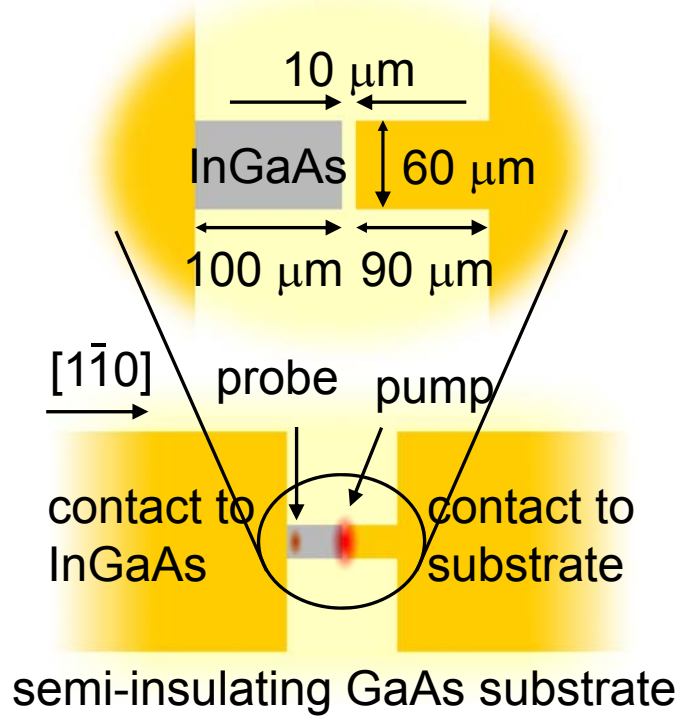
50 V cm<sup>-1</sup>



100 V cm<sup>-1</sup>



*Phys. Rev. Lett.* 93, 176601 (2004)



Precession at electron Larmor frequency of InGaAs

→ Faraday rotation signal due to electrons

The signal shows sign change with magnetic field

→ spins excited in the plane of the sample (along the effective magnetic field)

Electron spins are generated using picosecond electrical pulses

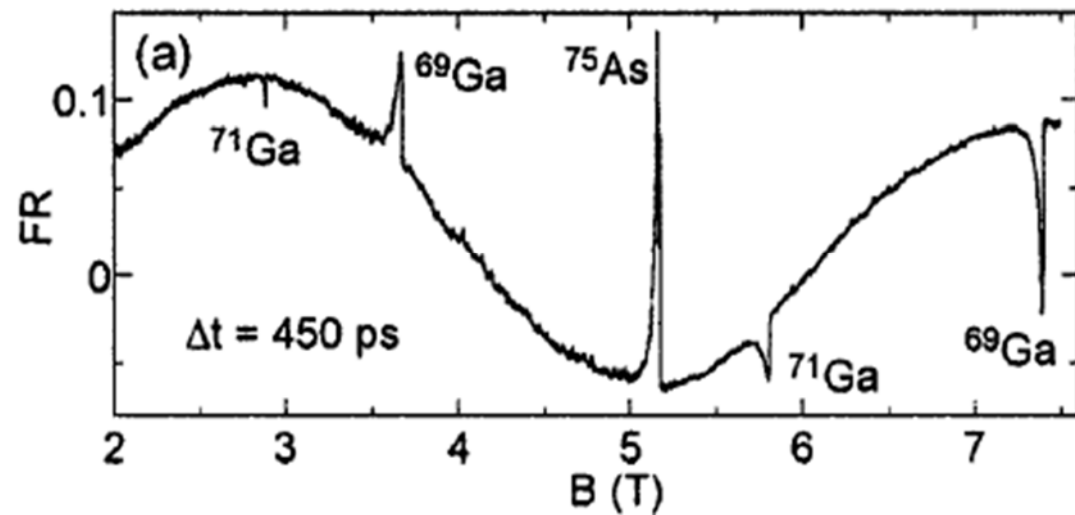
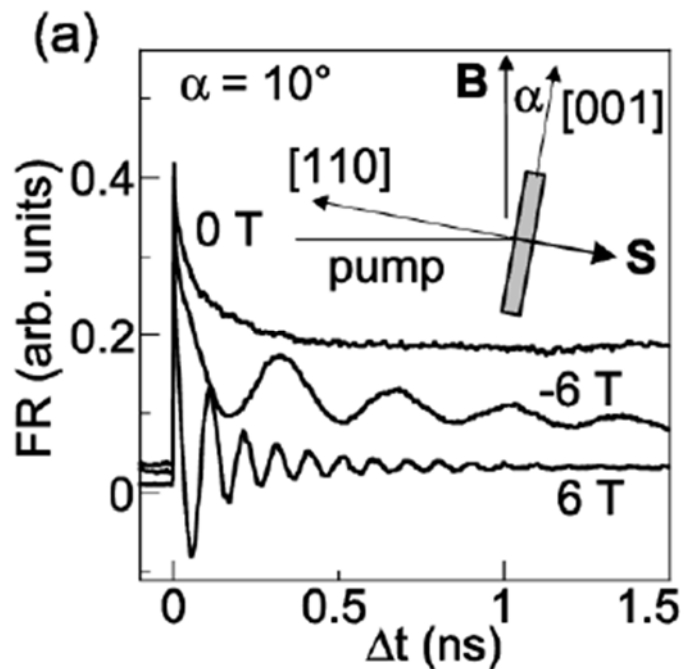
$$H = A\vec{I} \cdot \vec{S}$$

$\vec{I}$ : nuclear spin

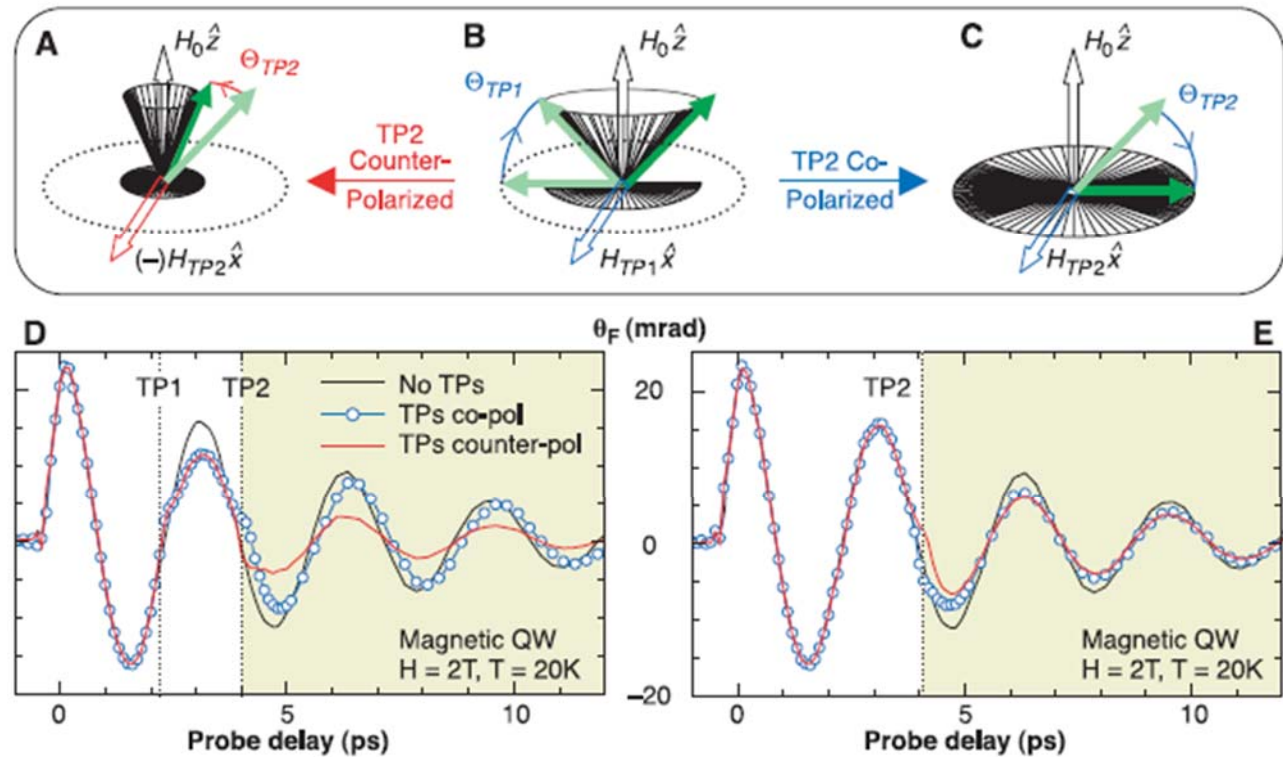
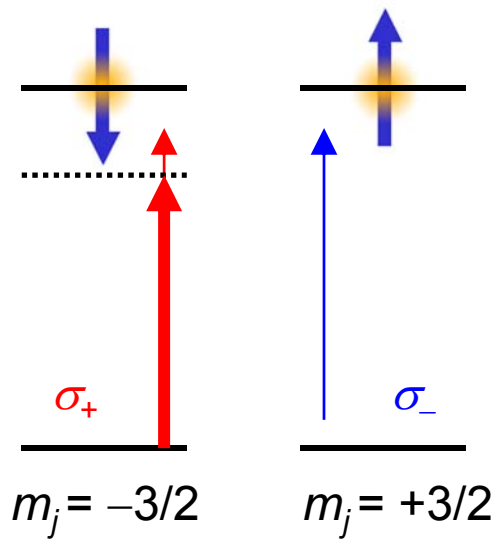
$\vec{S}$ : electron spin

Nuclear spin polarization acts as an effective magnetic field for electron spins

Dynamic nuclear polarization:  
spin injection causes nuclear spin polarization  
Lampel, PRL 20, 491 (1968)



Optical pulse below bandgap shifts the band gap (AC Stark effect)  
 Circularly polarized pulse shifts one of the spin subbands, causing spin splitting



## Topics covered in this talk

- Zeeman Hamiltonian
- Bloch sphere
- Larmor precession
- $T_1$ ,  $T_2$ , and  $T_2^*$
- Bloch equation

- Optical selection rules
- Time-resolved  
Kerr/Faraday rotation
- Hanle effect

- g-factor engineering
- spin-orbit interaction
- hyperfine interaction
- AC Stark effect