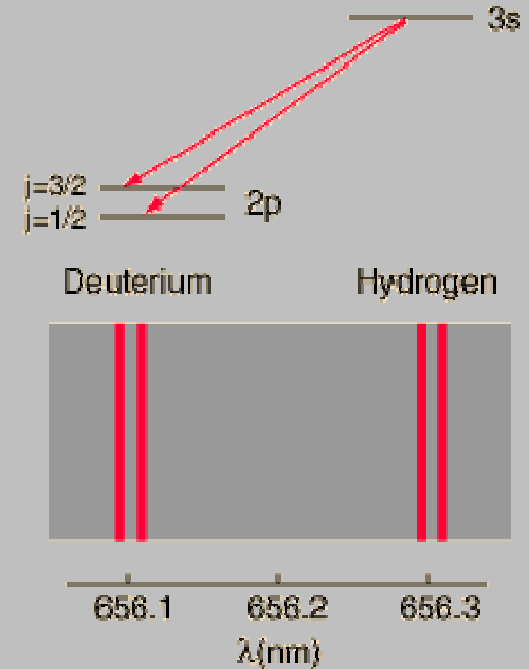
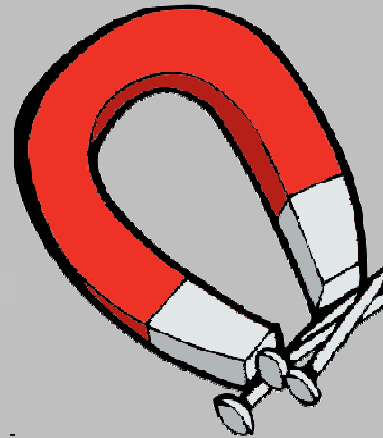


# Magnetism and Spin-Orbit Interaction: Some basics and examples



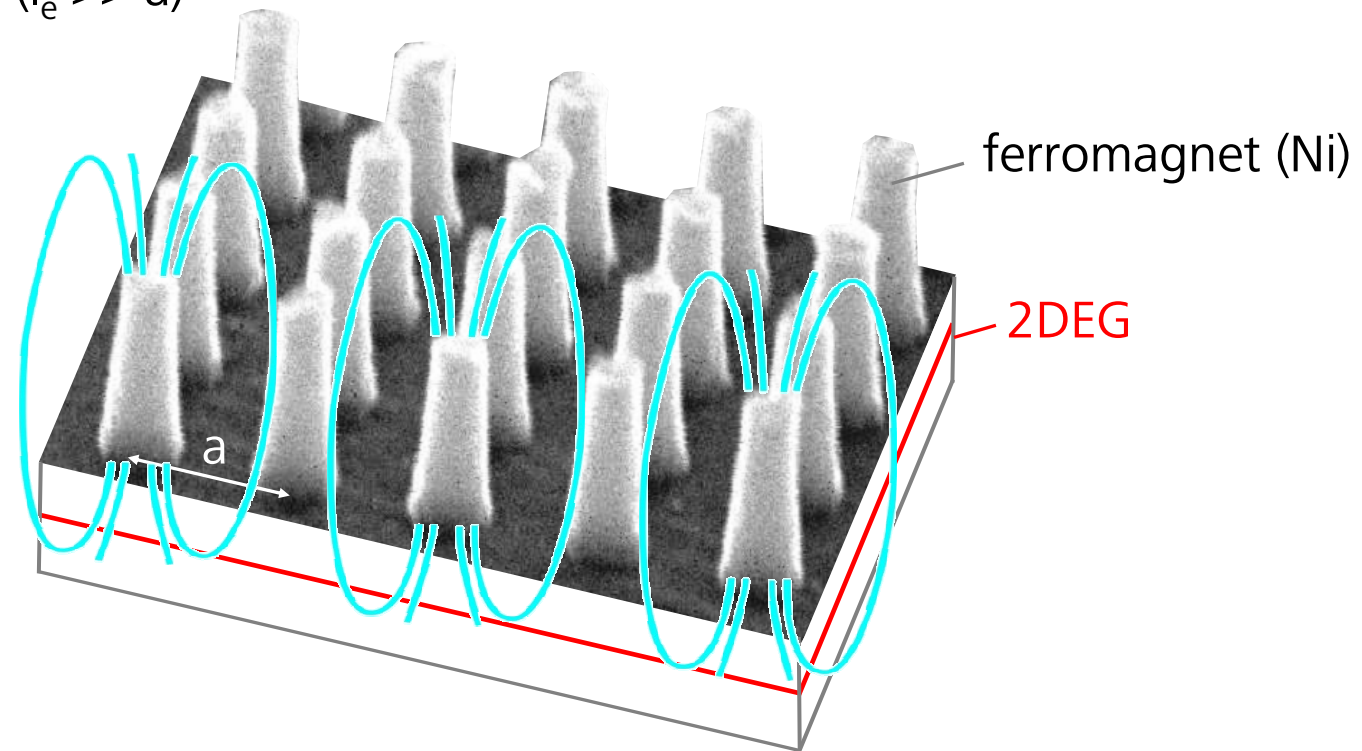
*Dieter Weiss*  
*Experimentelle und Angewandte Physik*



Universität Regensburg

# UR (Our initial) Motivation

Transport in a two-dimensional electron gas with superimposed periodic magnetic field ( $l_e \gg a$ )



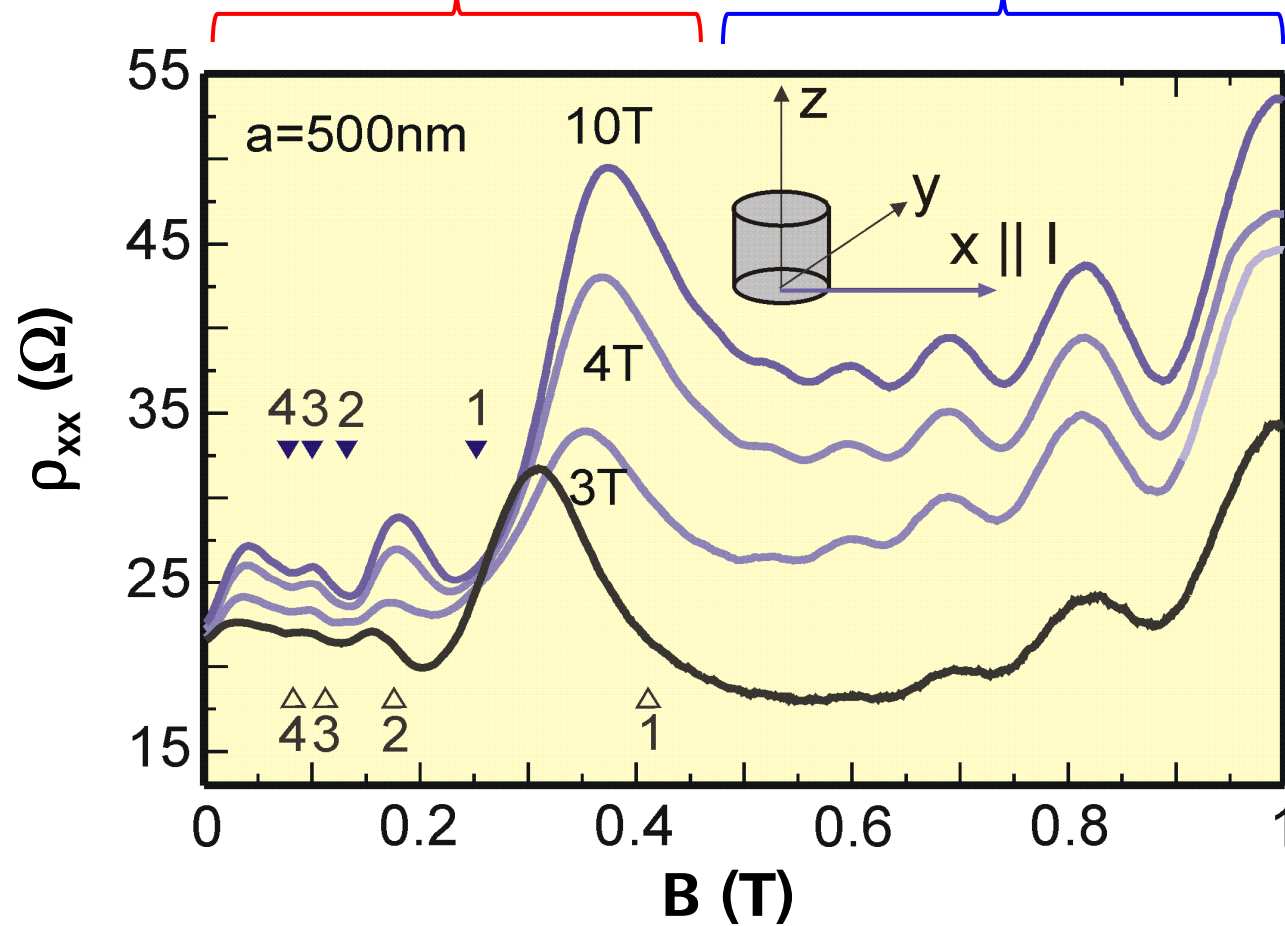
Magnetization pattern of ferromagnet determines stray field

# Transport in a periodic magnetic field

Commensurability (Weiss) Oscillations:

$$2R_C = \left(n \pm \frac{1}{4}\right) a$$

SdH oscillations



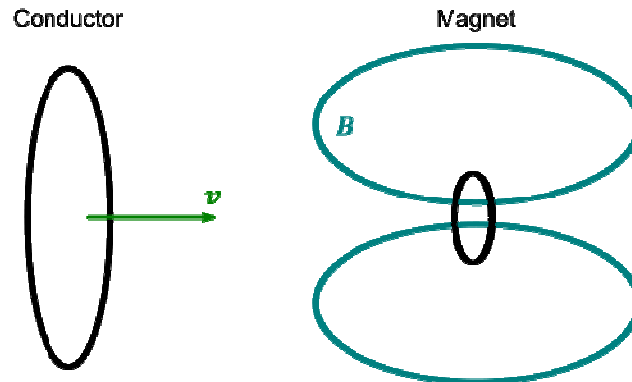


## A little bit of history

SciencephotoLIBRARY



**1820:** Electric current generates magnetic field



**1905:** On the electrodynamics of moving bodies  
Magnetic field stems from Lorentz contraction of moving charges

Hans Christian Ørsted



Albert Einstein





# Magnetism and Spin-Orbit Interaction:

## **Magnetism**

### Basic concepts

magnetic moments, susceptibility, paramagnetism, ferromagnetism  
exchange interaction, domains, magnetic anisotropy,

### Examples:

detection of (nanoscale) magnetization structure  
using Hall-magnetometry, Lorentz microscopy and MFM

## **Spin-Orbit interaction**

### Some basics

Rashba- and Dresselhaus contribution, SO-interaction and effective  
magnetic field

### Example:

Ferromagnet-Semiconductor Hybrids: TAMR involving epitaxial  
Fe/GaAs interfaces



# Magnetism and Spin-Orbit Interaction:

## Magnetism

### Basic concepts

magnetic moments, susceptibility, paramagnetism, ferromagnetism  
exchange interaction, domains, magnetic anisotropy,

### Examples:

detection of (nanoscale) magnetization structure  
using Hall-magnetometry, Lorentz microscopy and MFM

## Spin-Orbit interaction

### Some basics

Rashba- and Dresselhaus contribution, SO-interaction and effective  
magnetic field

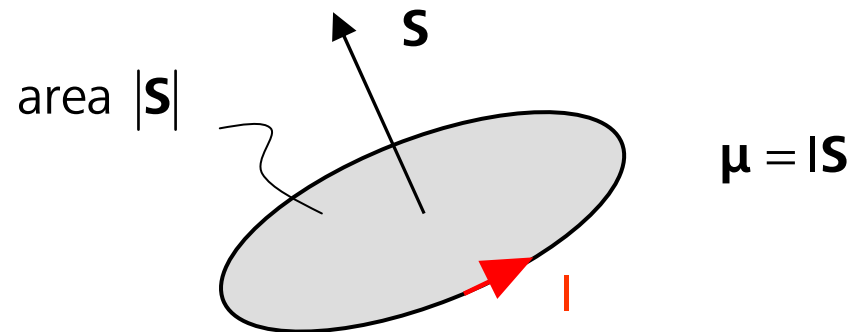
### Example:

Ferromagnet-Semiconductor Hybrids: TAMR involving epitaxial  
Fe/GaAs interfaces



# Magnetic moment and angular momentum

Current in a loop generates magnetic moment  $\mu$ , constitutes magnetic dipole



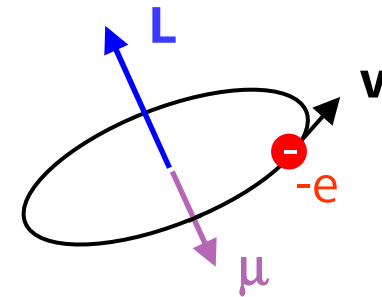
As mass is transported around the loop  $\Rightarrow \mu$  is connected to angular momentum

$$\mu = \gamma \mathbf{L} \quad [\text{Am}^2] \quad \gamma = \text{gyromagnetic ratio}$$

Size of atomic magnetic moment

$$I = \frac{-e}{T} = \frac{-ev}{2\pi r} \Rightarrow \mu = -\frac{1}{2} evr = -\frac{1}{2} \frac{e}{m} L = -\mu_B \frac{L}{\hbar}$$

$$\mu = -\frac{\mu_B}{\hbar} \mathbf{L} \quad \text{with} \quad \mu_B = \frac{e\hbar}{2m} = \text{Bohr Magnetron}$$



Magnetization  $\mathbf{M}$ : magnetic moments / volume  $[\text{A/m}]$



# Angular momentum: quantum mechanical picture

Total angular momentum of an atom is composed of angular momentum of all occupied electron states **L** and of all corresponding spins **S**

Total angular momentum (L-S coupling):

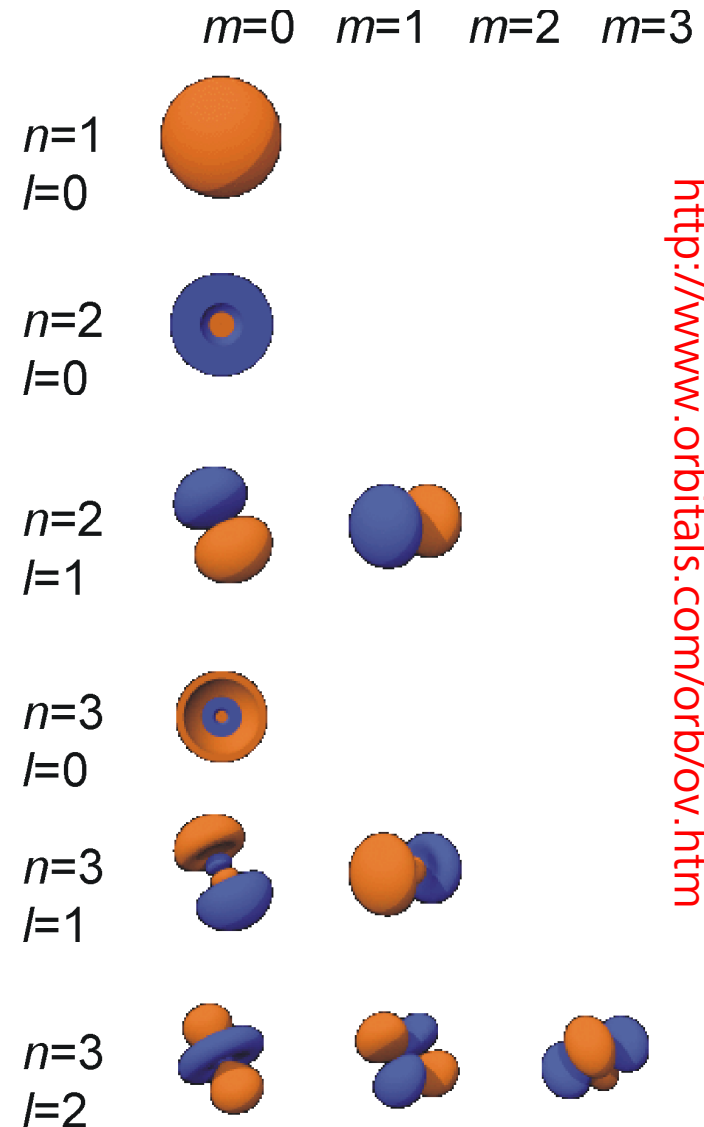
$$\mathbf{J} = \mathbf{L} + \mathbf{S}$$

Connection between  $\boldsymbol{\mu}$  and  $\mathbf{J}$

$$\boldsymbol{\mu} = -\frac{g\mu_B}{\hbar} \mathbf{J}$$

with Landé factor

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$



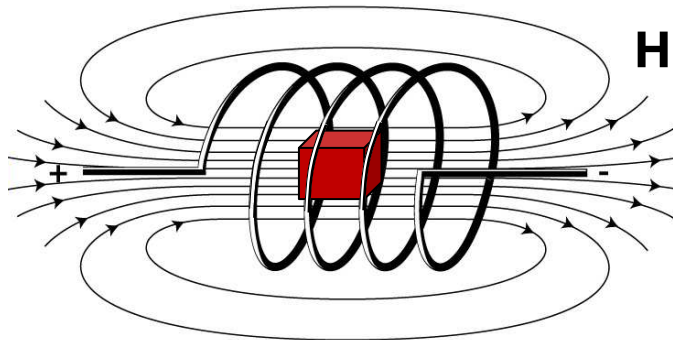
<http://www.orbitals.com/orb/ov.htm>





# Susceptibility

Connection between field  $H$  (e.g. generated by coil) and magnetization  $\mathbf{M}$ :



Linear materials:

$$\mathbf{M} = \chi \mathbf{H}$$

↑ magnetic susceptibility

diamagnetic if  $\chi < 0$   
paramagnetic if  $\chi > 0$

Connection between  $B$  and  $H$  field:  $\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$  (for  $M \ll H$ )

$$\Rightarrow \mathbf{B} = \mu_0 (\mathbf{H} + \chi \mathbf{H}) = \mu_0 (1 + \chi) \mathbf{H}$$

Not the whole story!

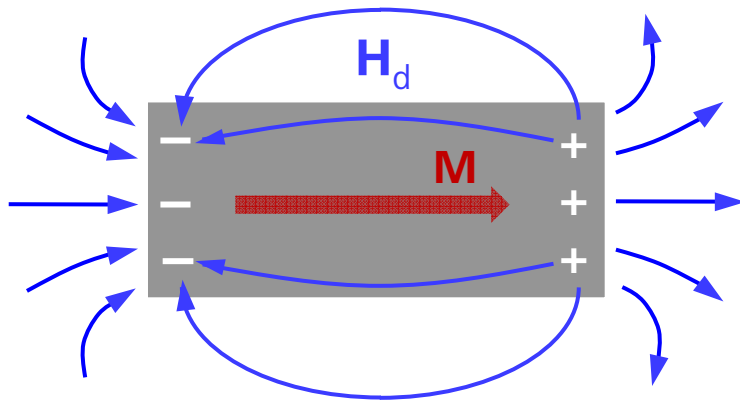
↑ permeability of free space

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{H}_d + \mathbf{M})$$

↑ demagnetizing field or stray field  
(important for ferromagnets)



## Demagnetizing field

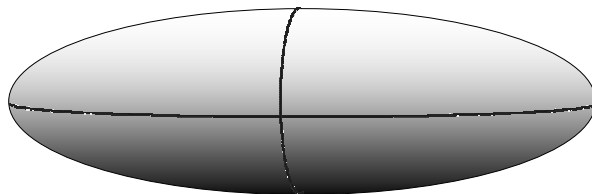


$$\nabla \cdot \mathbf{B} = \nabla \cdot [\mu_0(\mathbf{H}_d + \mathbf{M})] = 0$$

$$\nabla \cdot \mathbf{H}_d = -\nabla \cdot \mathbf{M}$$

Any divergence of  $\mathbf{M}$  creates a magnetic field – can be seen as sort of (bound) magnetic charges

Easy expression for strayfield (demagnetizing field)  $\mathbf{H}_d$  only for ellipsoids:



$$\mathbf{H}_d = - \begin{pmatrix} N_{xx} & 0 & 0 \\ 0 & N_{yy} & 0 \\ 0 & 0 & N_{zz} \end{pmatrix} \mathbf{M}$$

$$\text{with } N_{xx} + N_{yy} + N_{zz} = 1$$



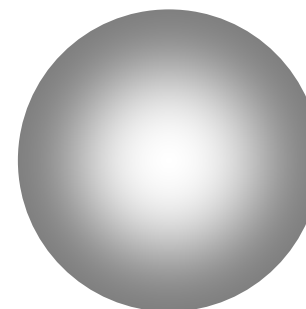
## Some examples:

$$N_{xx} = 0, N_{yy} = N_{zz} = \frac{1}{2}$$

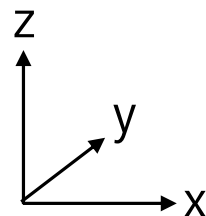


long cylindrical rod

$$N_{xx} = N_{yy} = N_{zz} = \frac{1}{3}$$



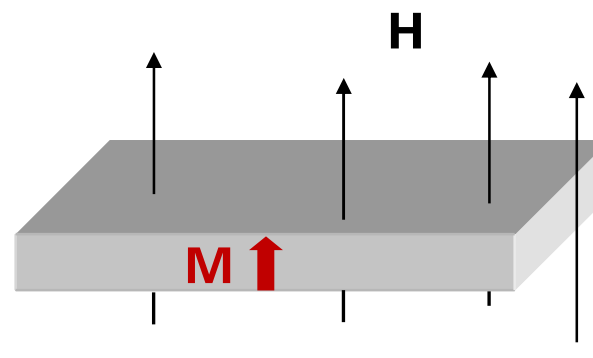
sphere



$$N_{xx} = N_{yy} = 0, N_{zz} = 1$$



flat plate



$$\mathbf{H}_d = -\mathbf{M} \Rightarrow$$

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{H}_d + \mathbf{M}) = \mu_0 \mathbf{H}$$

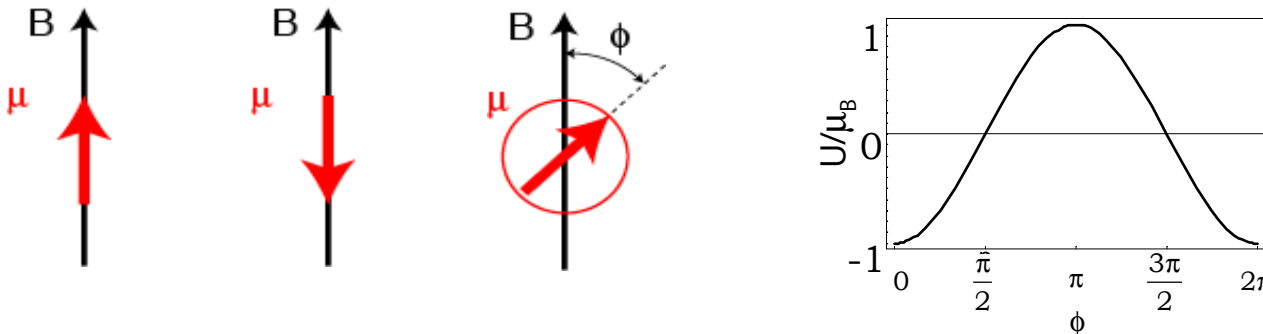


# Paramagnetism (of isolated magnetic moments)

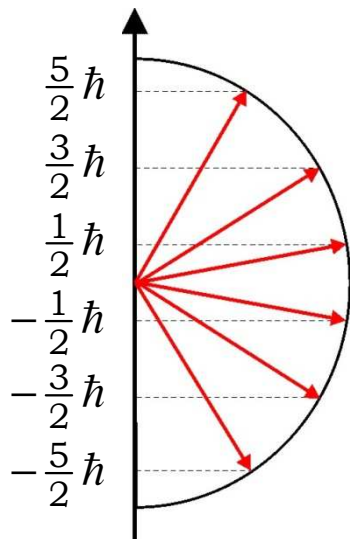
Requisite: Atoms with non-vanishing angular momentum  $J$

Energy of magnetic moment  $E = -\boldsymbol{\mu} \cdot \mathbf{B}$

**Classically:** arbitrary values and orientation of  $\boldsymbol{\mu}$



**Quantum mechanically:** discrete values and orientations of  $\boldsymbol{\mu}_J$ , e.g.  $J = 5/2$



$$\mu_z = -\frac{g\mu_B}{\hbar} J_z = -m_J g\mu_B \quad m_J = J, J-1, \dots, -(J-1), -J$$

$$J_z = m_J \hbar$$

$2J+1$  discrete values  
(and hence energy levels)

Associated energy levels  $E_{m_J} = m_J g\mu_B B$   
get thermally occupied

$$-\underbrace{\mu_z}_{\mu_B}$$



# Magnetization (thermal average)

Magnetization  $M = NgJ\mu_B B_J(x)$   $J$ : total angular momentum

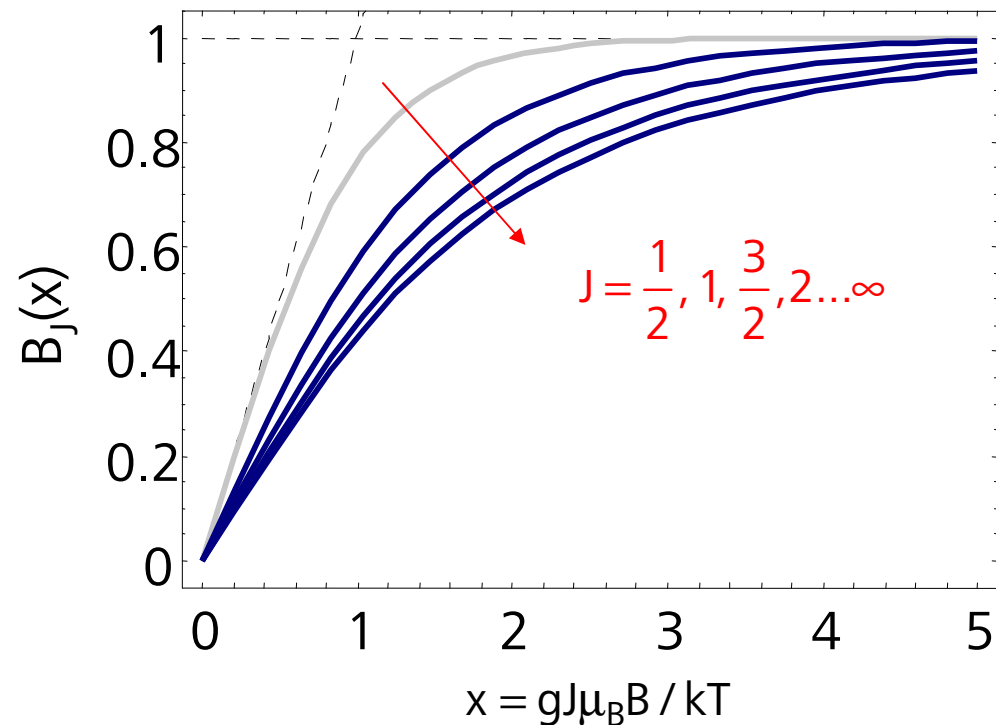
Brillouin function  $B_J(x) = \frac{2J+1}{2J} \coth\left(\frac{2J+1}{2J}x\right) - \frac{1}{2J} \coth\left(\frac{x}{2J}\right)$ ; with  $x = \frac{gJ\mu_B B}{k_B T}$

Saturation:

for  $x \gg 1$ :  $B_J \rightarrow 1$

and  $M \rightarrow NgJ\mu_B = M_S$

$N$ : Density of magnetic atoms



"High - T":

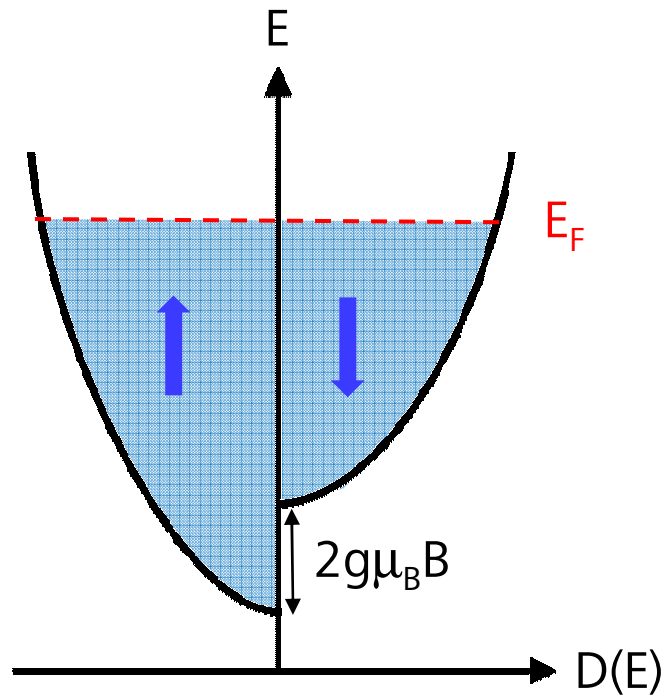
$$x \ll 1 \Rightarrow \coth(x) \approx \frac{1}{x} + \frac{x}{3} - \dots$$

$$\Rightarrow B_J \approx \frac{J+1}{J} \frac{x}{3}$$

$$M = NgJ\mu_B B_J(x) \approx N \frac{g^2 J(J+1)\mu_B^2}{3k_B T} B = N \frac{g^2 J(J+1)\mu_B^2}{3k_B T} \mu_0 H \Rightarrow \chi = \frac{M}{H} \approx N \frac{g^2 J(J+1)\mu_B^2 \mu_0}{3k_B T}$$



# Paramagnetism (of free electrons)



$$\text{Magnetization: } M = (N_{\uparrow} - N_{\downarrow})\mu_B = \Delta N\mu_B$$

$$\Delta N = \frac{1}{2} D(E_F) \cdot 2g\mu_B B = \frac{3}{2} \frac{N}{E_F} g\mu_B B$$

$$D(E_F) = \frac{3}{2} \frac{N}{E_F} \text{ in 3D}$$

Resulting magnetization:

$$M = \frac{3}{2} \frac{N}{E_F} g\mu_B^2 B = \underbrace{\frac{3}{2} \frac{N}{k_B T_F} g\mu_B^2 \mu_0 H}_{\chi}$$

Taking into account diamagnetism of free electrons ( $M_{\text{dia}} = -\frac{1}{3}M_{\text{para}}$ )

$$\chi = \frac{N}{k_B T_F} g\mu_B^2 \mu_0$$

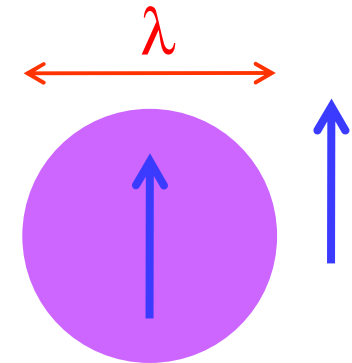
Independent of temperature!



## Exchange Interaction (a la Heisenberg)

Exchange due to Pauli exclusion principle and Coulomb repulsion

An electron pushes a second electron with the same spin orientation out of a region with diameter  $\lambda \rightarrow$  exchange hole forms



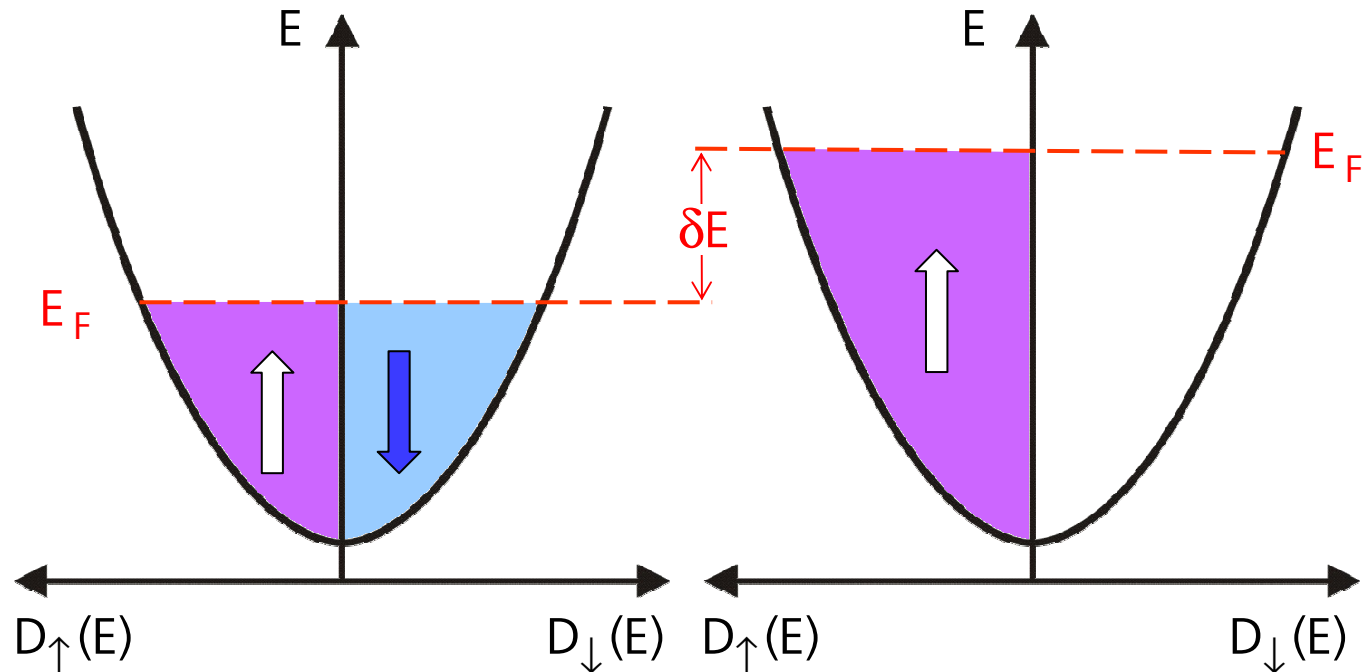
Exchange hole

Consequences of spin arrangement for total energy

- For parallel orientation, e.g.,  $|\uparrow\uparrow\rangle$  potential energy gets reduced compared to anti-parallel arrangement, e.g.,  $|\uparrow\downarrow\rangle$  since Coulomb-repulsion is reduced due to larger average interparticle distance.
- On the other hand kinetic energy is larger for  $|\uparrow\uparrow\rangle$  since Fermi-energy increases due to fixed electron number.

delicate interplay!

## Increase of kinetic energy (example)

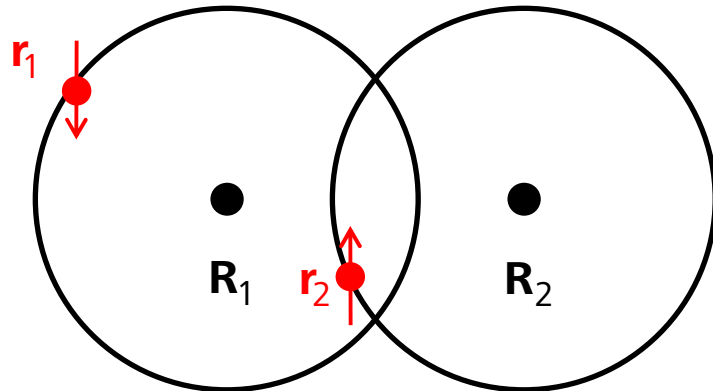


Fermi energy  $E_F$  of a free electron gas increases if, for fixed electron density only one spin species is permitted





## Exchange Interaction (a la Heisenberg)



Two electron wave function of hydrogen molecule like system:

$$\hat{H}\psi = -\frac{\hbar^2}{2m}(\nabla_1^2 + \nabla_2^2)\psi + V(\mathbf{r}_1, \mathbf{r}_2)\psi = E\psi$$

Fermi-Dirac statistics requires that

$$\psi(\mathbf{r}_1, \mathbf{r}_2) = -\psi(\mathbf{r}_2, \mathbf{r}_1) \text{ exchange of electrons}$$

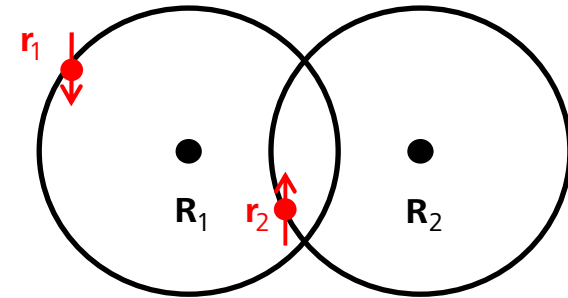
Construction of  $\Psi$  with Heitler-London approximation:

Spin of both electrons anti-parallel (Singulett,  $S=0$ )

$$\psi_s(\mathbf{r}_1, \mathbf{r}_2) = \underbrace{[\varphi_1(\mathbf{r}_1)\varphi_2(\mathbf{r}_2) + \varphi_1(\mathbf{r}_2)\varphi_2(\mathbf{r}_1)]}_{\text{symmetric}} \underbrace{\chi_s}_{\text{anti-symmetric under particle exchange}}$$

Spin of both electrons parallel (Triplett,  $S=1$ )

$$\psi_T(\mathbf{r}_1, \mathbf{r}_2) = \underbrace{[\varphi_1(\mathbf{r}_1)\varphi_2(\mathbf{r}_2) - \varphi_1(\mathbf{r}_2)\varphi_2(\mathbf{r}_1)]}_{\text{anti-symmetric}} \underbrace{\chi_T}_{\text{symmetric under particle exchange}}$$

Spin wave function  $\chi$ 

	$S$	$m_s$	$\chi$	$\mathbf{S}_1 \cdot \mathbf{S}_2$	
Triplet	1	1	$ \uparrow\uparrow\rangle$	$\frac{1}{4}$	$\chi_T$ symmetric
	1	0	$ \uparrow\downarrow\rangle +  \downarrow\uparrow\rangle$	$\frac{1}{4}$	
	1	-1	$ \downarrow\downarrow\rangle$	$\frac{1}{4}$	
Singlett	0	0	$ \uparrow\downarrow\rangle -  \downarrow\uparrow\rangle$	$-\frac{3}{4}$	$\chi_S$ anti-symmetric

see, e.g. Stephen Blundell *Magnetism in Condensed Matter* Oxford University Press



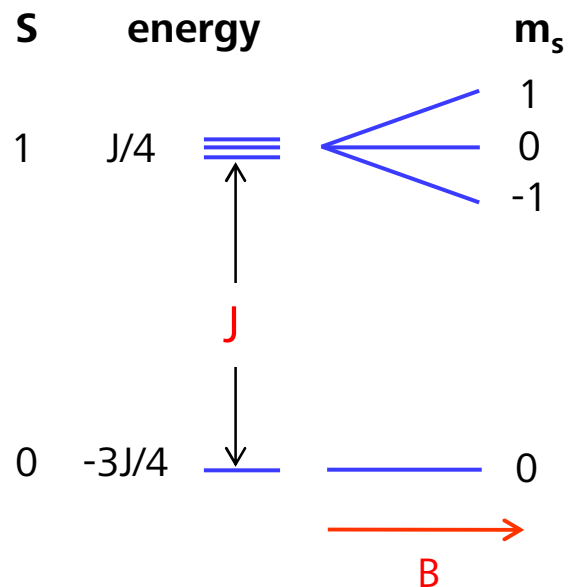
# Exchange splitting

exclusively due to the exchange of the two electrons

$\psi_S(\mathbf{r}_1, \mathbf{r}_2)$  and  $\psi_T(\mathbf{r}_1, \mathbf{r}_2)$  have different energy  $E_S$  and  $E_T$

$$E_S - E_T = \frac{\langle \psi_S | \hat{H} | \psi_S \rangle}{\langle \psi_S | \psi_S \rangle} - \frac{\langle \psi_T | \hat{H} | \psi_T \rangle}{\langle \psi_T | \psi_T \rangle} \dots\dots\dots \text{Ashcroft, Mermin}$$

$$= 2 \int d\mathbf{r}_1 d\mathbf{r}_2 \left[ \phi_1(\mathbf{r}_1) \phi_2(\mathbf{r}_2) \right] \left( \frac{e^2}{|\mathbf{r}_1 - \mathbf{r}_2|} + \frac{e^2}{|\mathbf{R}_1 - \mathbf{R}_2|} - \frac{e^2}{|\mathbf{r}_1 - \mathbf{R}_1|} - \frac{e^2}{|\mathbf{r}_2 - \mathbf{R}_2|} \right) \left[ \phi_1(\mathbf{r}_1) \phi_2(\mathbf{r}_2) \right]$$



exchange integral  $J$

sign of  $J$  determines spin ground state

$J > 0 \Rightarrow E_S > E_T \Rightarrow$  spins parallel

$J < 0 \Rightarrow E_S < E_T \Rightarrow$  spins anti-parallel



## Heisenberg Hamiltonian

Expressing spin Hamiltonian with Eigenvalues  $E_S$  and  $E_T$  in terms of  $\mathbf{S}_1$  and  $\mathbf{S}_2$ :

$$\hat{\mathbf{H}} = \frac{1}{4}(E_S + 3E_T) - (E_S - E_T)\hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2$$
$$\hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2 = \begin{cases} +\frac{1}{4} & S = 1 \\ -\frac{3}{4} & S = 0 \end{cases}$$

using that  $\hat{\mathbf{S}}^2 = S(S + 1)$

$\hat{\mathbf{H}}$  has Eigenvalue  $E_T$  for triplett ( $\uparrow\uparrow$  or  $\downarrow\downarrow$ ) and  
 $E_S$  for singulett ( $\uparrow\downarrow$  or  $\downarrow\uparrow$ )

Shift of origin results in usual form of Heisenberg Hamiltonian

$$\hat{\mathbf{H}} = -(E_S - E_T)\hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2 = -J\hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2$$

Generalisation for many spins:

$$\hat{\mathbf{H}} = -\sum_{i,j} J_{ij}\hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j$$

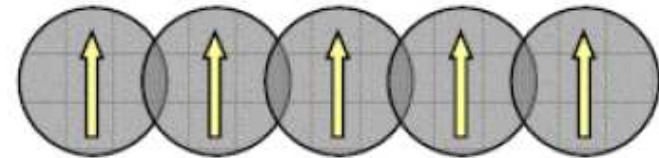
Exchange integral between spin  $i$  and spin  $j$



# Forms of exchange interaction

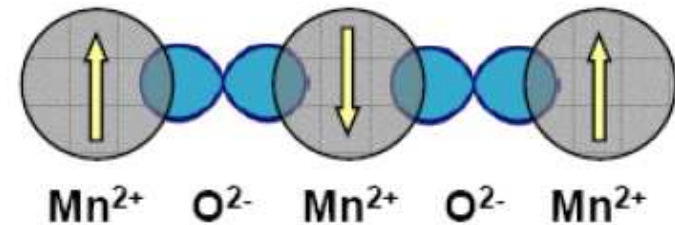
## Direct exchange interaction

from direct overlap



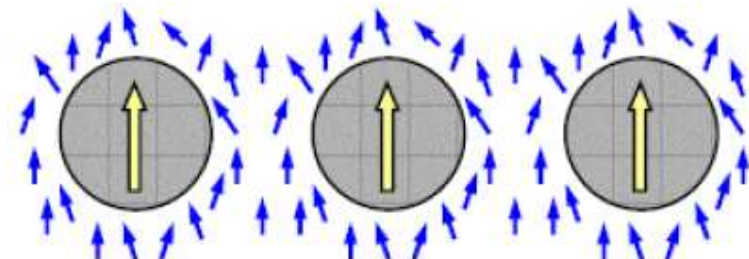
## Superexchange

mediated by paramagnetic atoms or ions



## RKKY Interaction

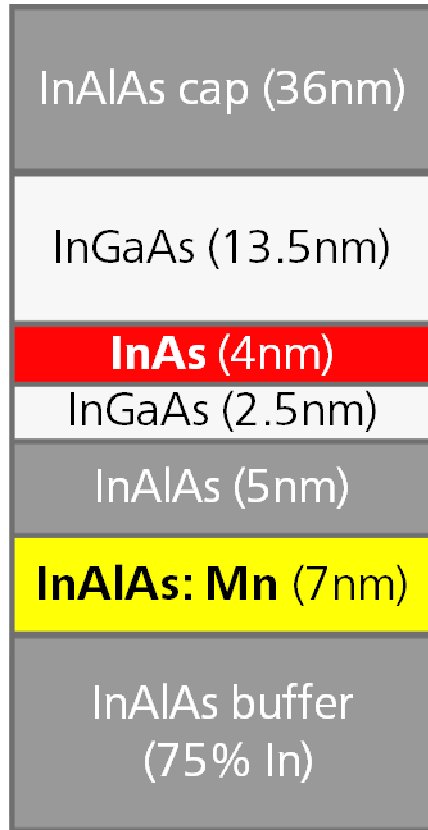
(After M.A. Rudermann, C. Kittel, T. Kasuya, K. Yosida)  
coupling mediated by mobile charge carriers



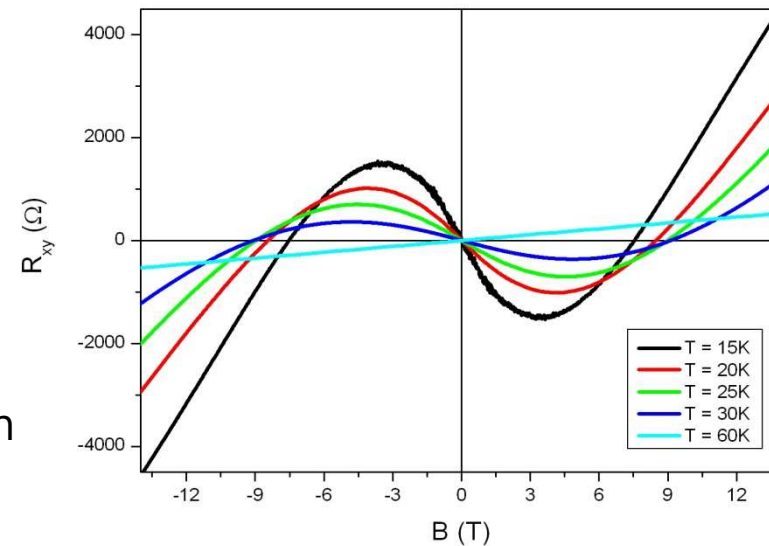
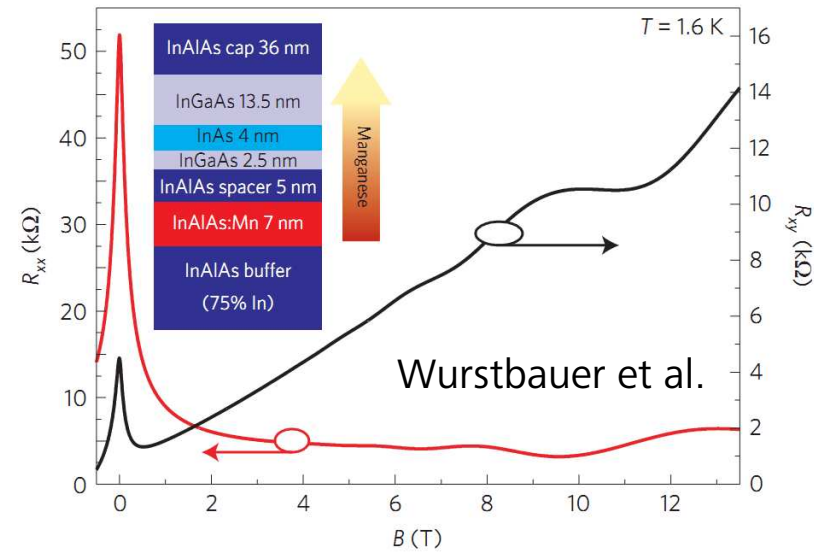


# Example: p-d exchange interaction in a 2DHG

Mn ( $S=5/2$ ) in two-dimensional hole gas



Nature Physics  
6, 955 (2010)



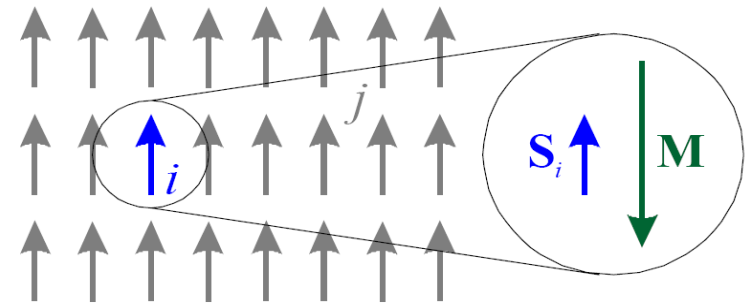
see: **WP-20** Anomalous Hall effect in Mn doped, p-type InAs quantum wells  
C. Wensauer, D. Vogel et al.



# Weiss molecular field

Pierre Weiss 1907

**Idea:** Replace interaction of a given spin with all other spins by interaction with an **effective field** (molecular field)



Sum of all  $\mathbf{S}_j$  act on one particular spin  $\mathbf{S}_i$

magnetic moment  $\boldsymbol{\mu}_i$  of spin  $\mathbf{S}_i$

$$E_{\text{ex}} = -2\mathbf{S}_i \cdot \sum_j J_{ij} \mathbf{S}_j$$

$$\boldsymbol{\mu}_i = \frac{-g\mu_B \mathbf{S}_i}{\hbar} \Rightarrow \mathbf{S}_i = -\frac{\boldsymbol{\mu}_i \hbar}{g\mu_B}$$

$$E_{\text{ex}} = 2 \frac{\boldsymbol{\mu}_i \hbar}{g\mu_B} \cdot \sum_j J_{ij} \mathbf{S}_j = -\boldsymbol{\mu}_i \cdot \underbrace{\left( \frac{2\hbar^2}{(g\mu_B)^2} \langle \boldsymbol{\mu}_i \rangle \sum_j J_{ij} \right)}_{\text{Weiss molecular field } \mathbf{B}_M} = -\boldsymbol{\mu}_i \cdot \mathbf{B}_M = \lambda \mathbf{M}$$

$$\vec{S}_j \text{ replaced by constant moment: } \langle \boldsymbol{\mu}_j \rangle = -g\mu_B \langle \mathbf{S}_j \rangle / \hbar \Rightarrow \langle \mathbf{S}_j \rangle = -\hbar \langle \boldsymbol{\mu}_j \rangle / g\mu_B$$

Magnetic moments get aligned in an effective field  $\mathbf{B}_{\text{eff}} = \mathbf{B}_0 + \mathbf{B}_M \gg \mathbf{B}_0$



# Ferromagnetism within the molecular field picture

Molecular field  $B_m = \lambda M$  allows to map ferromagnetism onto paramagnetism

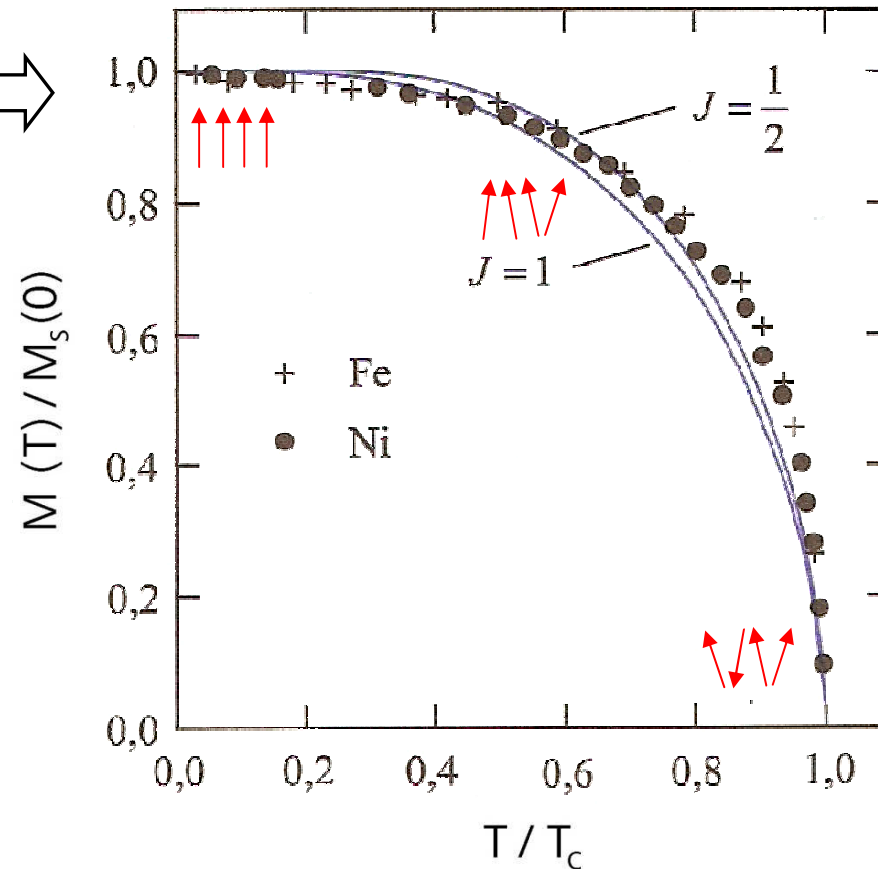
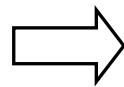
$$M = Ng_J \mu_B \left[ \frac{2J+1}{2J} \coth\left(\frac{2J+1}{2J} x\right) - \frac{1}{2J} \coth\left(\frac{x}{2J}\right) \right] \quad \text{with } x = \frac{g_J \mu_B (B + \lambda M)}{kT}$$

M on both sides of equation:  
graphical or numerical solution

$$T_C = \frac{g_J \mu_B (J+1) \lambda M_S}{3k_B}$$

Für  $T_C \sim 10^3 \text{K}$  und  $J = 1/2 \Rightarrow$

$$B_M = \lambda M_S \sim 1500 \text{T}$$







## Band magnetism and Stoner criterion

Heisenberg Model unable to describe all aspects of ferromagnetism e.g. moment of iron is  $2.2 \mu_B$ . Stoner picture:

Redistribution of spins: Energy cost  $E_{\text{kin}}$

$$\Delta E_{\text{kin}} = \frac{1}{2} D(E_F) \delta E^2$$

Resulting magnetization

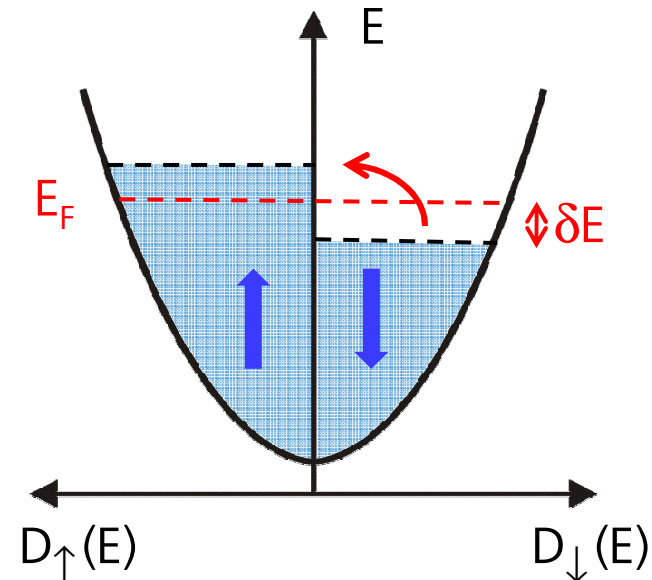
$$M = (n_{\uparrow} - n_{\downarrow}) \mu_B = D(E_F) \mu_B \delta E$$

Energy of electron's magnetization  $dM$  in effective (Weiss) field  $B_{\text{eff}}$  of other electrons

$$dE_{\text{pot}} = -dM \cdot B_{\text{eff}} \rightarrow \Delta E_{\text{pot}} = -\int_0^M dM' \lambda M' = -\lambda \frac{M^2}{2} = -\frac{1}{2} \lambda \mu_B^2 D(E_F)^2 \delta E^2$$

Energy gain if  $\Delta E_{\text{kin}} + \Delta E_{\text{pot}} = \frac{1}{2} D(E_F) \delta E^2 [1 - \lambda \mu_B^2 D(E_F)] < 0$

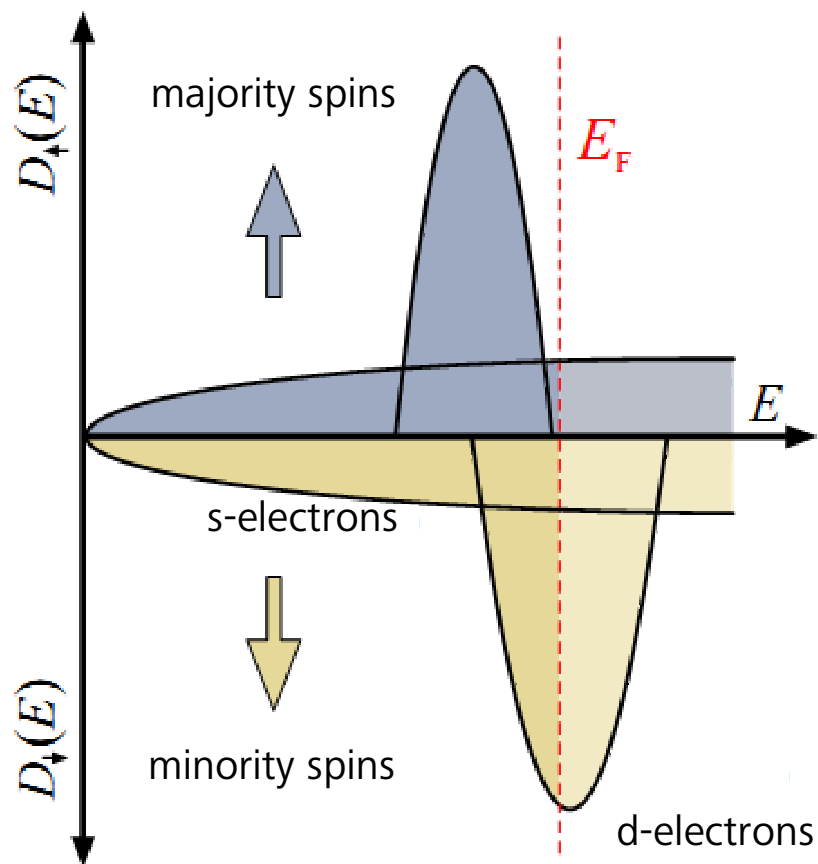
Ferromagnetism occurs if  $\lambda \mu_B^2 \cdot D(E_F) \geq 1$  **Stoner criterion** (holds for Fe, Ni, Co)



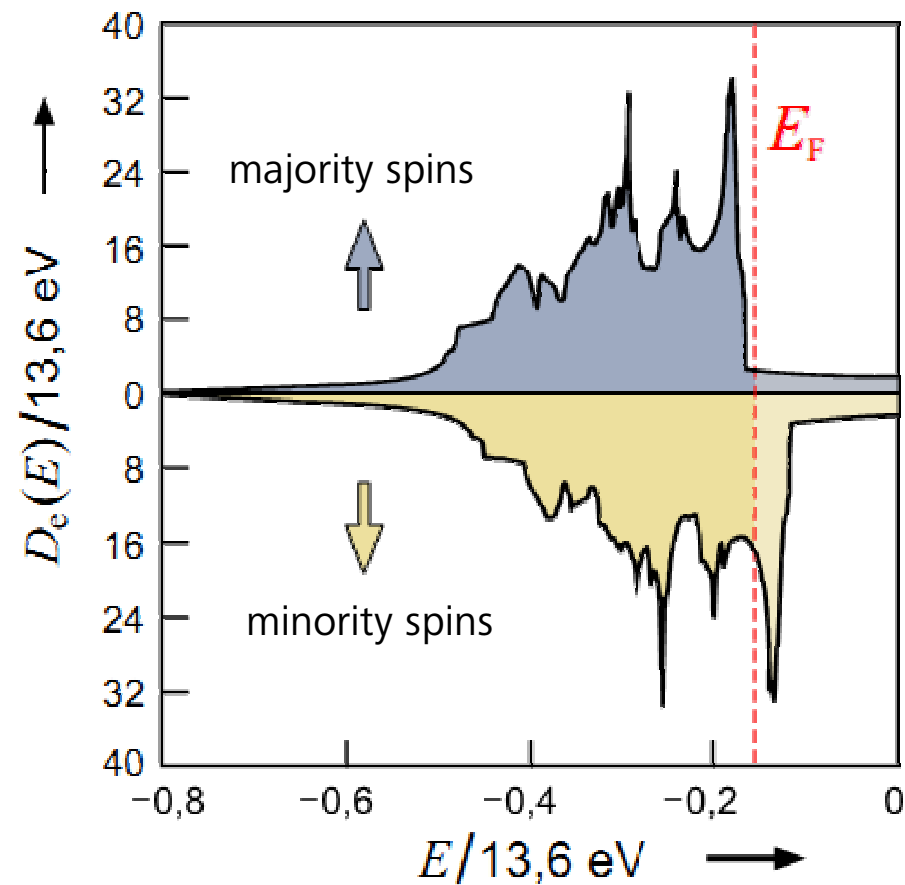


# Ferromagnet: Density of States $D(E)$

### Schematic



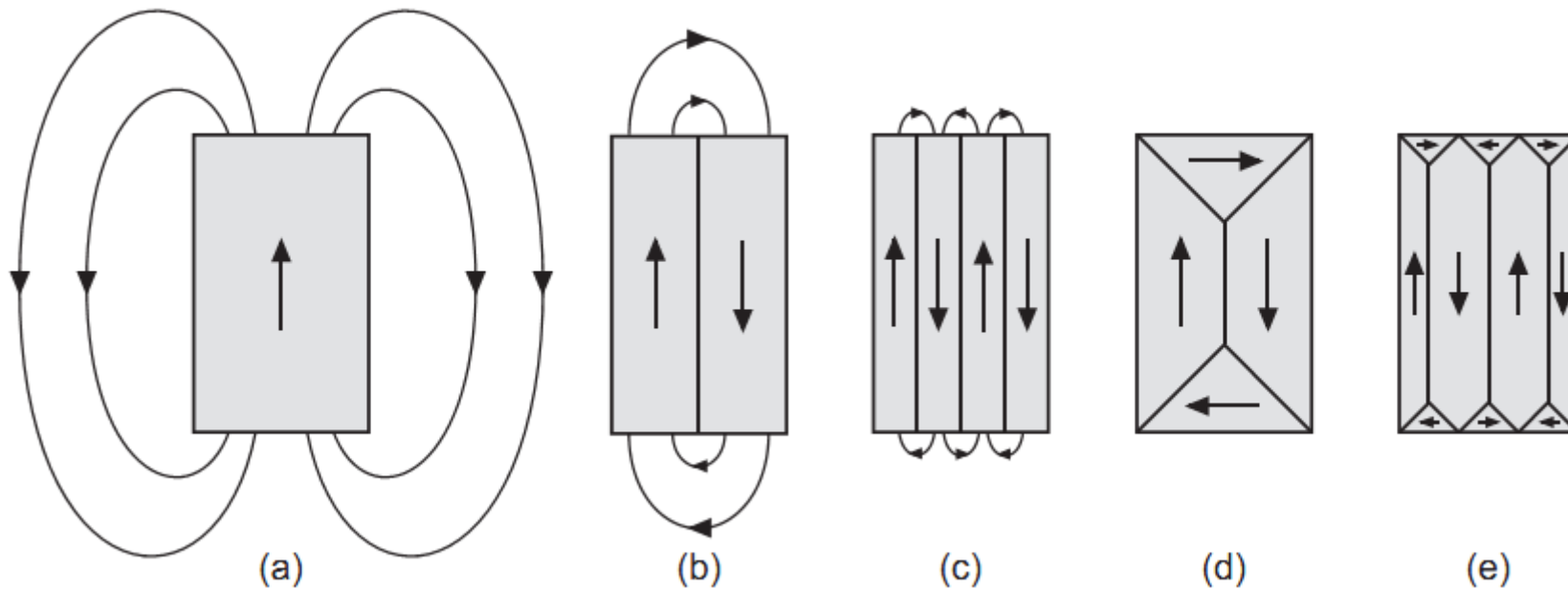
### Nickel





# Magnetic Domains

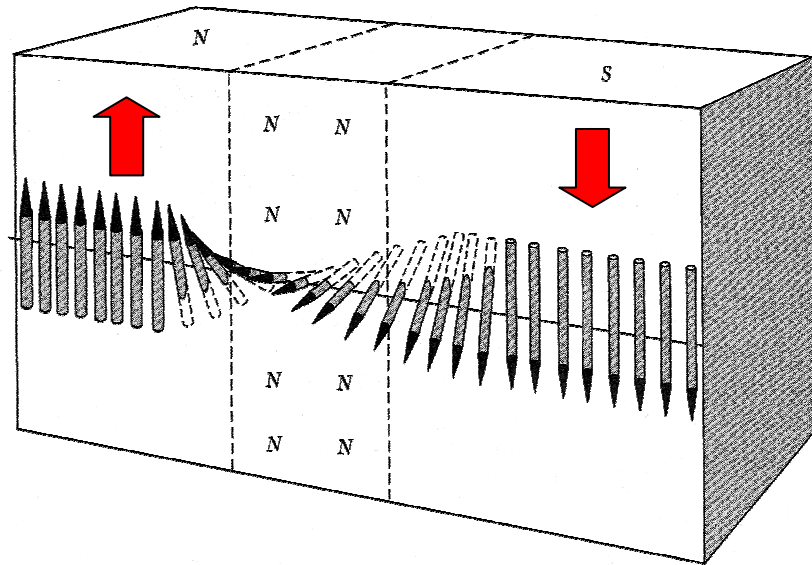
Energy of stray field  $\mathbf{H}_d$ : 
$$E_d = \frac{1}{2} \mu_0 \int_{\text{space}} \mathbf{H}_d^2 dV = -\frac{1}{2} \mu_0 \int_{\text{sample}} \mathbf{H}_d \cdot \mathbf{M} dV$$



Formation of domains reduces stray field energy!



# Width of a Bloch wall



Classical Heisenberg Hamiltonian

$$E_{\text{ex}} = -2J\mathbf{S}_1 \cdot \mathbf{S}_2 = -2JS^2 \cos \theta$$

$$\theta = 0 \rightarrow E_{\text{ex}} = -2JS^2$$

Energy to rotate spin by angle  $\theta$

$$\theta \neq 0 \rightarrow \Delta E_{\text{ex}} = JS^2 \theta^2 \quad \text{for } \theta \ll 1$$

Spin rotation by  $\pi$  over  $N$  sites  $\Rightarrow \theta = \frac{\pi}{N} \Rightarrow$  energy cost per line:  $\Delta E_{\text{ex}} = JS^2 \frac{\pi^2}{N}$

$$\text{Energy per unit area } \varepsilon_{\text{ex}} = \frac{\Delta E_{\text{ex}}}{a^2} = JS^2 \frac{\pi^2}{Na^2}$$

$a$ : lattice constant

$\Rightarrow$  best  $N \rightarrow \infty?$



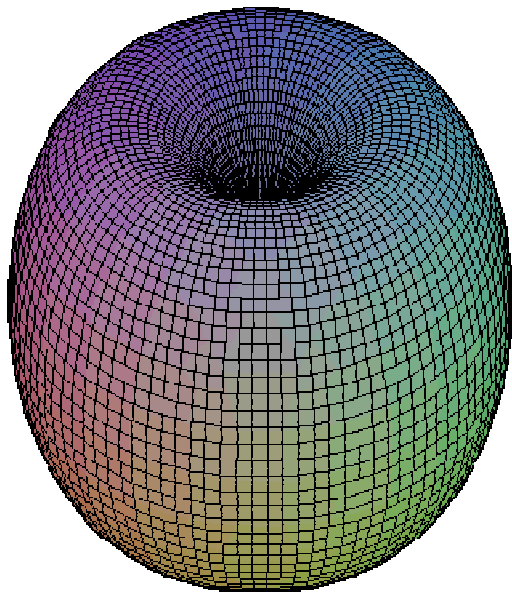
## Magnetocrystalline anisotropy

Ferromagnetic crystals have easy and hard axes. Along some crystallographic directions it is easy to magnetize the crystal, along others harder. Reflects the crystal symmetry.

Extra energy associated with magnetocrystalline anisotropy:

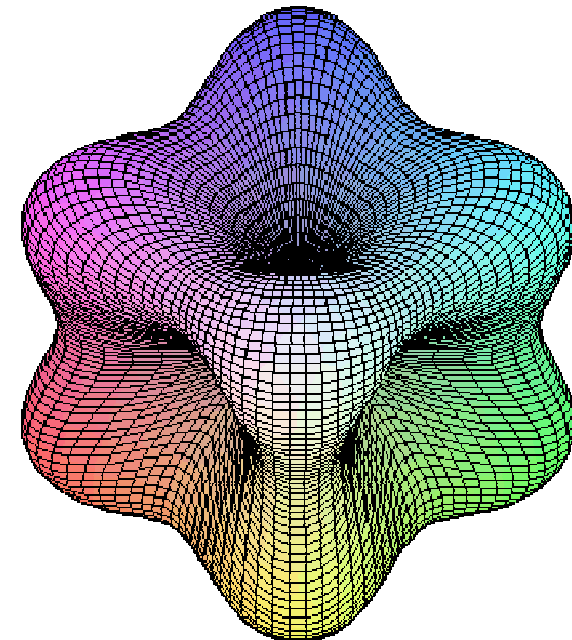
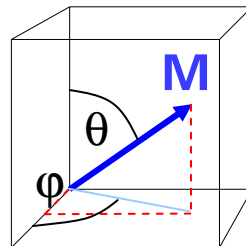
**uniaxial anisotropy (e.g. Co)**

$$E = K_1 \sin^2(\theta) + K_2 \sin^4(\theta) + \dots$$

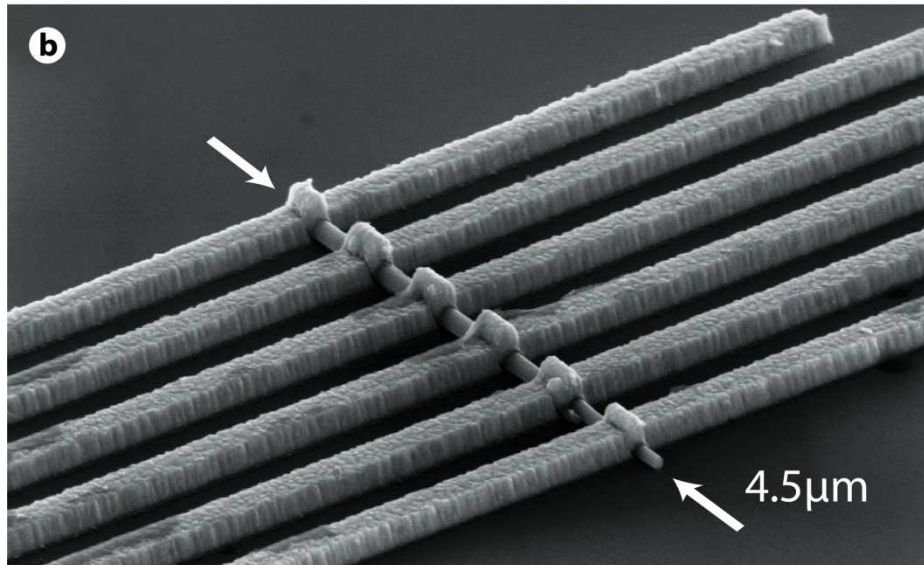
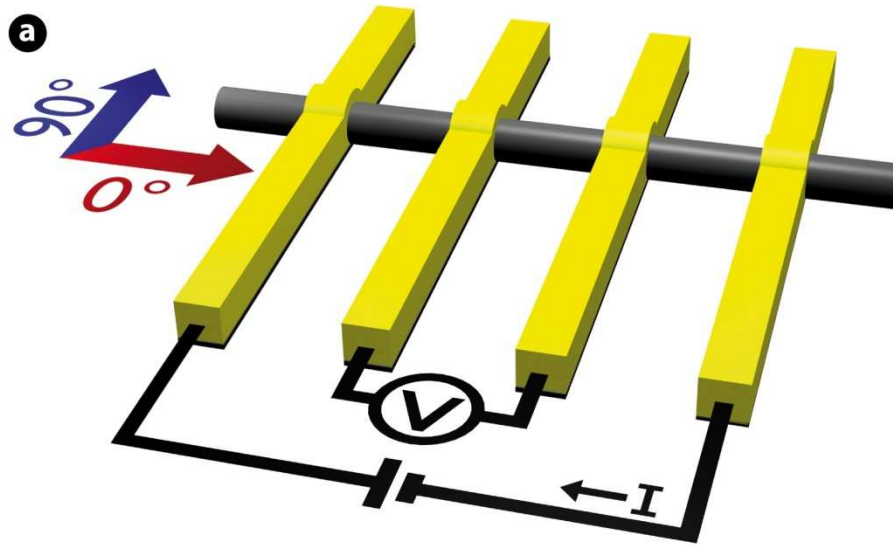


**cubic anisotropy (e.g. Fe)**

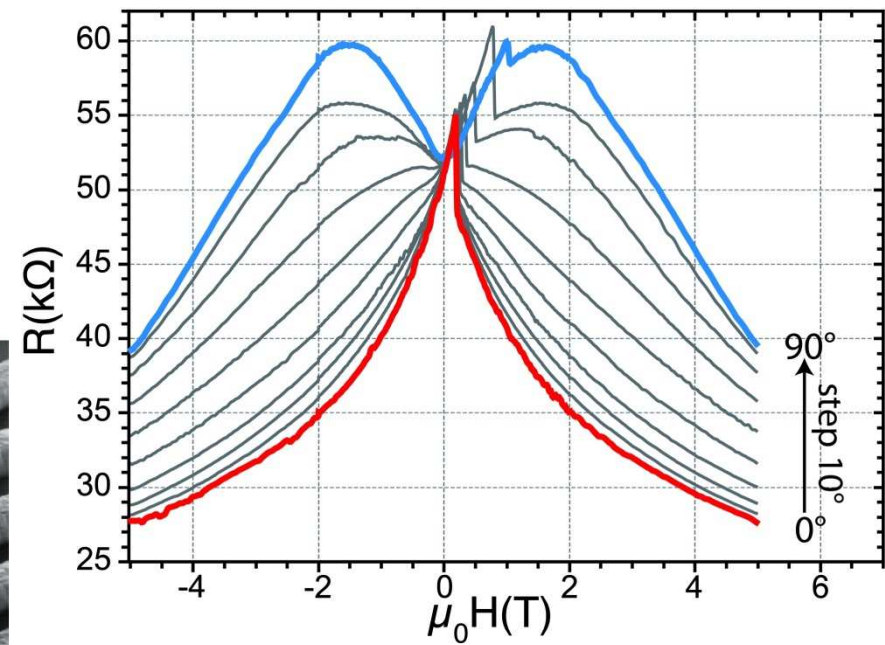
$$E = K_1 \left( \frac{1}{4} \sin^2(\theta) \sin^2(2\phi) + \cos^2(\theta) \right) \sin^2(\theta) + \dots$$



## Example: GaMnAs core-shell nanowires



system with very strong uniaxial anisotropy



see: **WP-22**, C. Butschkow et al.



## Width of a Bloch wall, continued

assume simple form of anisotropy energy

$$E_{\text{ani}} = K \sin^2(\theta); \quad K > 0$$

energy cost of rotation out of easy direction

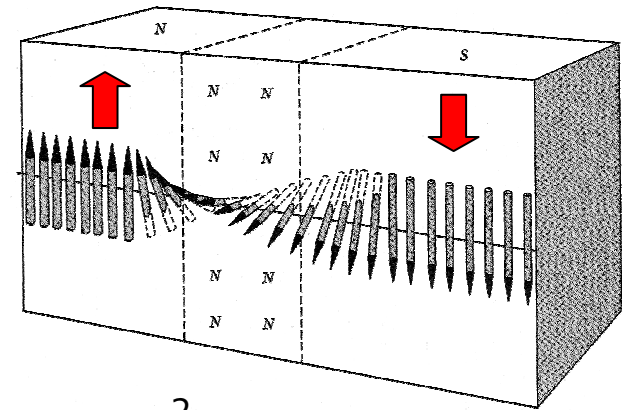
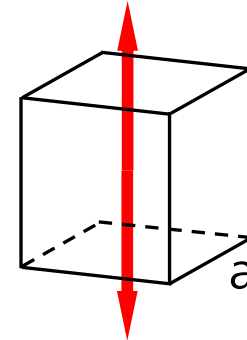
$$\sum_{i=1}^N K \sin^2 \theta_i \approx \frac{N}{\pi} \int_0^{\pi} K \sin^2 \theta d\theta = \frac{NK}{2}$$

$$\text{energy per unit area} = \frac{NKa}{2}$$

$$\text{exchange energy} + \text{anisotropy energy } \epsilon_{\text{BW}} = \frac{NKa}{2} + JS^2 \frac{\pi^2}{Na^2}$$

$$\text{From } d\epsilon_{\text{BW}} / dN = 0 \Rightarrow N = \pi S \sqrt{\frac{2J}{Ka^3}} \Rightarrow Na = \pi S \sqrt{\frac{2J}{Ka}}$$

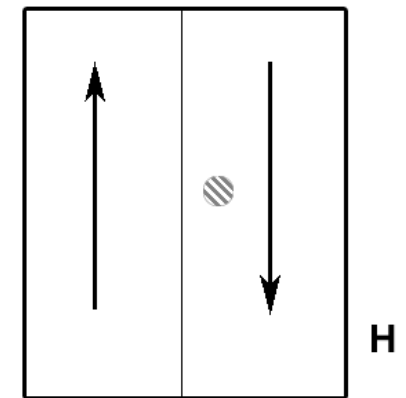
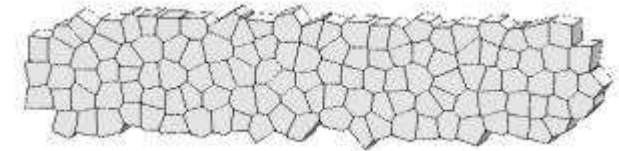
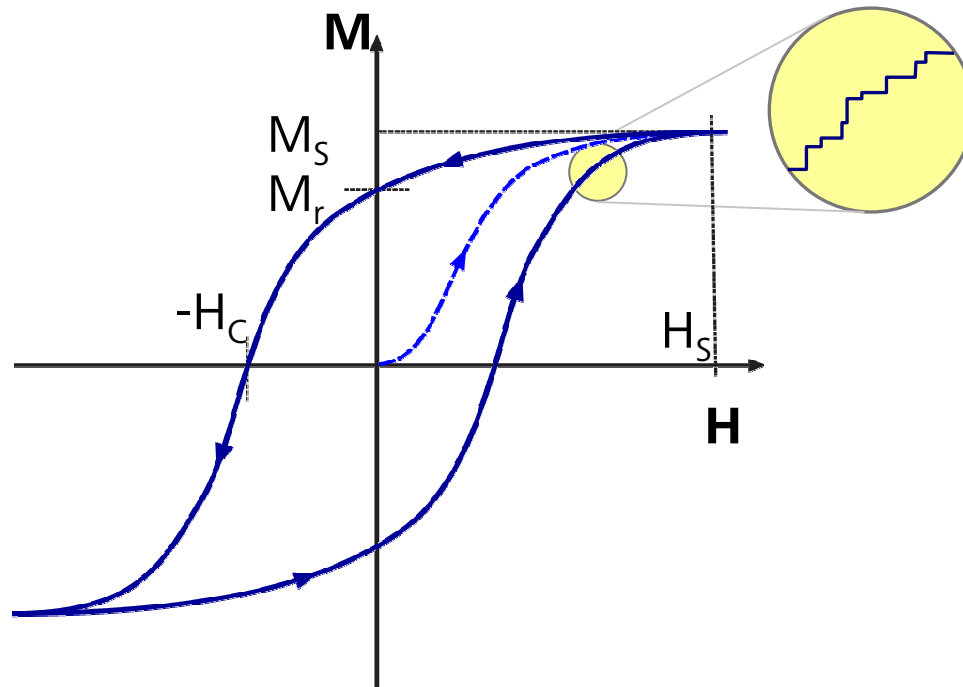
domain wall width





# Reversal of magnetization: Hysteresis

macroscopic bulk material consisting of many domains

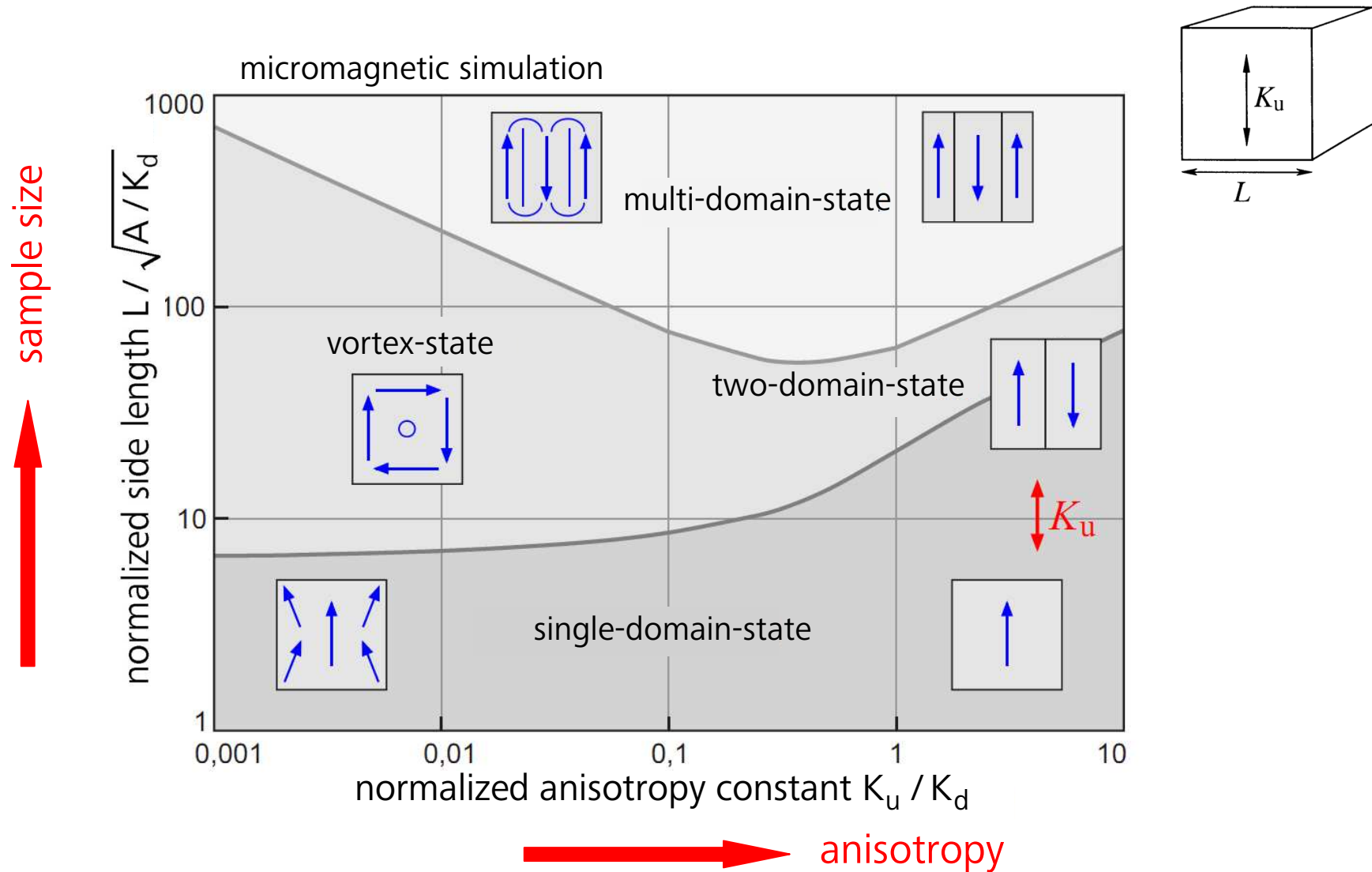


## Processes

1. Shift of domain walls (reversible and irreversible)
2. Domain rotation towards nearest easy axis
3. Coherent rotation at high fields



# Magnetization of a cube with uni-axial anisotropy





# Magnetism and Spin-Orbit Interaction:

## Magnetism

### Basic concepts

magnetic moments, susceptibility, paramagnetism, ferromagnetism  
exchange interaction, domains, magnetic anisotropy,

### Examples:

detection of (nanoscale) magnetization structure  
using Hall-magnetometry, Lorentz microscopy and MFM

## Spin-Orbit interaction

### Some basics

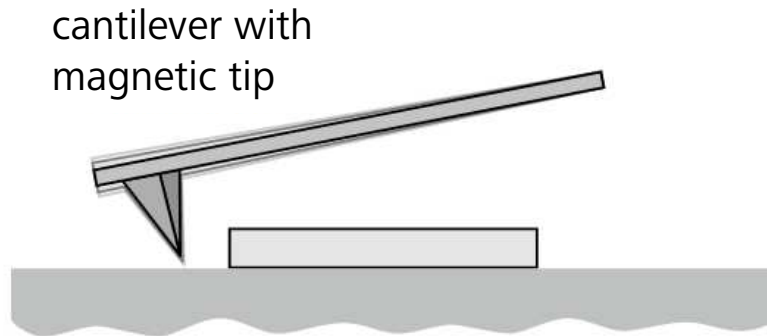
Rashba- and Dresselhaus contribution, SO-interaction and effective  
magnetic field

### Example:

Ferromagnet-Semiconductor Hybrids: TAMR involving epitaxial  
Fe/GaAs interfaces

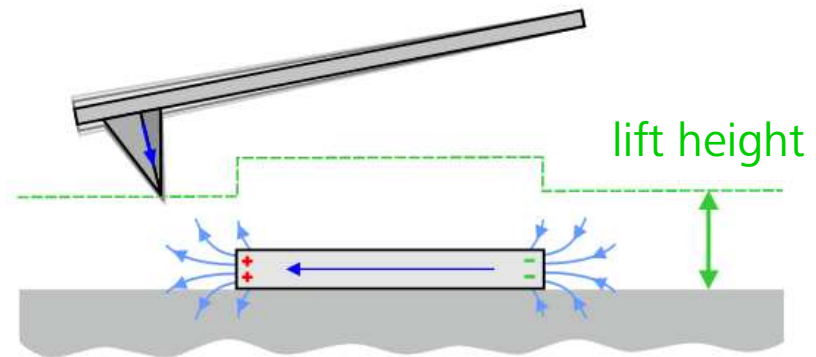


# Measurement of domains: magnetic force microscopy



cantilever with magnetic tip

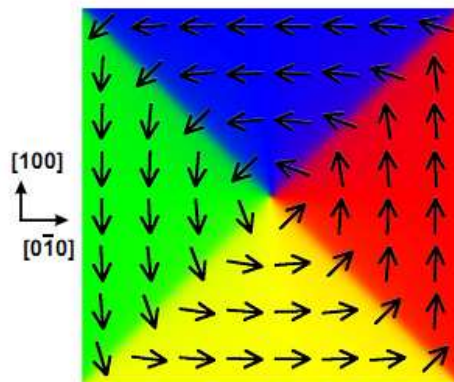
1. topography scan



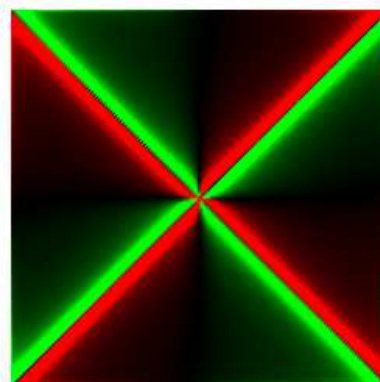
lift height

2. MFM scan (lift mode)

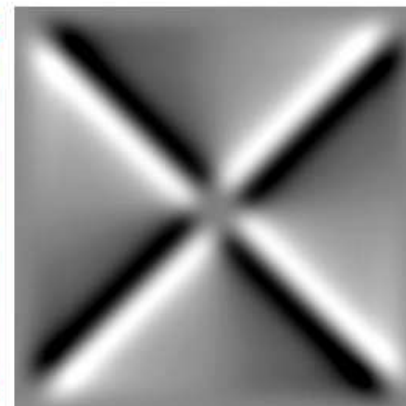
MFM measures „magnetic charges“



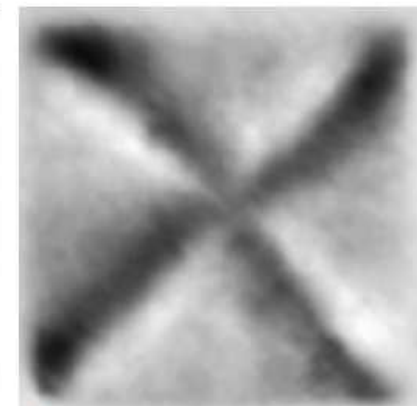
magnetization  $\mathbf{M}$



$\text{div } \mathbf{M}$



sim. MFM picture

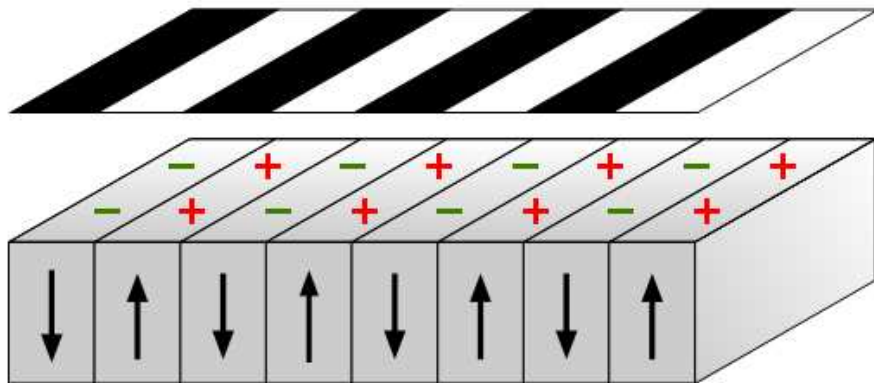


real MFM picture

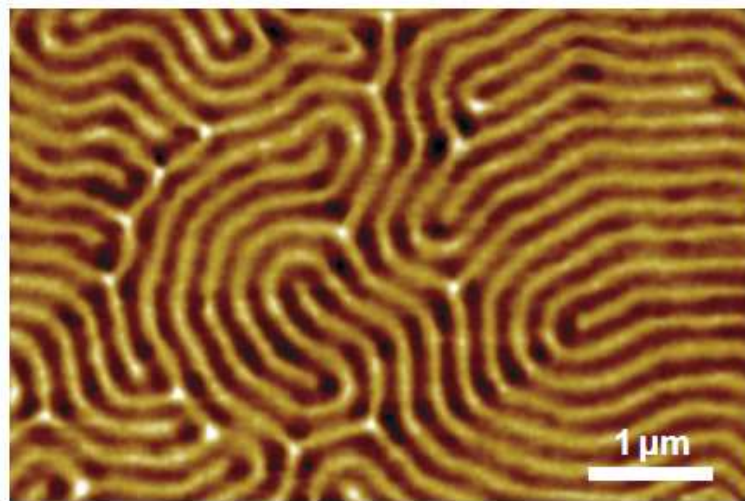
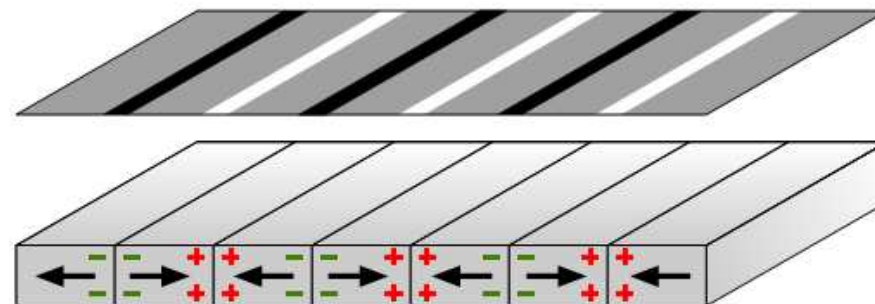


# Domains, measured by MFM

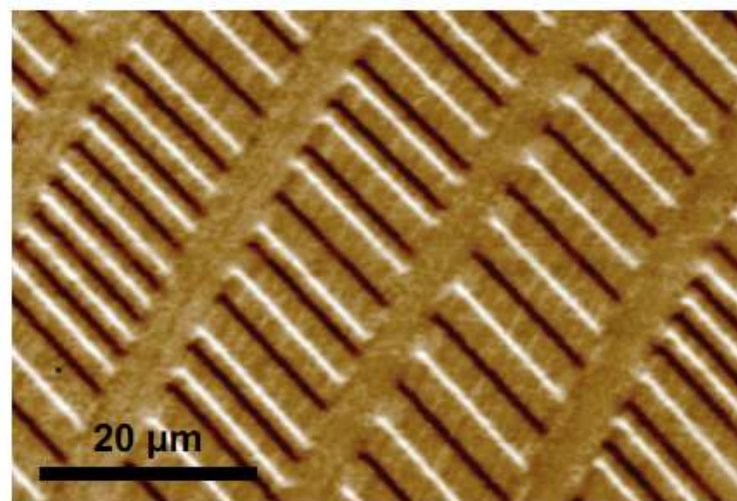
Perpendicular magnetization



in-plane magnetization



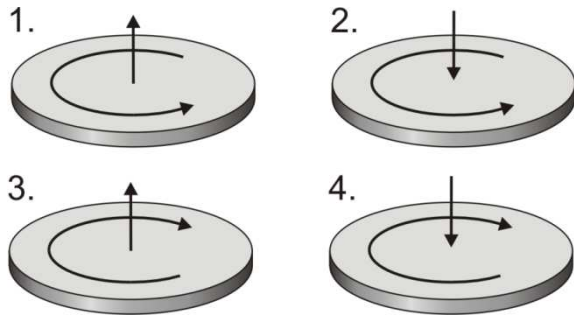
4Å Fe / 4Å Gd (75 layers)



50 Mbyte hard drive



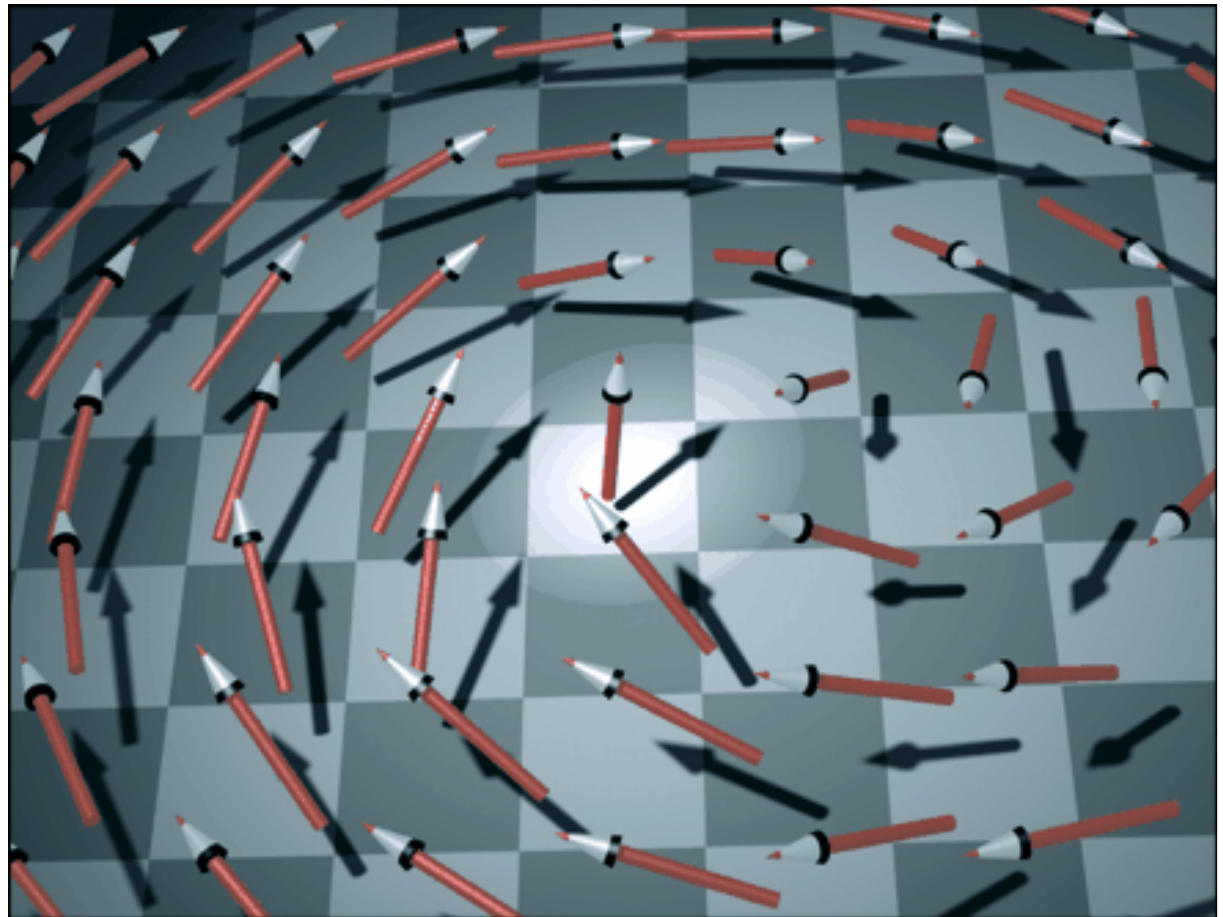
## Between multidomain and single-domain state: vortex structure



4 ground state configurations

in-plane and out-of plane magnetization component can be switched independently!

Picture: Ref <sup>3)</sup>



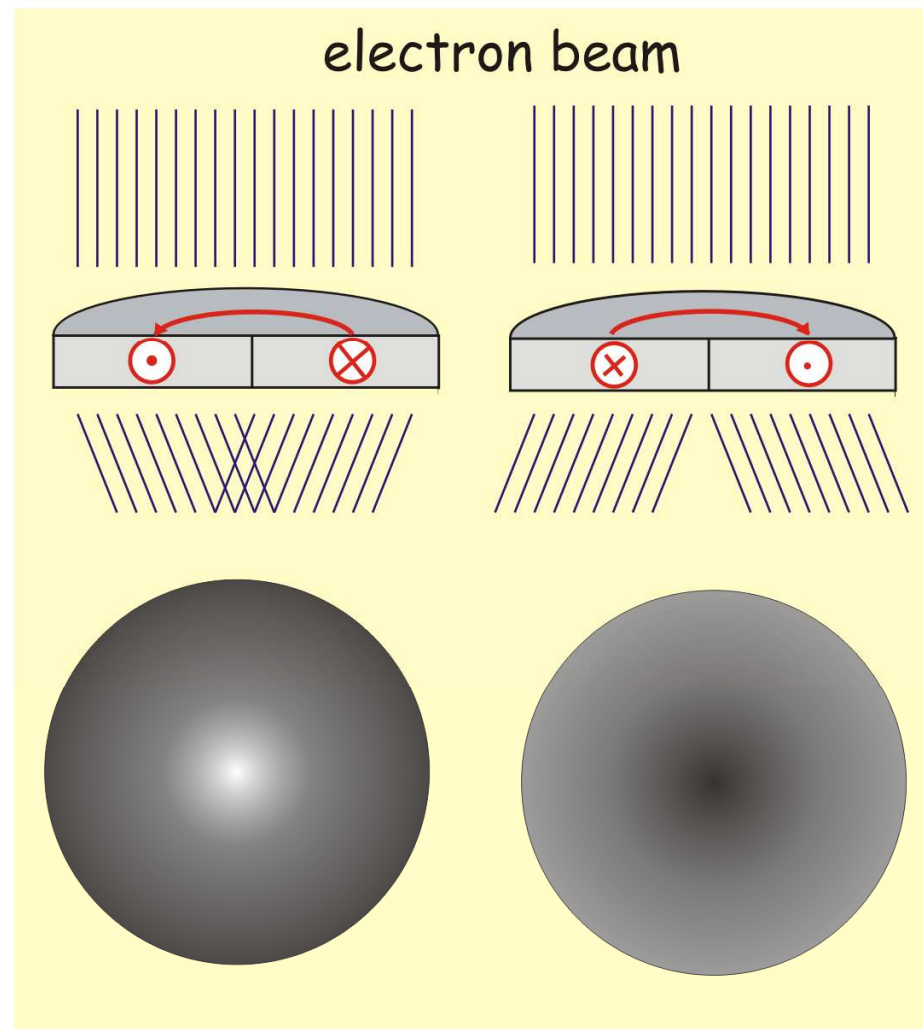
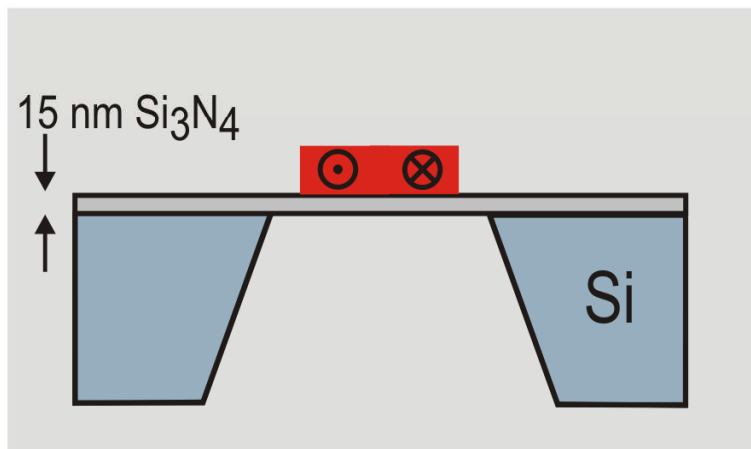
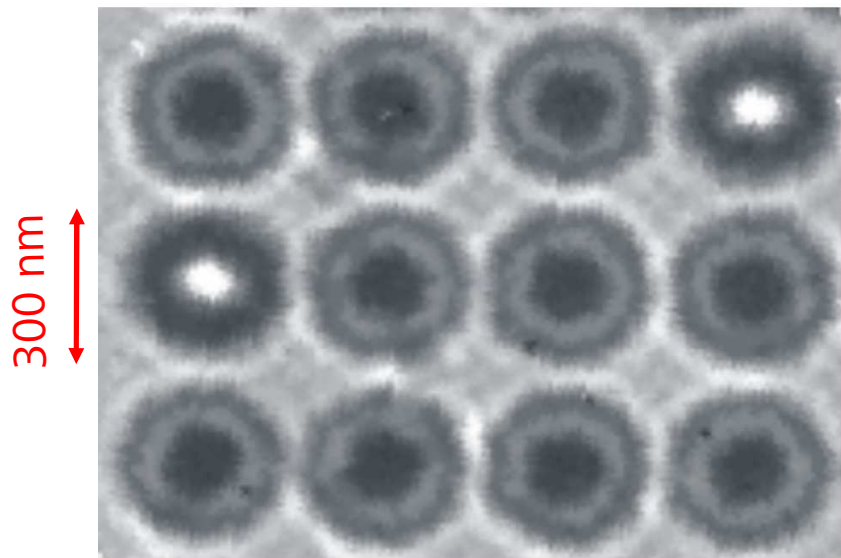
1) J. Raabe, R. Pulwey et al., J. Appl. Phys. **88**, 4437 (2000)

2) T. Shinjo et al., Science 289, 930 (2000)

3) A. Wachowiak et al., Science **298**, 577 (2002)

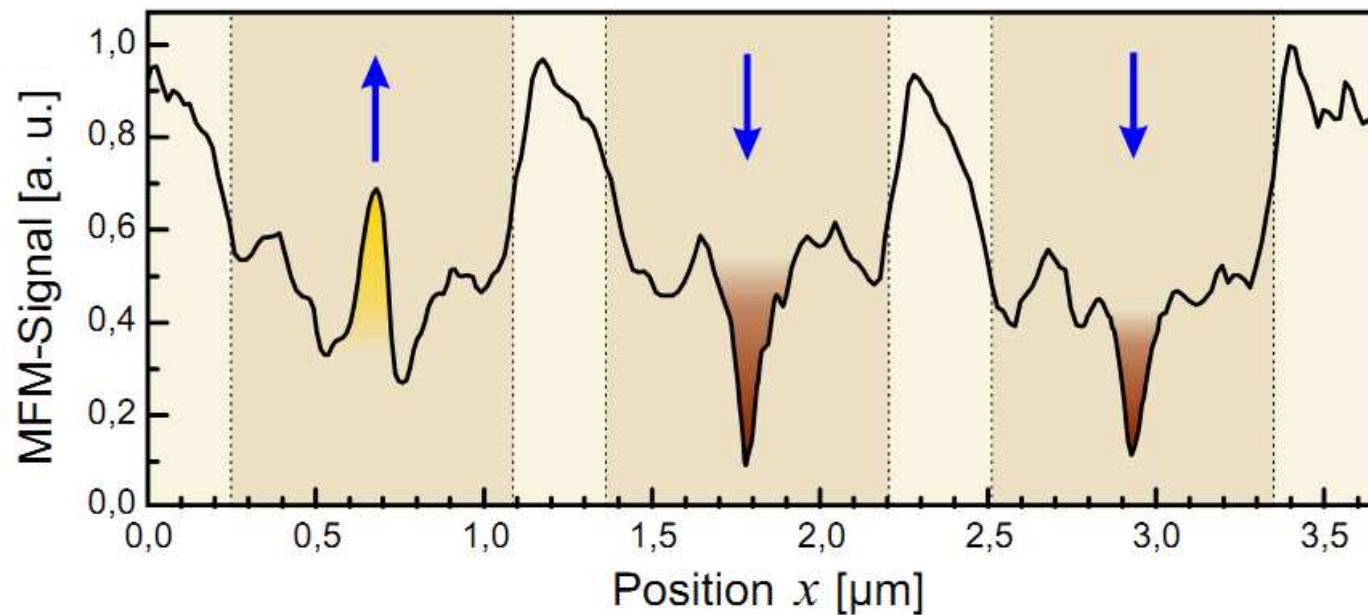
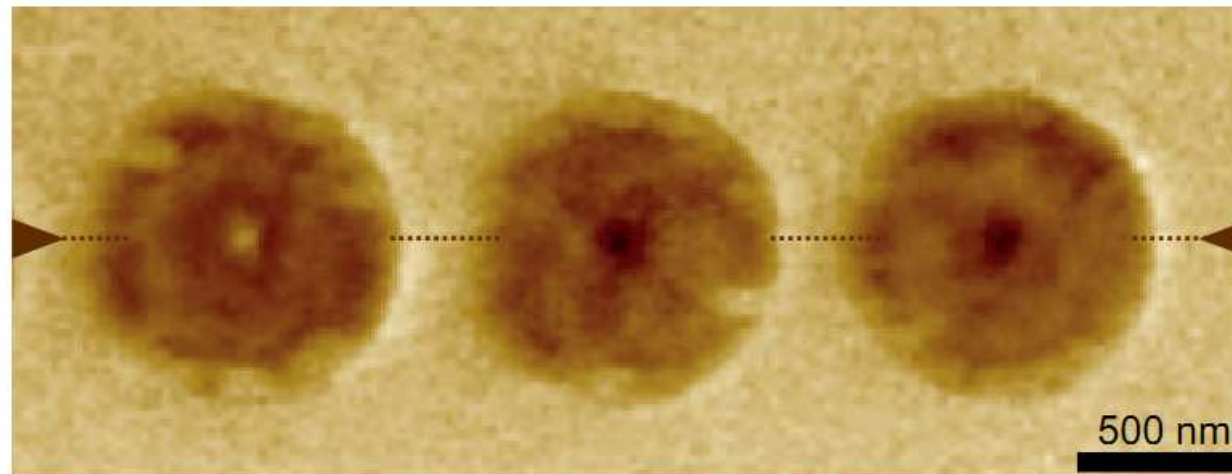


# Detection of in-plane vortex



Lorentz-microscopy probes in-plane magnetization...

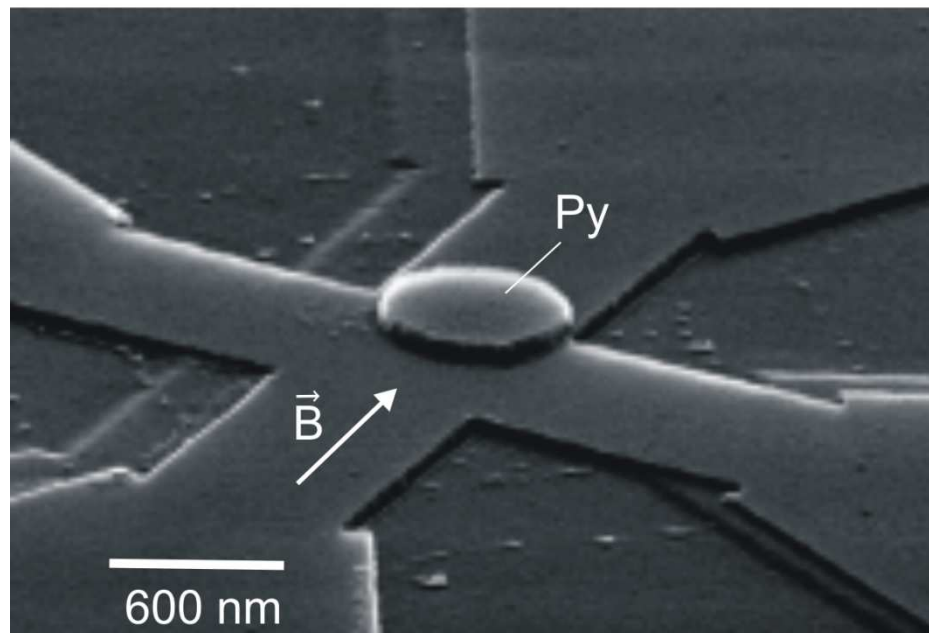
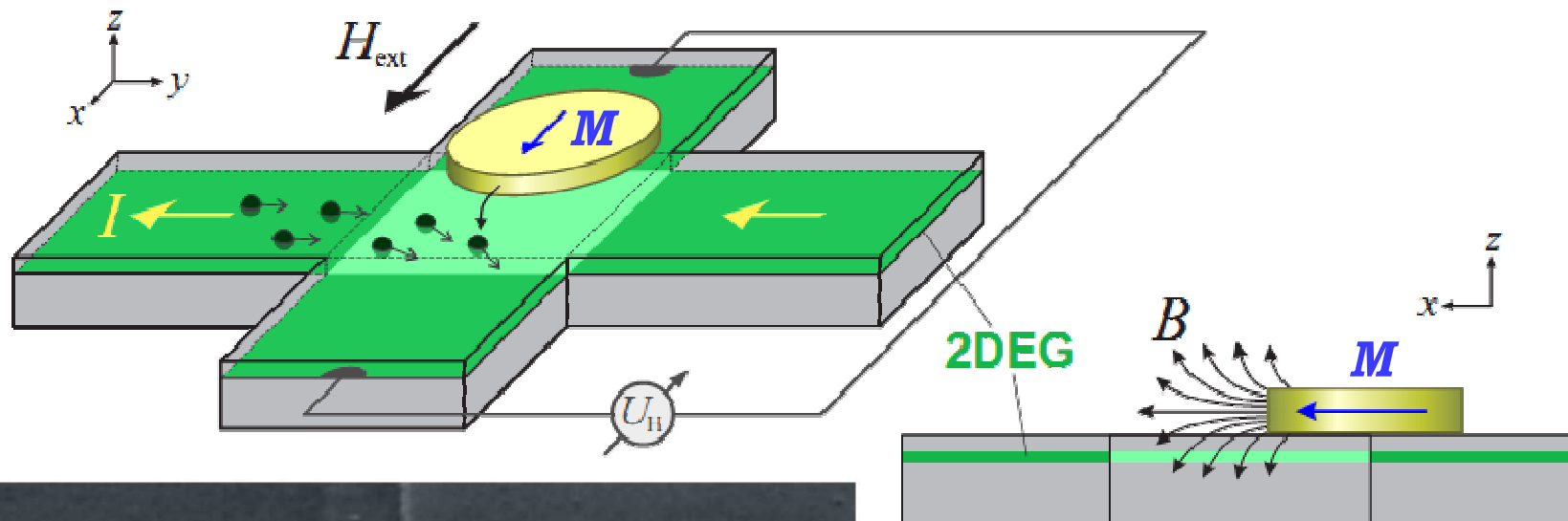
## Detection of vortex singularity





## Another technique: Micro Hall magnetometry

allows to measure hysteresis traces



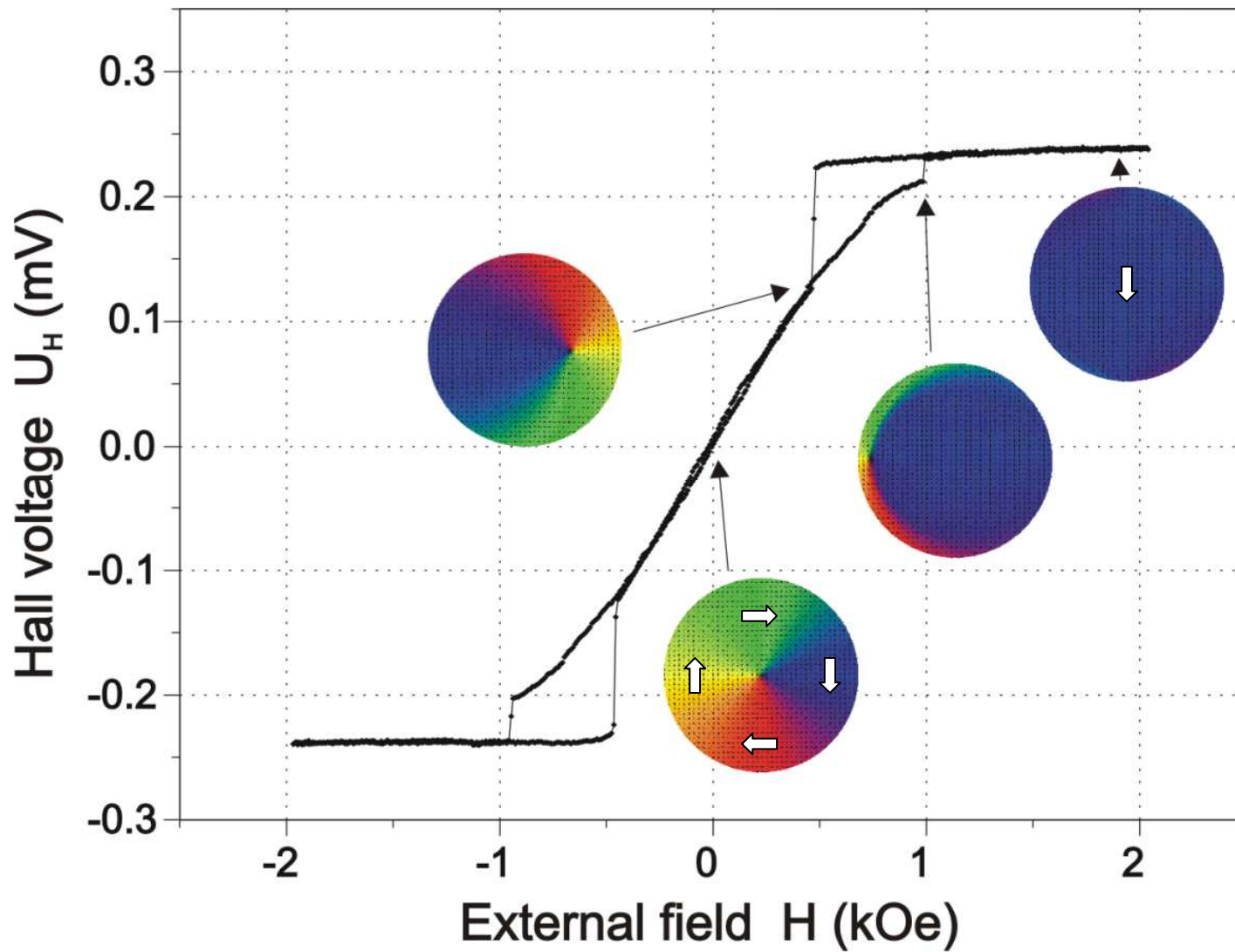
Measures directly the stray field, averaged over the central area of the Hall junction

$$U_H = \frac{I}{en_s} \langle B \rangle$$



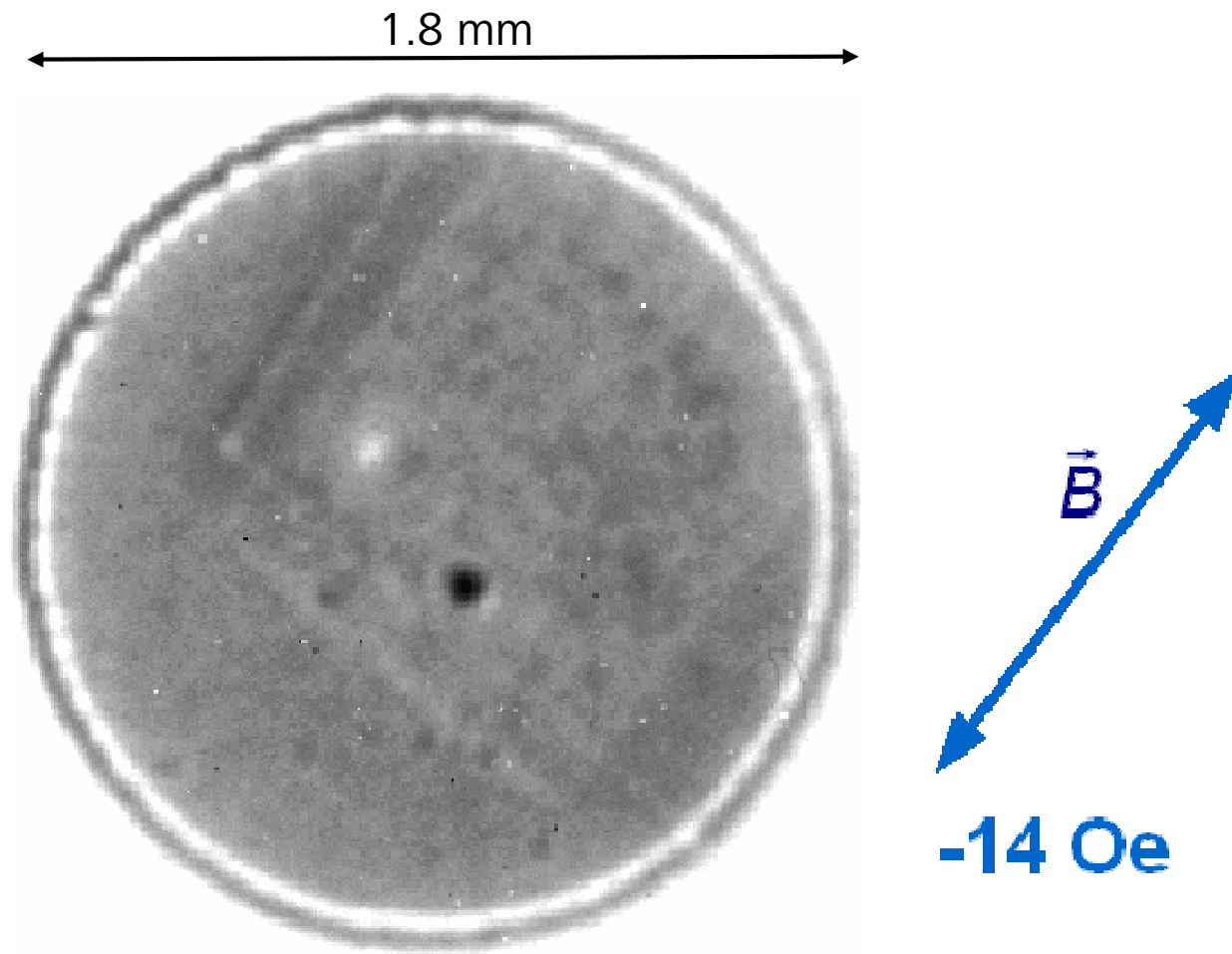


# Hysteresis trace of a vortex state





# Vortex-Golf





# Magnetism and Spin-Orbit Interaction:

## Magnetism

### Basic concepts

magnetic moments, susceptibility, paramagnetism, ferromagnetism  
exchange interaction, domains, magnetic anisotropy,

### Examples:

detection of (nanoscale) magnetization structure  
using Hall-magnetometry, Lorentz microscopy and MFM

## Spin-Orbit interaction

### Some basics

Rashba- and Dresselhaus contribution, SO-interaction and effective  
magnetic field

### Example:

Ferromagnet-Semiconductor Hybrids: TAMR involving epitaxial  
Fe/GaAs interfaces



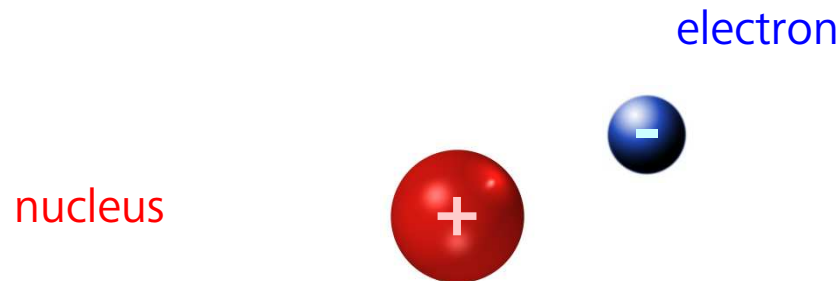
# Origin of spin-orbit (SO) interaction

Spin-orbit interaction

$$E = -\mu_B \mathbf{B}_{\text{eff}}$$

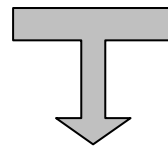
due to orbital motion

magnetic moment of electron



$$\hat{H}_{\text{SO}} = -\mu_B \hat{\boldsymbol{\sigma}} \cdot \left[ \frac{\mathbf{E} \times \mathbf{p}}{2mc^2} \right]$$

vector of Pauli spin matrices

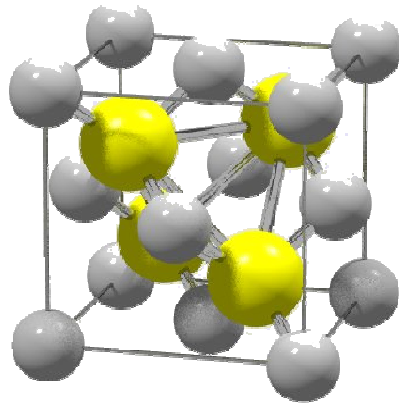


$$\mathbf{B}_{\text{eff}} = \frac{\mathbf{E} \times \mathbf{p}}{2mc^2}$$

$$\hat{H}_{\text{Zeeman}} = -\mu_B \hat{\boldsymbol{\sigma}} \cdot \mathbf{B}$$

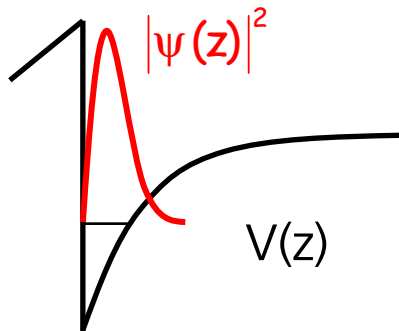
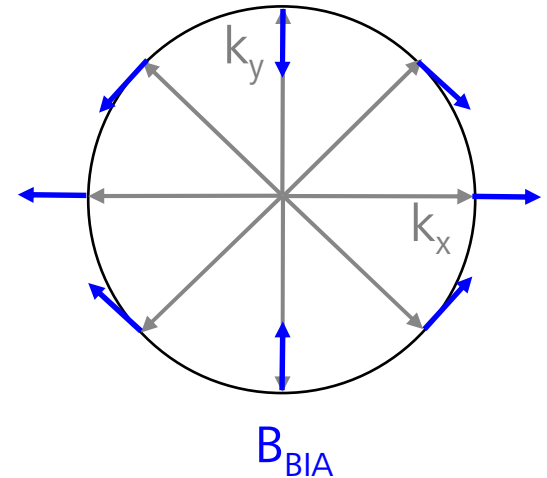


# Electric field E in solids



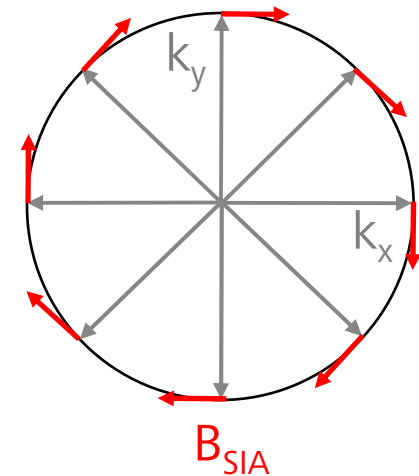
Bulk inversion asymmetry (BIA)  
Lack of inversion symmetry in  
III-V semiconductors  
"Dresselhaus contribution  $\gamma$ "

$$\mathbf{B}_{\text{BIA}} \propto \gamma \begin{pmatrix} k_x \\ -k_y \end{pmatrix}$$



Structure inversion asymmetry (SIA)  
due to macroscopic confining potential:  
"Rashba contribution  $\alpha$ ". Tunable by  
external electric field!

$$\mathbf{B}_{\text{SIA}} \propto \alpha \begin{pmatrix} k_y \\ -k_x \end{pmatrix}$$

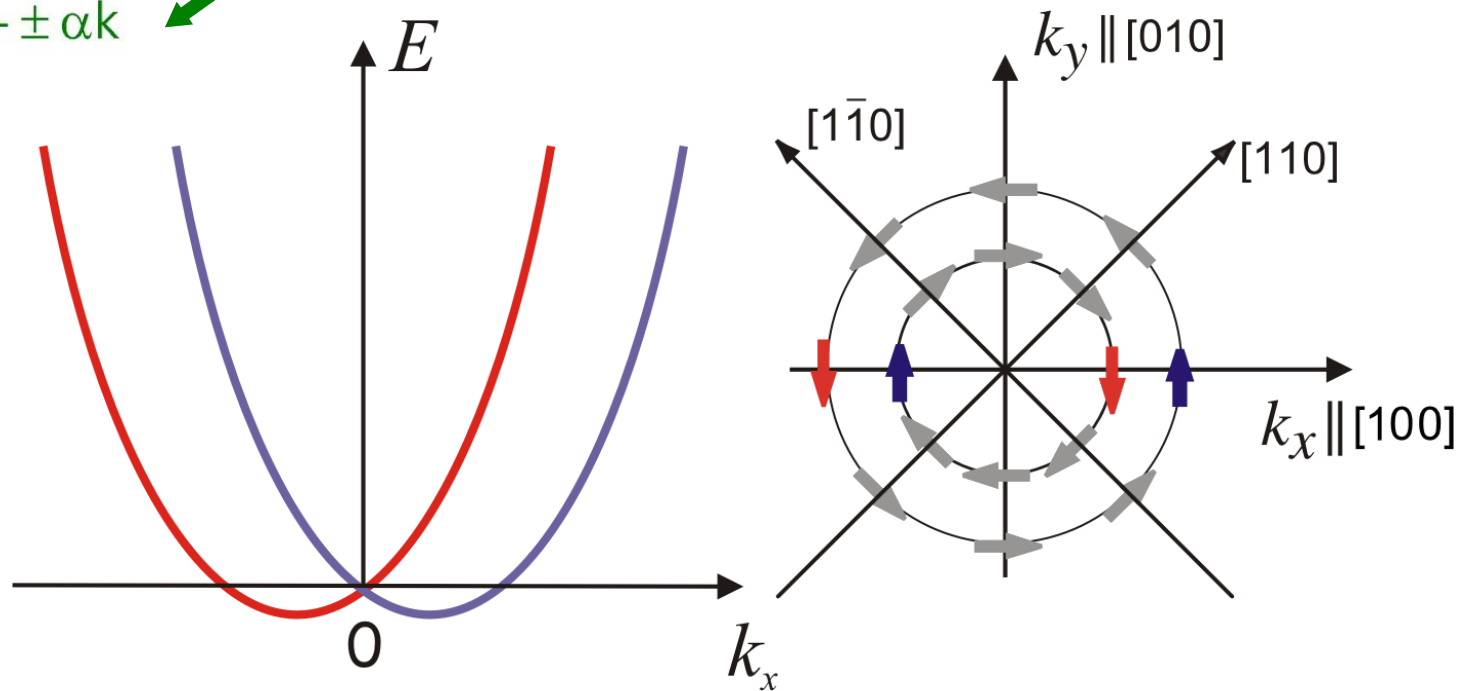




# SO interaction in 2DEG: Rashba & Dresselhaus terms

$$\hat{H} = \frac{\hbar^2 k^2}{2m} + \hat{H}_{SO} \quad \text{with} \quad \begin{array}{l} \text{tunable by gate voltage} \\ \text{Pauli spin matrix} \end{array}$$
$$\hat{H}_{SO} = \underbrace{\alpha(\sigma_x k_y - \sigma_y k_x)}_{\text{Rashba}} + \underbrace{\gamma(\sigma_x k_x - \sigma_y k_y)}_{\text{Dresselhaus}}$$

$$E = \frac{\hbar^2 k^2}{2m} \pm \alpha k$$



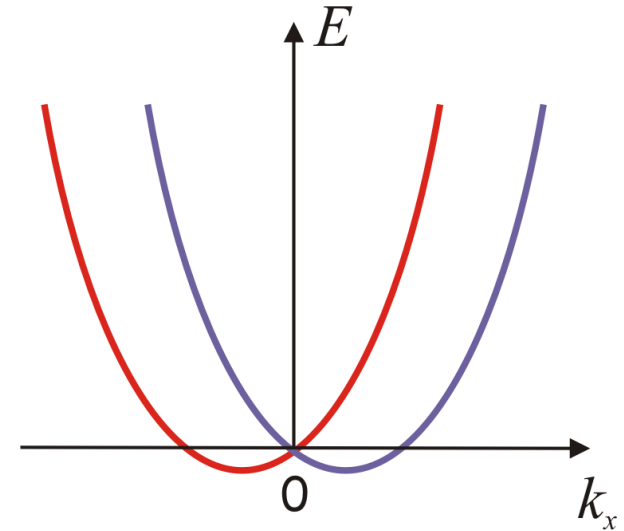


## Calculation of Eigenvalue

$$E = \frac{\hbar^2 k^2}{2m} \pm \alpha k$$

$$\hat{H}_{SO}^{\text{Rashba}} = \alpha(\sigma_x k_y - \sigma_y k_x)$$

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \quad \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



Pauli Spin matrices

$$\hat{H}_{SO}^{\text{Rashba}} = \alpha(\sigma_x k_y - \sigma_y k_x) =$$

$$= \begin{pmatrix} 0 & \alpha k_y \\ \alpha k_y & 0 \end{pmatrix} - \begin{pmatrix} 0 & -i\alpha k_x \\ i\alpha k_x & 0 \end{pmatrix} = \begin{pmatrix} 0 & \alpha(k_y + ik_x) \\ \alpha(k_y - ik_x) & 0 \end{pmatrix}$$

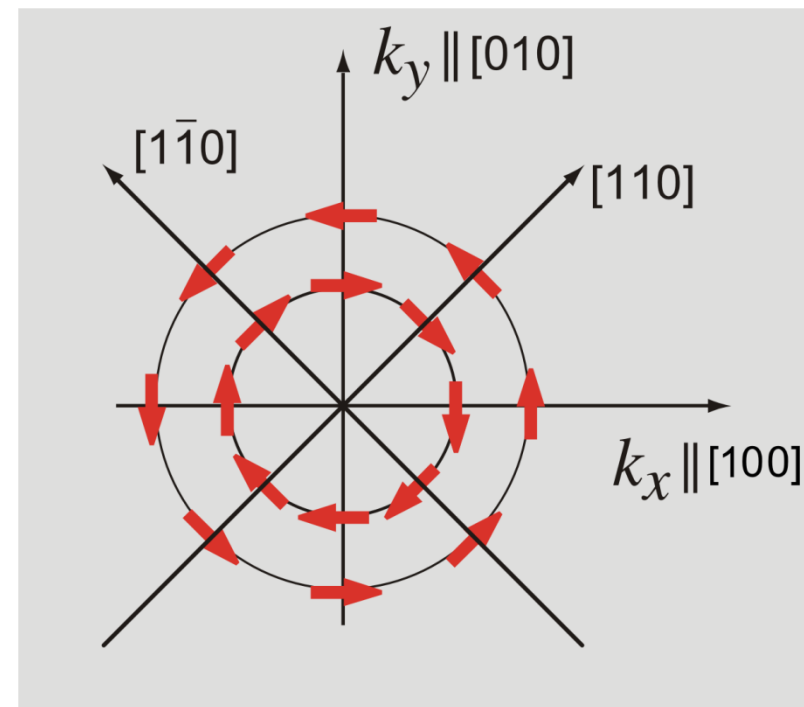
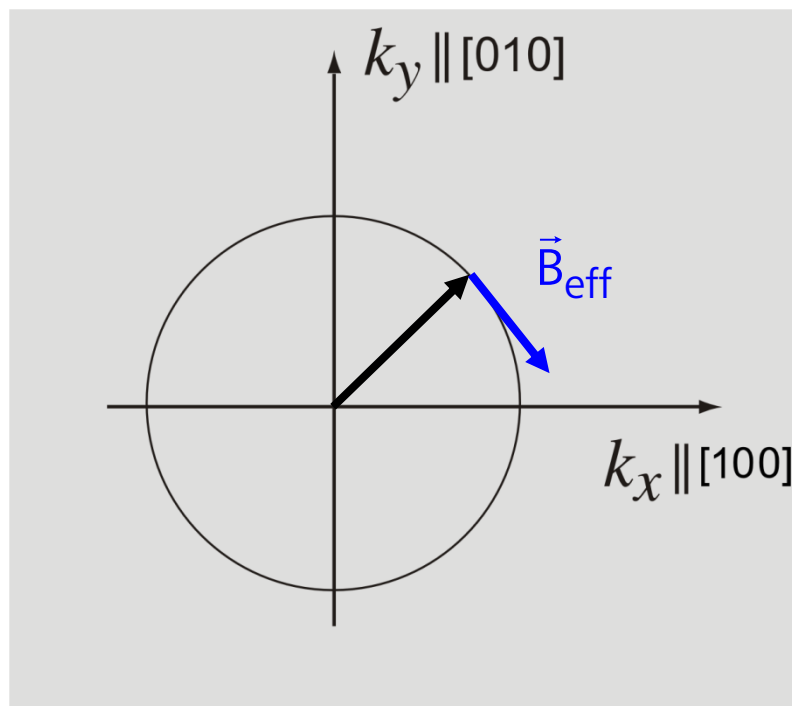
$$\text{Eigenvalues of the matrix : } \pm \alpha \sqrt{k_x^2 + k_y^2} = \pm \alpha k_{\parallel}$$



## Description of zero-field spin splitting by $\vec{B}_{\text{eff}}$

$$\hat{H}_{\text{SO}} = \underbrace{\alpha(\sigma_x k_y - \sigma_y k_x)}_{\text{Rashba}} + \underbrace{\gamma(\sigma_x k_x - \sigma_y k_y)}_{\text{Dresselhaus}} \sim \hat{\sigma} \cdot \vec{B}_{\text{eff}}; \quad \hat{\sigma} \cdot \vec{B}_{\text{eff}} = \sigma_x B_x + \sigma_y B_y + \sigma_z B_z$$

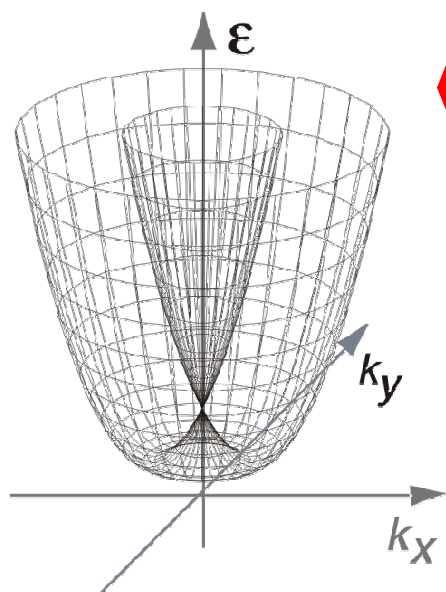
Comparison of coefficients. E.g. only Rashba contribution:  $\vec{B}_{\text{eff}} \propto \alpha \begin{pmatrix} k_y \\ -k_x \end{pmatrix}$





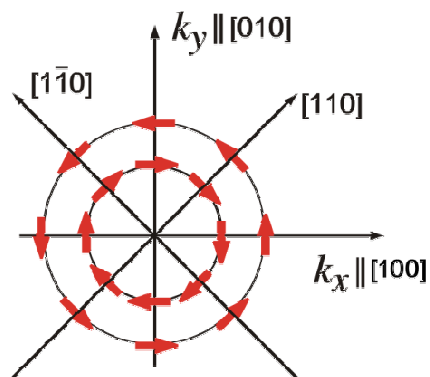
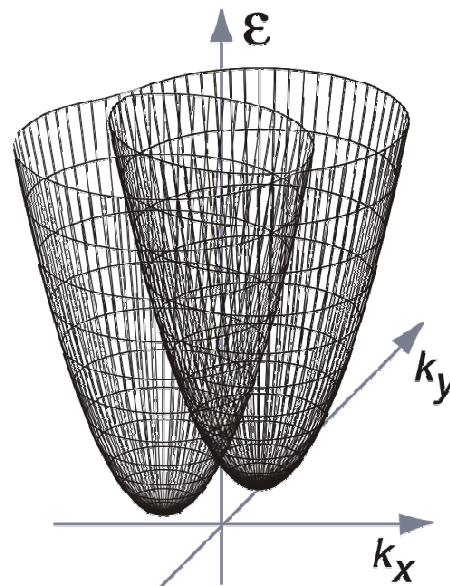


# Presence of Rashba & Dresselhaus contributions

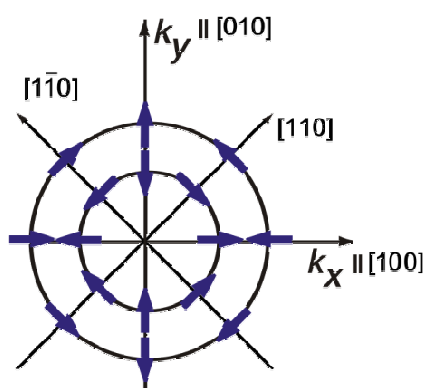


← Rashba or Dresselhaus

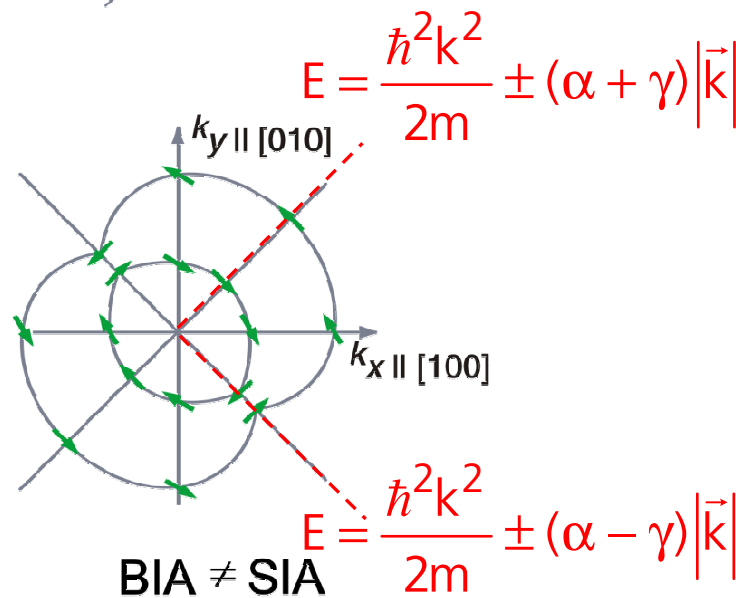
Rashba and Dresselhaus



BIA=0  
SIA≠0



BIA≠0  
SIA=0

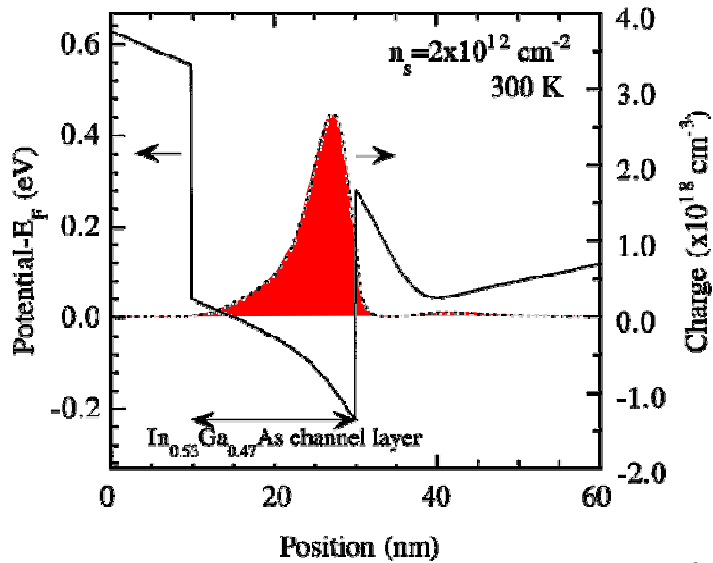
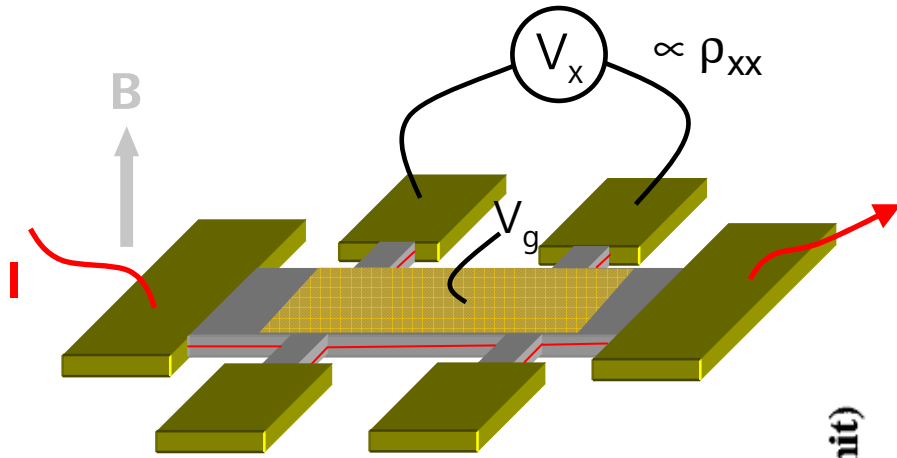


BIA ≠ SIA

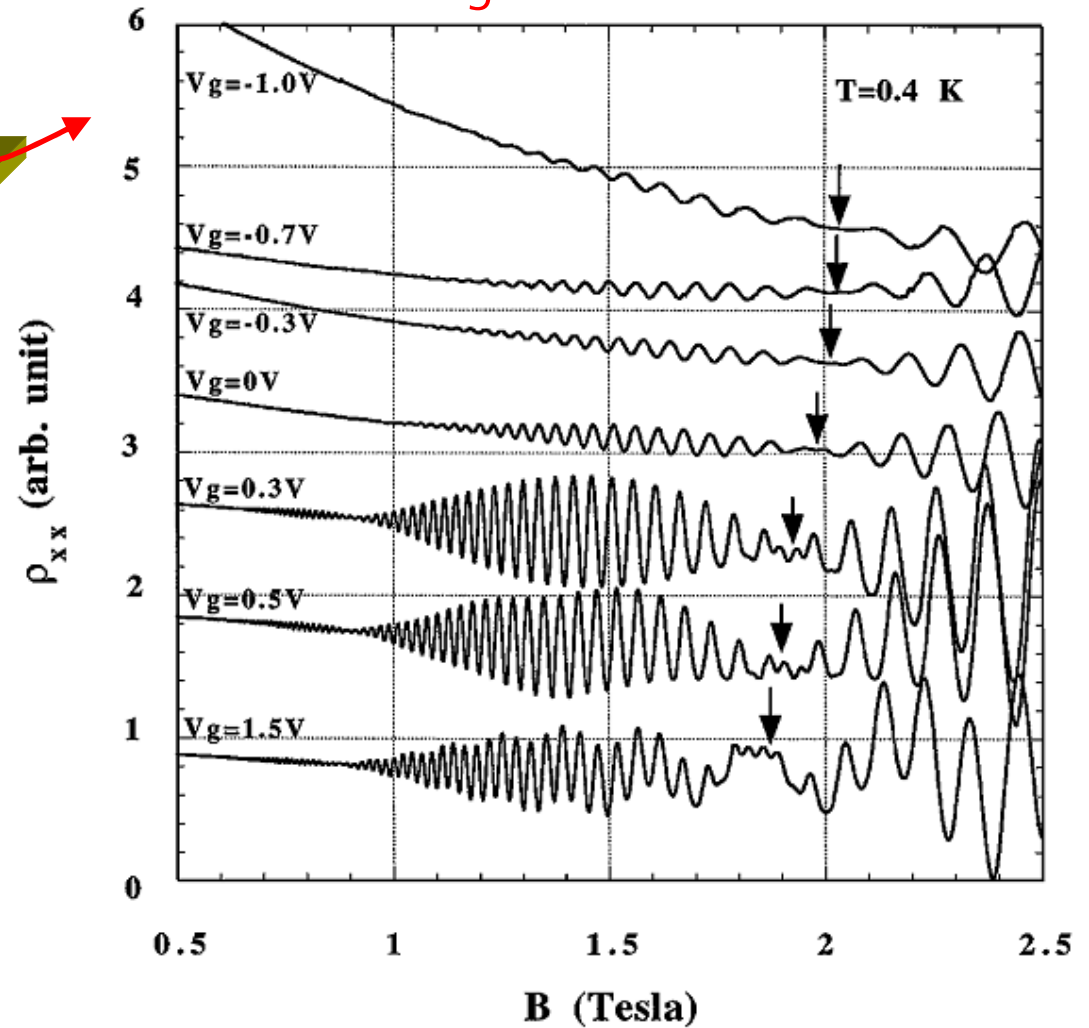
$$E = \frac{\hbar^2 k^2}{2m} \pm (\alpha + \gamma) |\vec{k}|$$

$$E = \frac{\hbar^2 k^2}{2m} \pm (\alpha - \gamma) |\vec{k}|$$

# SO-interaction in a InGaAs quantum well

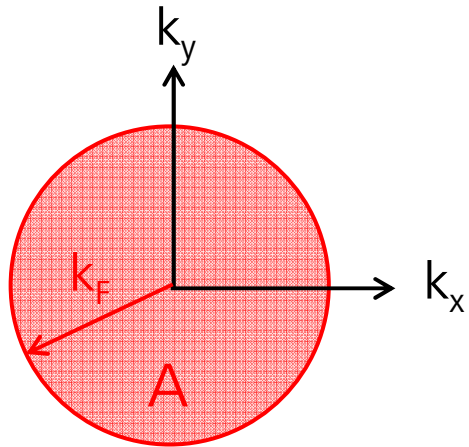


## Beating of SdH-oscillations





# Quantum oscillations (SdH) reflect k-space area

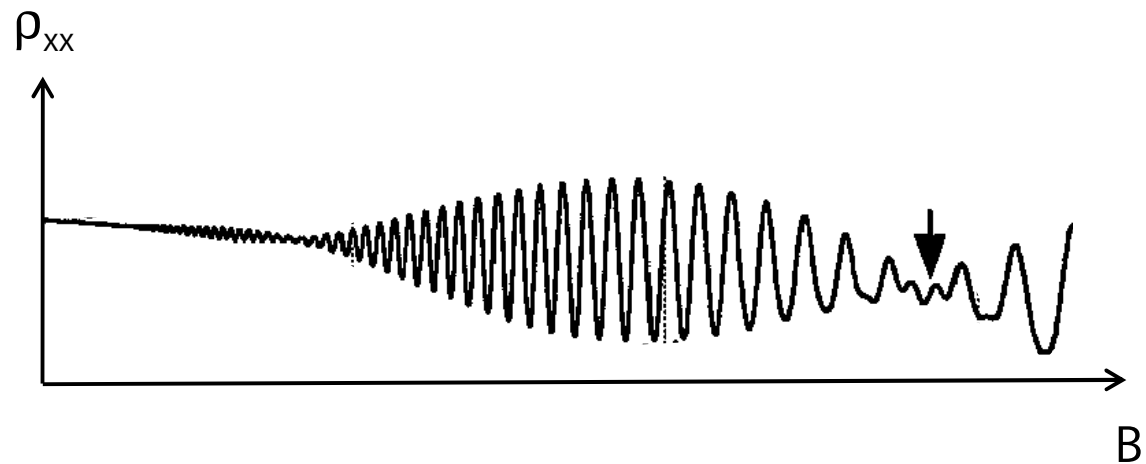
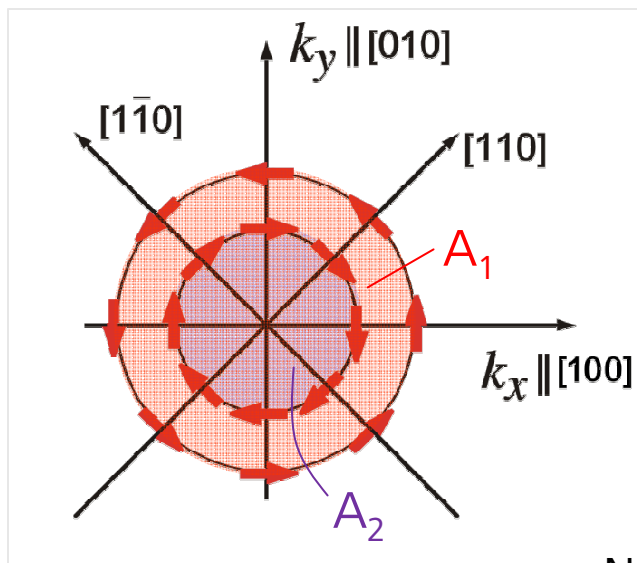


Periodicity of Shubnikov-de Haas (SdH) oscillations

$$\Delta\left(\frac{1}{B}\right) = \frac{2\pi e}{\hbar A}$$

Note that  $A = \pi k_F^2 = 2\pi^2 n_s$

Origin of beating: two periodicities due to two k-space areas  $A_1$  and  $A_2$

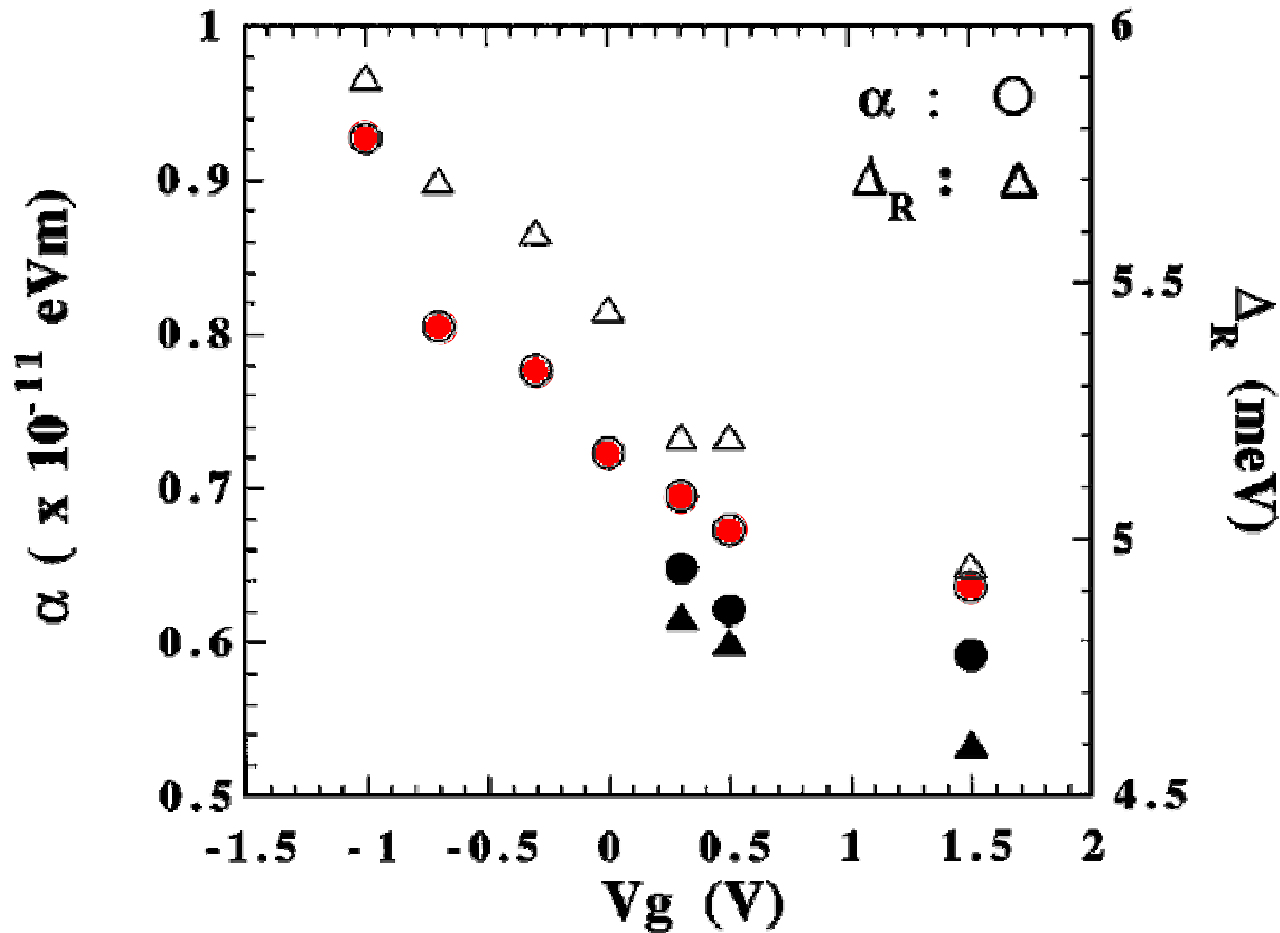


Nitta et al., Phys. Rev. Lett **78**, 1335 (1997)



# Tuning of Rashba coefficient $a$ by gate voltage $V_g$

Corresponds to tuning of spin orbit field, i.e.,  $\vec{B}_{\text{eff}} = \alpha \begin{pmatrix} k_y \\ -k_x \end{pmatrix}$





# Magnetism and Spin-Orbit Interaction:

## Magnetism

### Basic concepts

magnetic moments, susceptibility, paramagnetism, ferromagnetism  
exchange interaction, domains, magnetic anisotropy,

### Examples:

detection of (nanoscale) magnetization structure  
using Hall-magnetometry, Lorentz microscopy and MFM

## Spin-Orbit interaction

### Some basics

Rashba- and Dresselhaus contribution, SO-interaction and effective  
magnetic field

### Example:

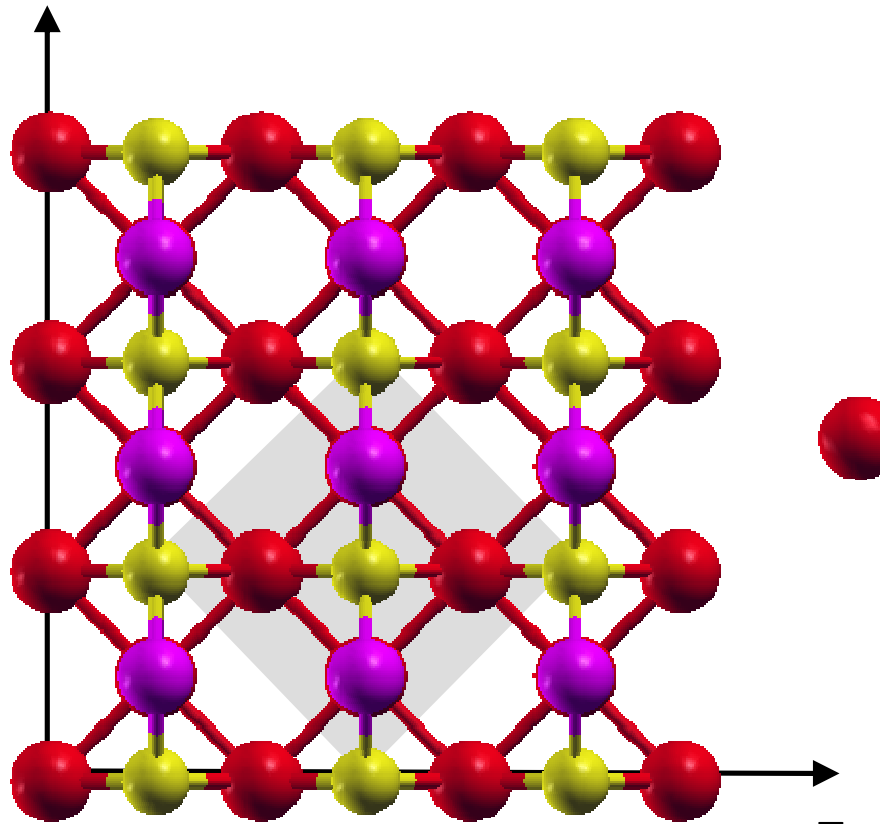
Ferromagnet-Semiconductor Hybrids: TAMR involving epitaxial  
Fe/GaAs interfaces



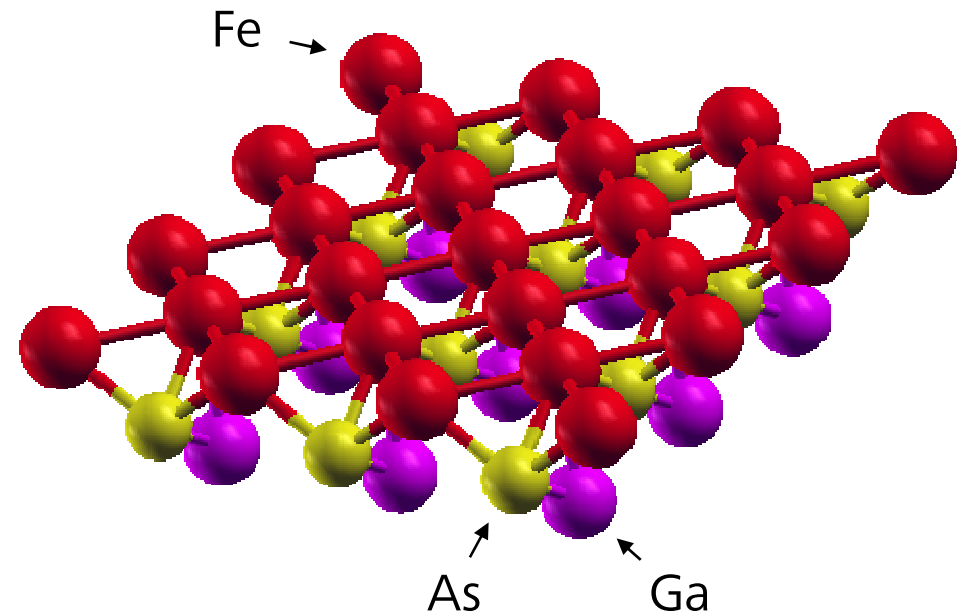
# Spin-orbit interaction plays also role in tunneling

Epitaxial Fe-GaAs interface

[110]



[ $\bar{1}\bar{1}0$ ]

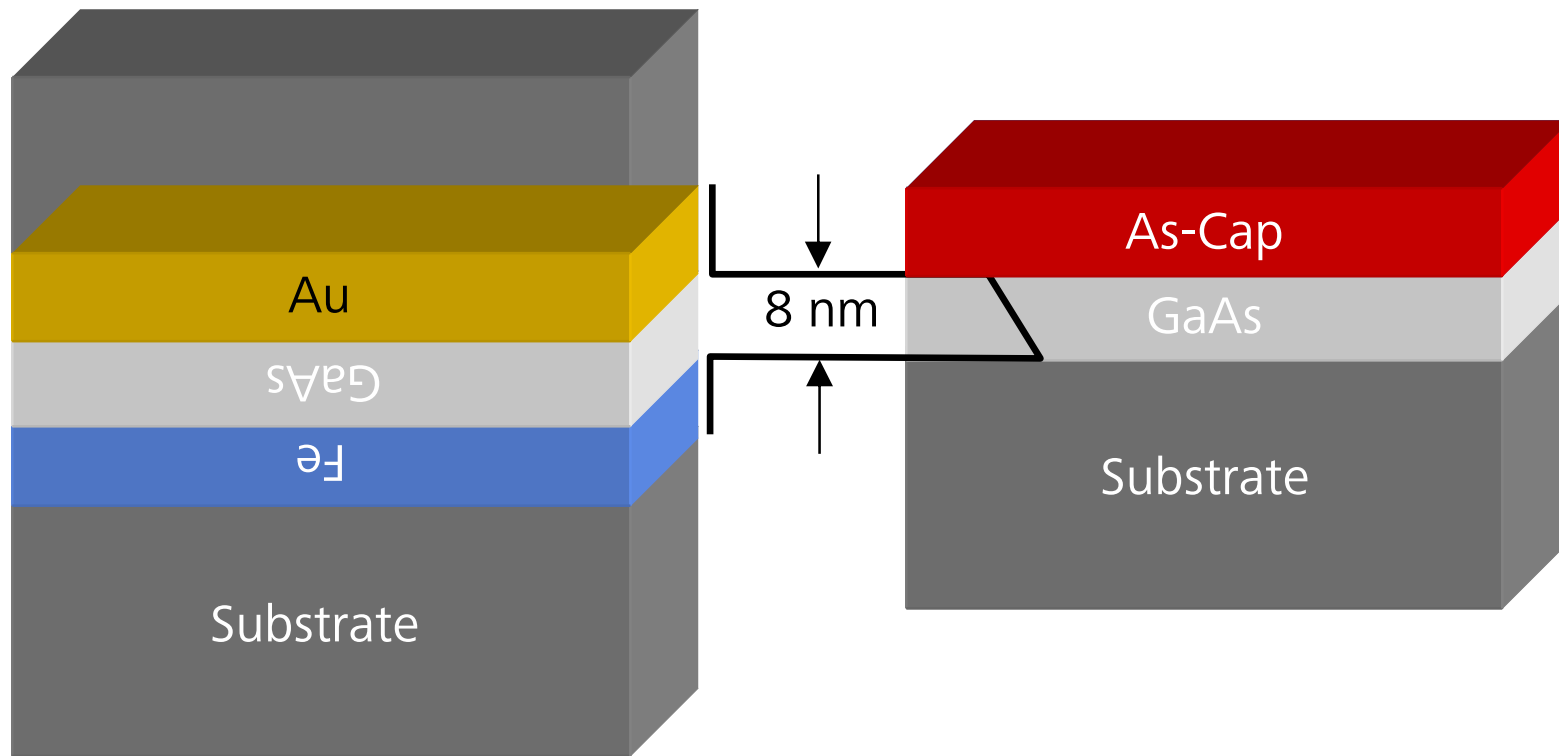


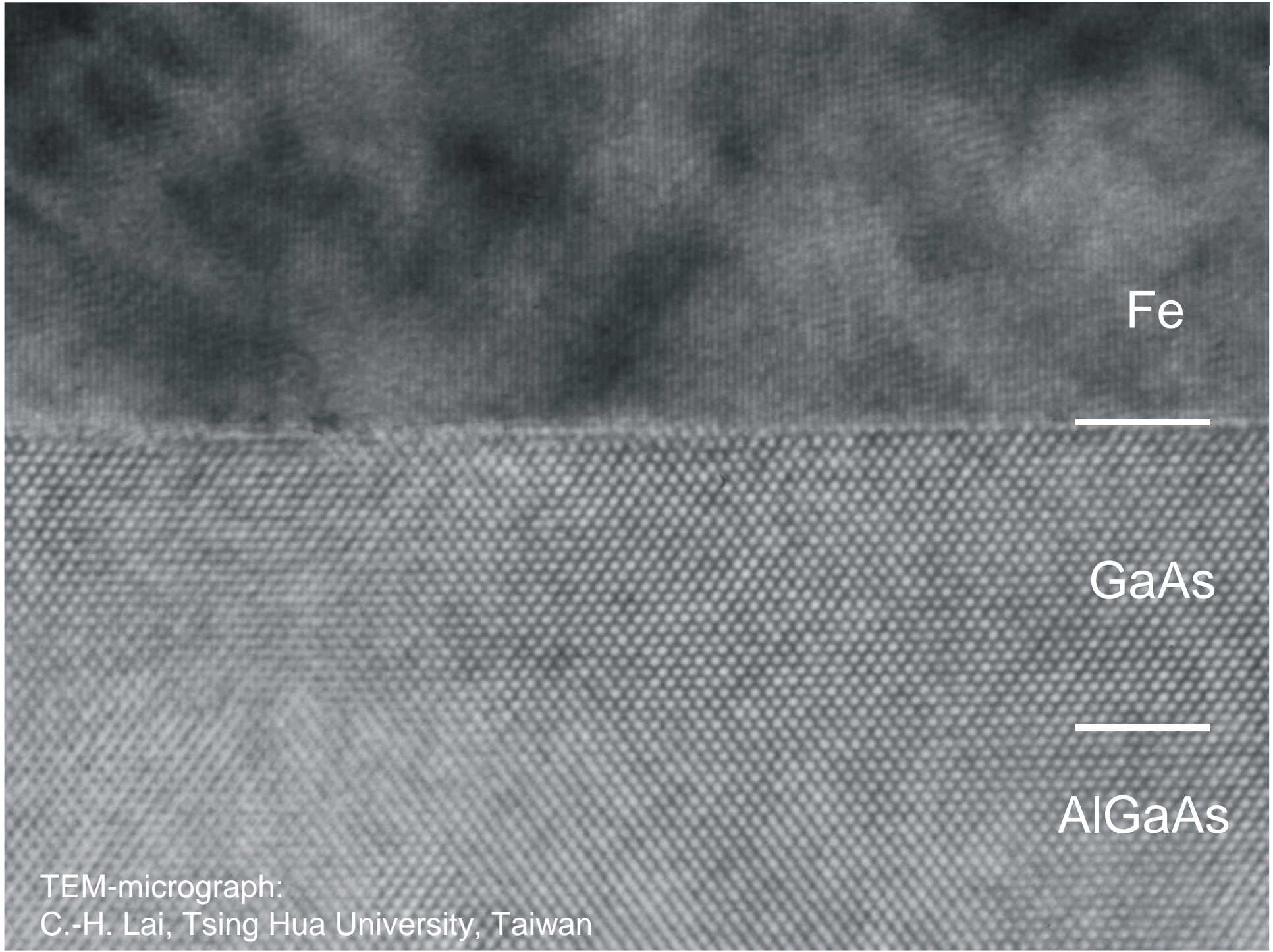
$a_{\text{GaAs}} = 5.653 \text{ \AA}$

$a_{\text{Fe}} = 2.867 \text{ \AA}$



# Device fabrication





Fe

GaAs

AlGaAs

TEM-micrograph:  
C.-H. Lai, Tsing Hua University, Taiwan

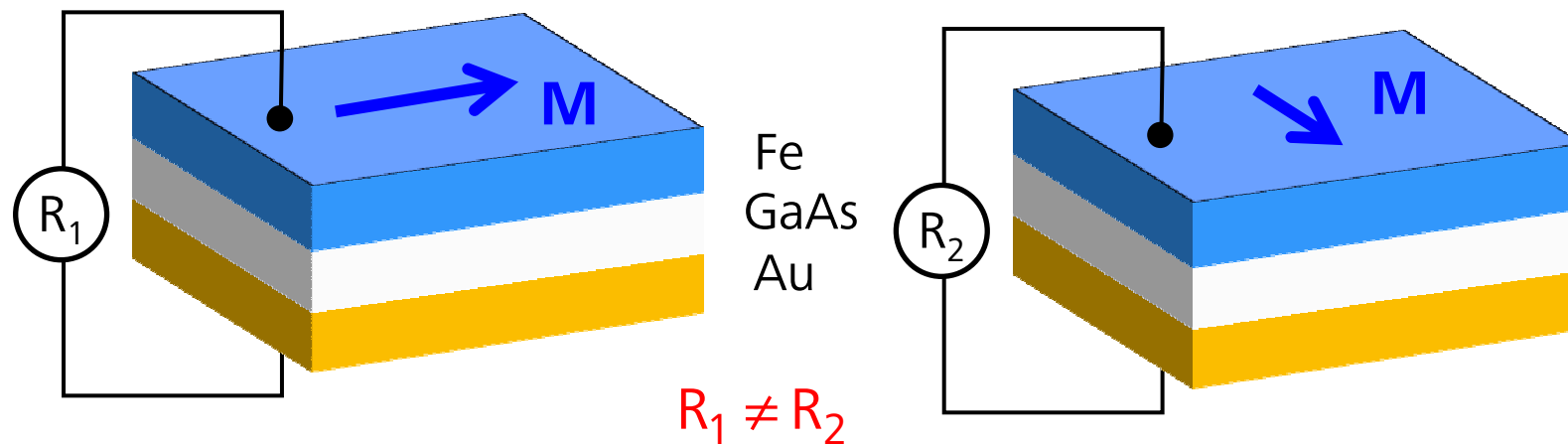




# TAMR: Tunneling Anisotropic Magnetoresistance

Are always two ferromagnetic layers necessary to see a magnetization dependent resistance?

Our model system: Fe/GaAs/Au with epitaxial Fe/GaAs interface

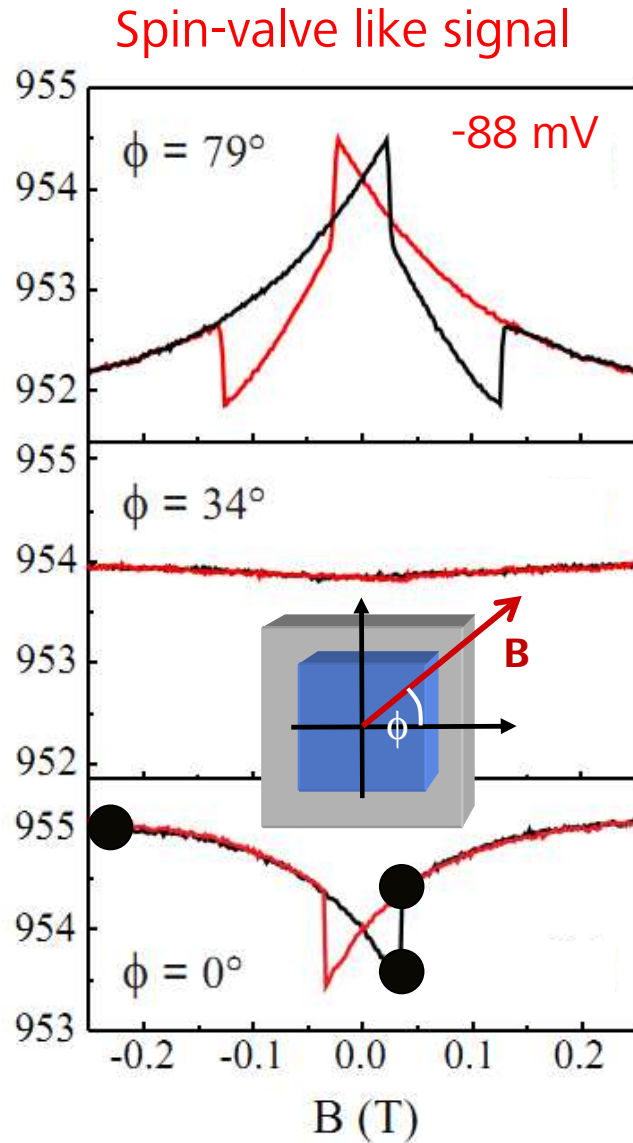


See also:

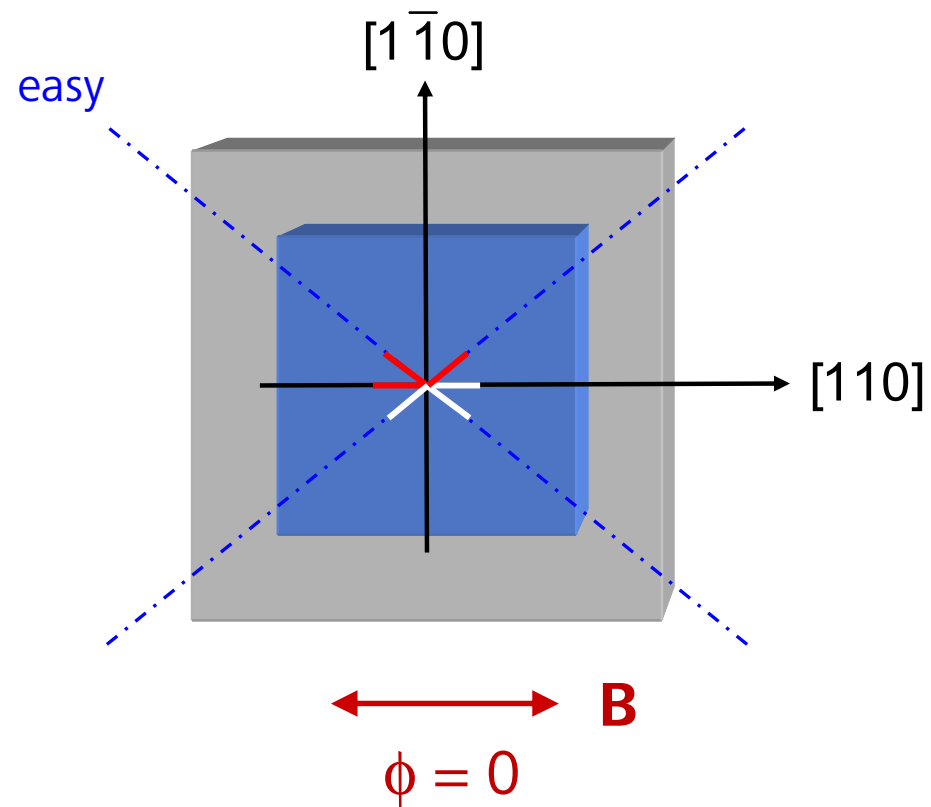
Gould et al. PRL **93**, 117203 (2004)

(Ga,Mn)As/ $\text{Al}_2\text{O}_3$ /Au

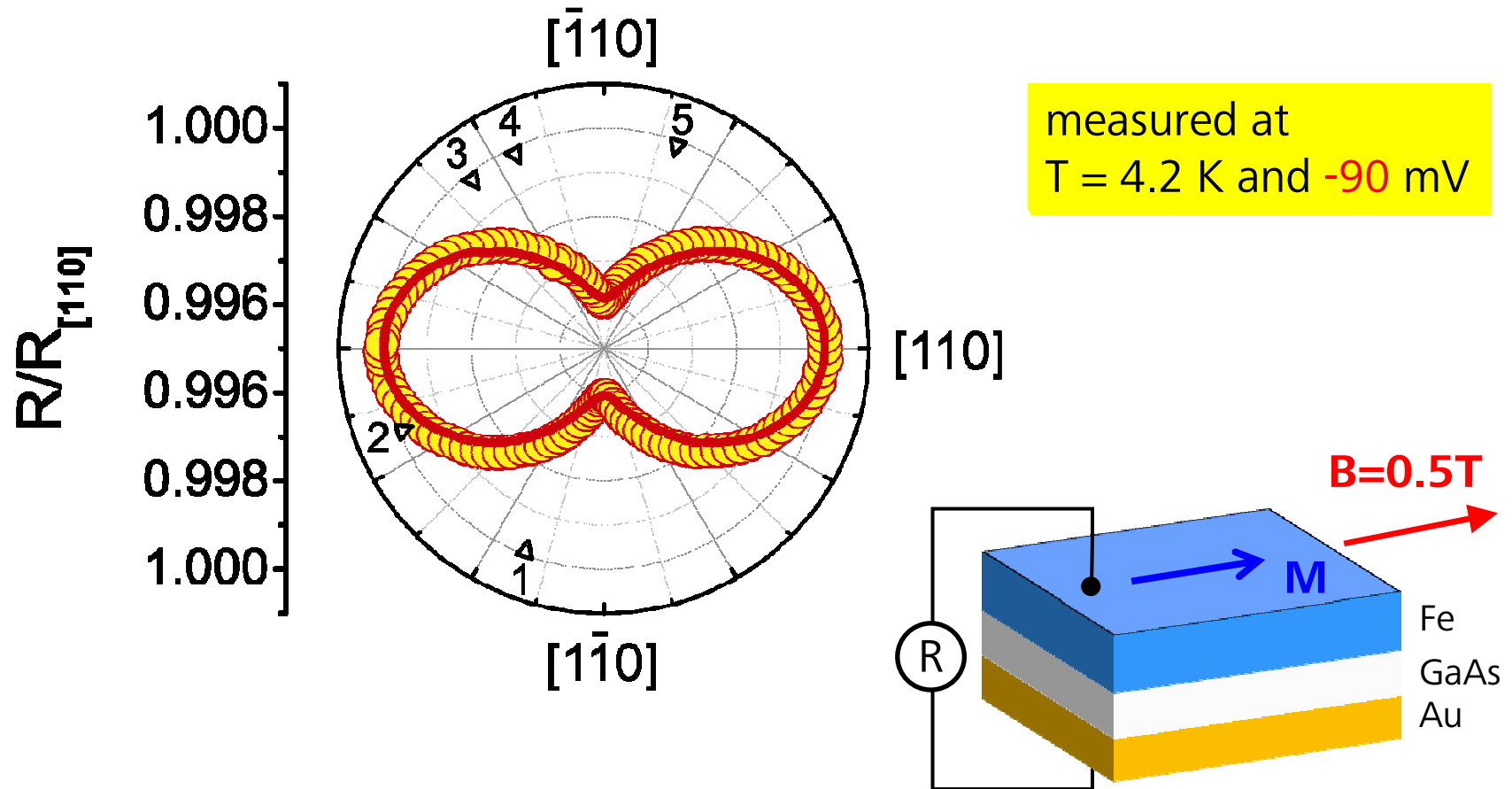
# Tunneling magnetoresistance: **B** dependence



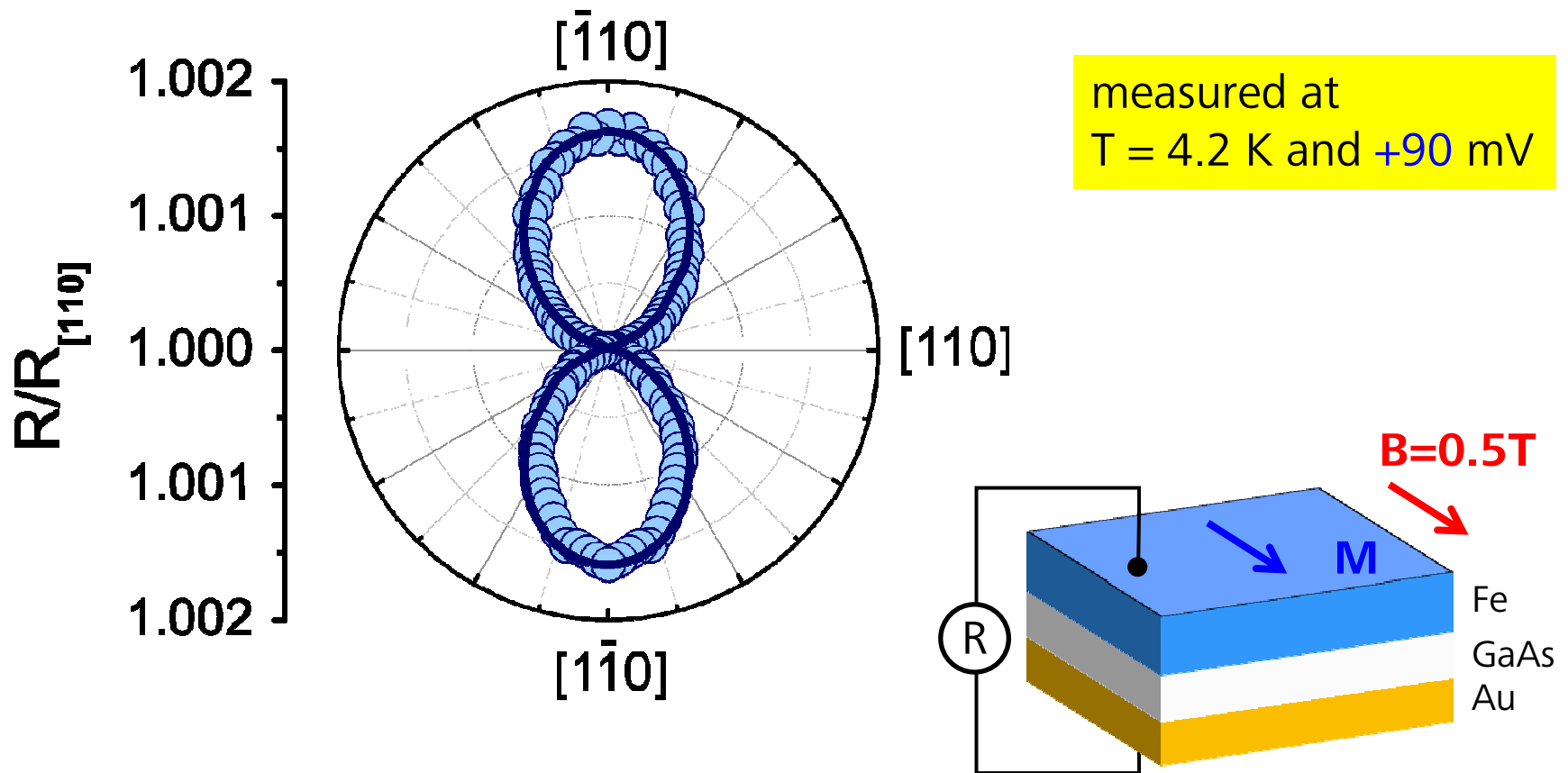
Double step/single step switching due to magnetic anisotropy  
 Fe on GaAs: Cubic + uniaxial anisotropy



## Co/Fe(epi)/GaAs(8nm)/Au



## Co/Fe(epi)/GaAs(8nm)/Au



A. Matos-Abiague & J. Fabian, PRB **79**, 155303 (2009)

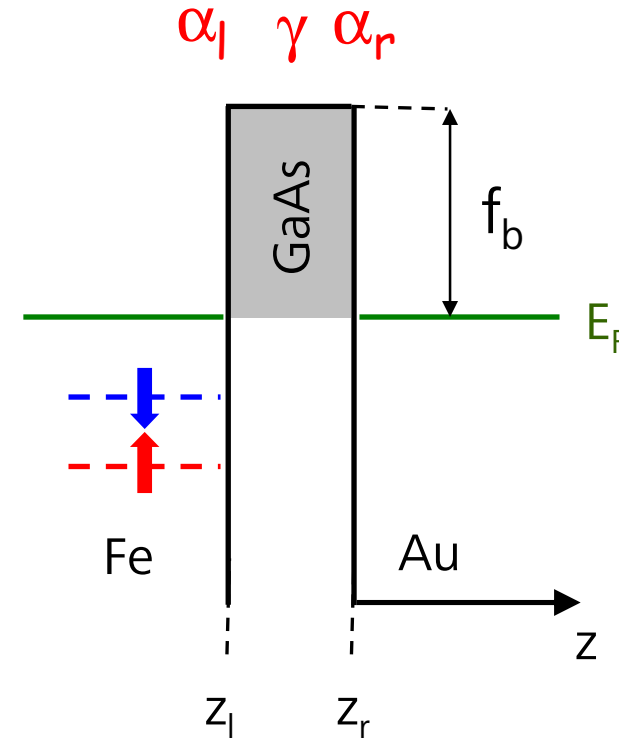
$$H = H_0 + H_z + H_{BR} + H_D$$

$$H_0 = -\frac{\hbar^2}{2} \nabla \left[ \frac{1}{m(z)} \nabla \right] + V(z)$$

$$H_{BR} = \frac{1}{\hbar} \sum_{i=l,r} \alpha_i (\sigma_x p_y - \sigma_y p_x) \delta(z - z_i)$$

$$H_z = -\frac{\Delta(z)}{2} \mathbf{n} \cdot \boldsymbol{\sigma}$$

$$H_D = \frac{1}{\hbar} (\sigma_x p_x - \sigma_y p_y) \frac{\partial}{\partial z} \left( \gamma(z) \frac{\partial}{\partial z} \right)$$

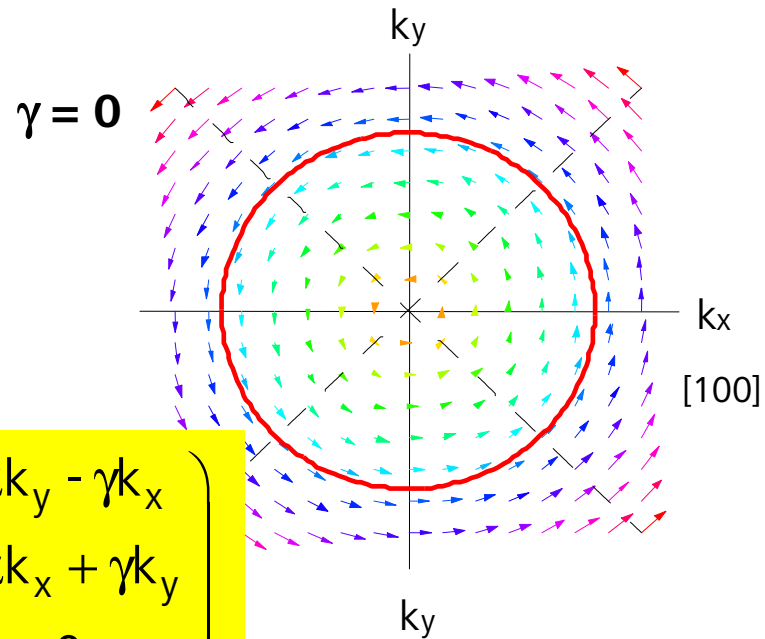
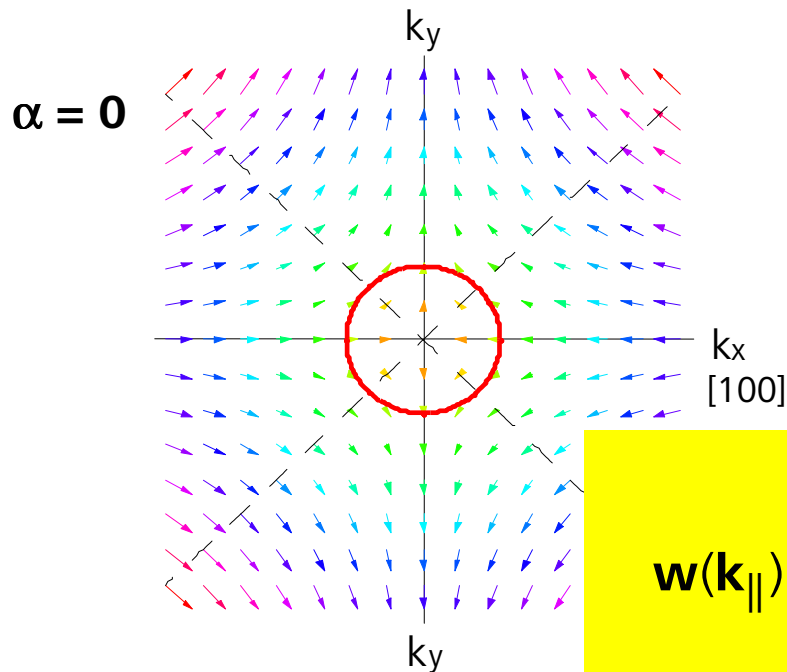


$$I = \frac{e}{(2\pi)^3 \hbar} \sum_{\sigma=-1,1} \int dE d^2 k_{\parallel} T_{\sigma}(E, k_{\parallel}) [f_l(E) - f_r(E)]$$

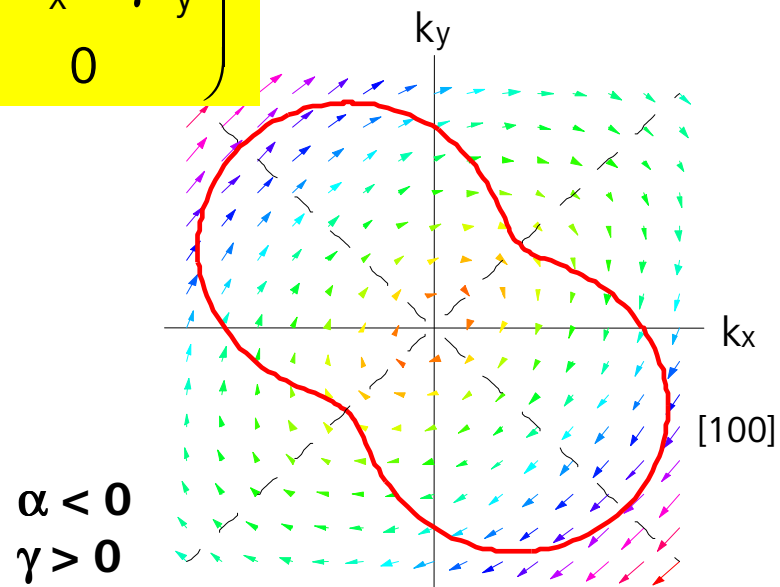
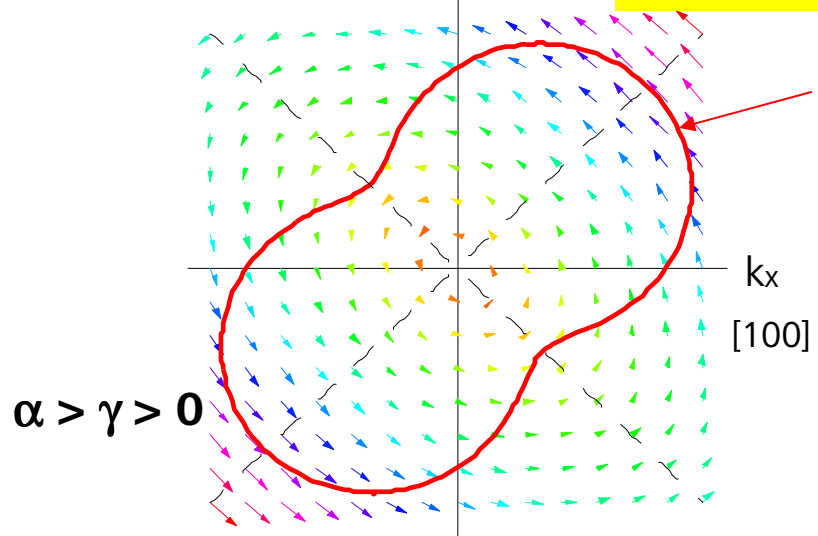
↑  
particle transmissivity



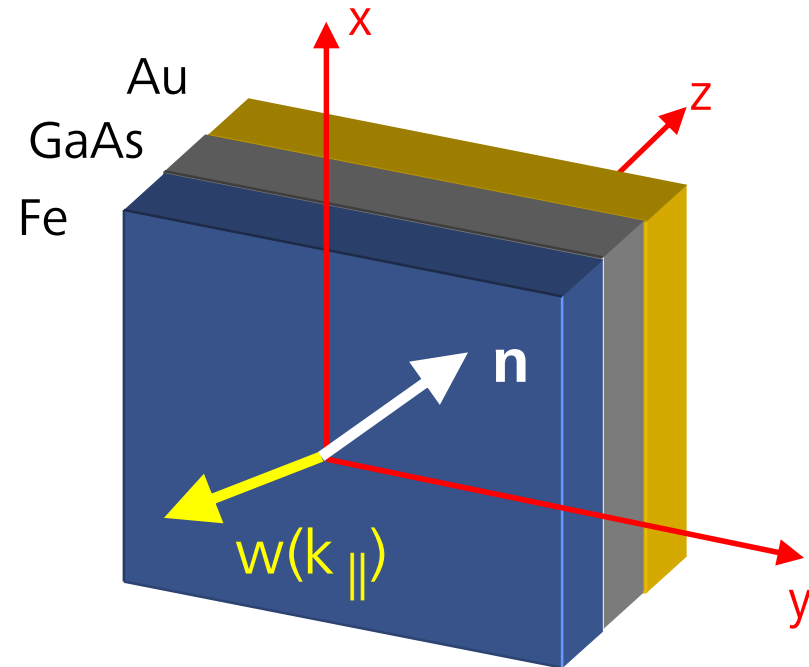
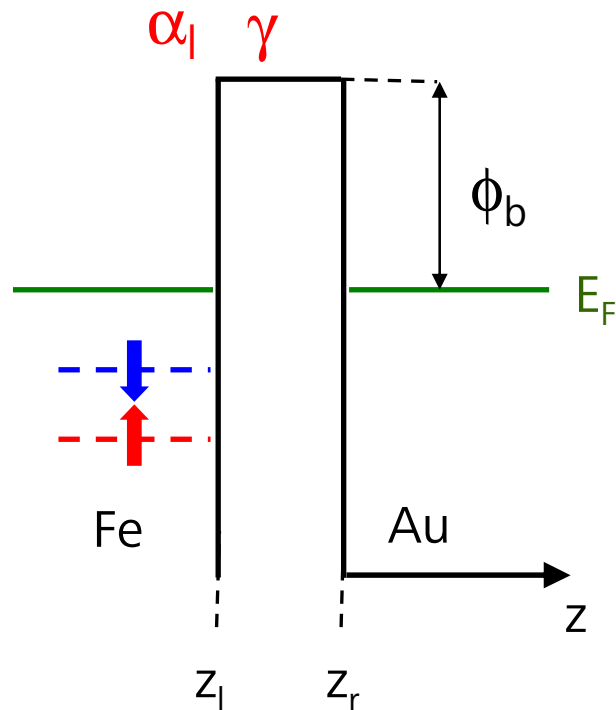
# Spin-orbit field $\mathbf{w}(\mathbf{k})$



$$\mathbf{w}(\mathbf{k}_{\parallel}) = \begin{pmatrix} \alpha k_y - \gamma k_x \\ -\alpha k_x + \gamma k_y \\ 0 \end{pmatrix}$$



# Origin of anisotropic resistance: superposition of Rashba & Dresselhaus SO-contribution



Anisotropy determined by

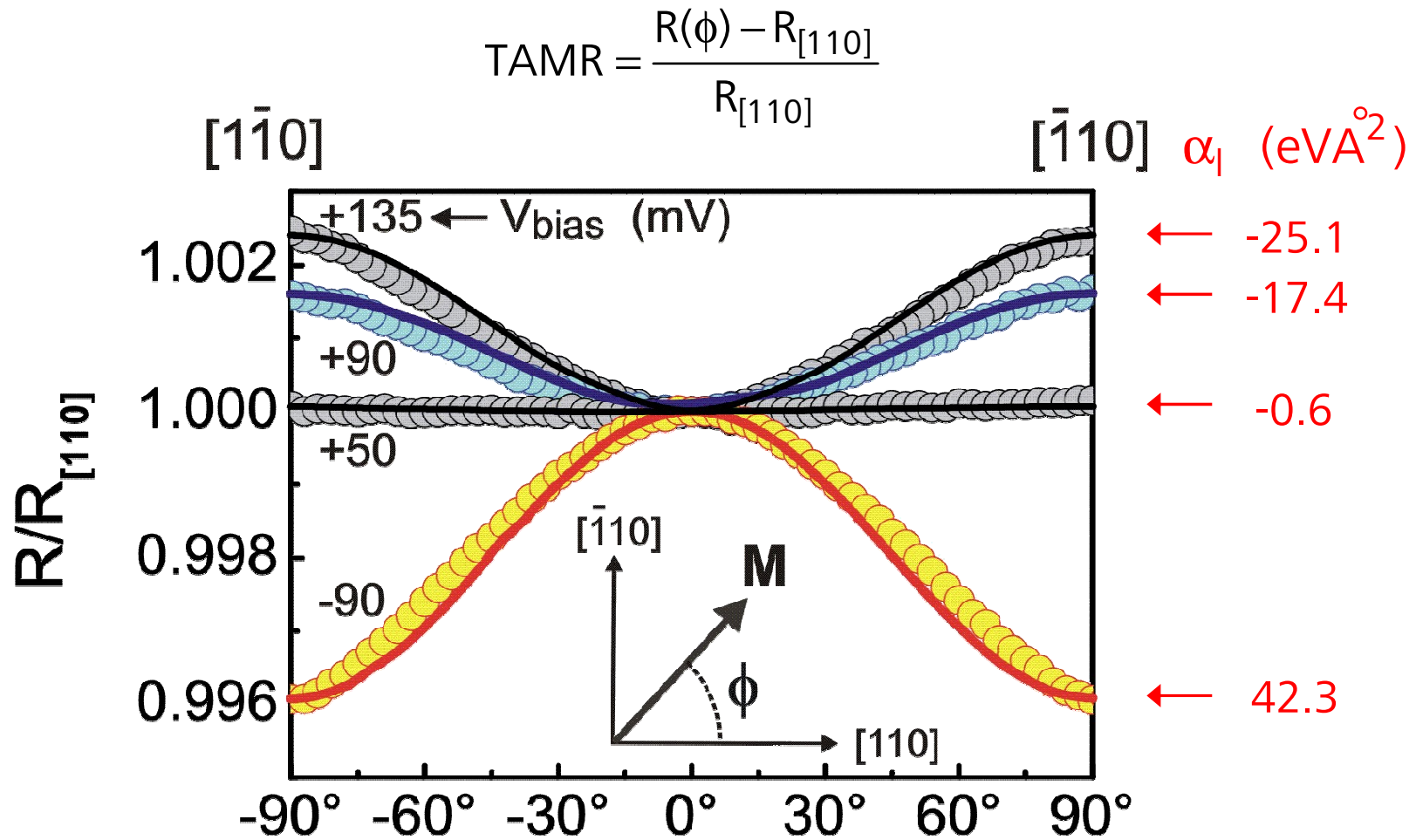
$$T(\mathbf{k}_{\parallel}) \approx f([\mathbf{n} \cdot \mathbf{w}(\mathbf{k}_{\parallel})]^2) \rightarrow R / R_{[110]} - 1 \sim \alpha\gamma(\cos 2\phi - 1)$$

Anisotropy vanishes for  $\alpha\gamma \rightarrow 0$

$$\mathbf{w}(\mathbf{k}_{\parallel}) = \begin{pmatrix} \alpha k_y - \gamma k_x \\ -\alpha k_x + \gamma k_y \\ 0 \end{pmatrix}$$



# Angular dependence of TAMR: bias dependence



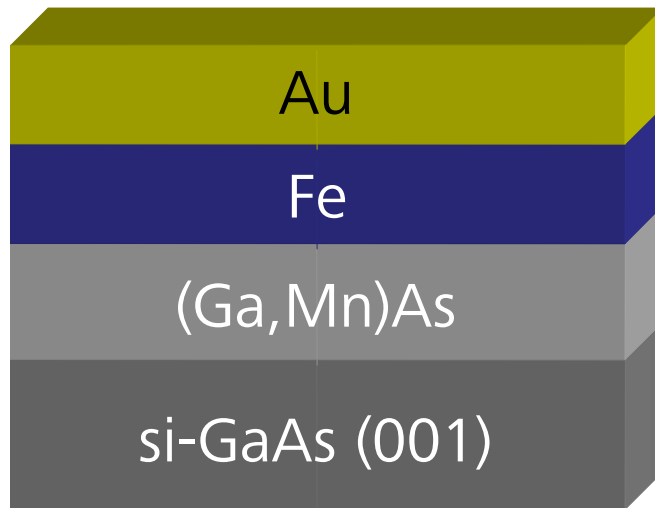
$\gamma$  in GaAs: 24 eVÅ<sup>3</sup>

J. Moser et al., Phys. Rev. Lett. **99**, 056601, 2007  
See also: T. Uemura, Appl. Phys. Lett. **98**, 102503 (2011)



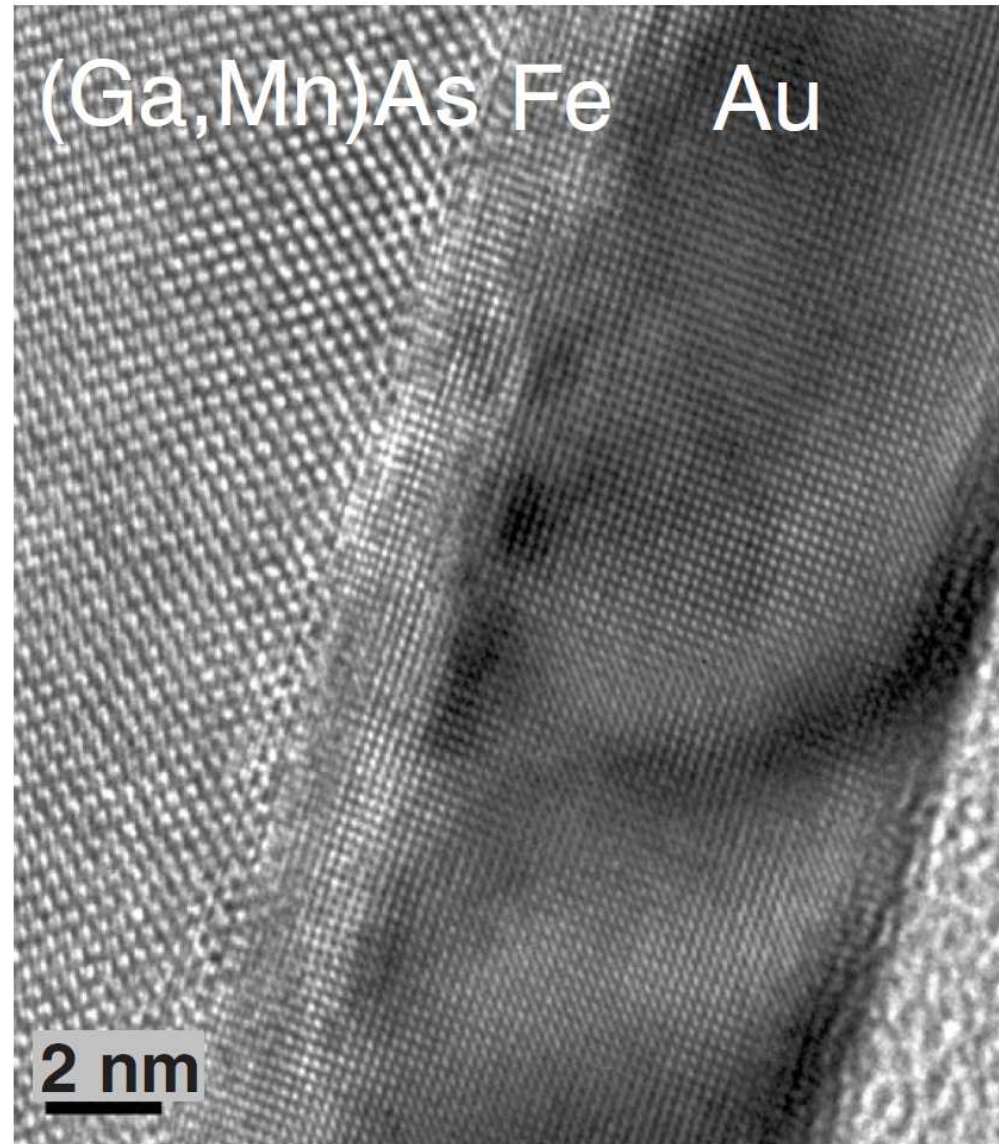


# Fe on (Ga,Mn)As

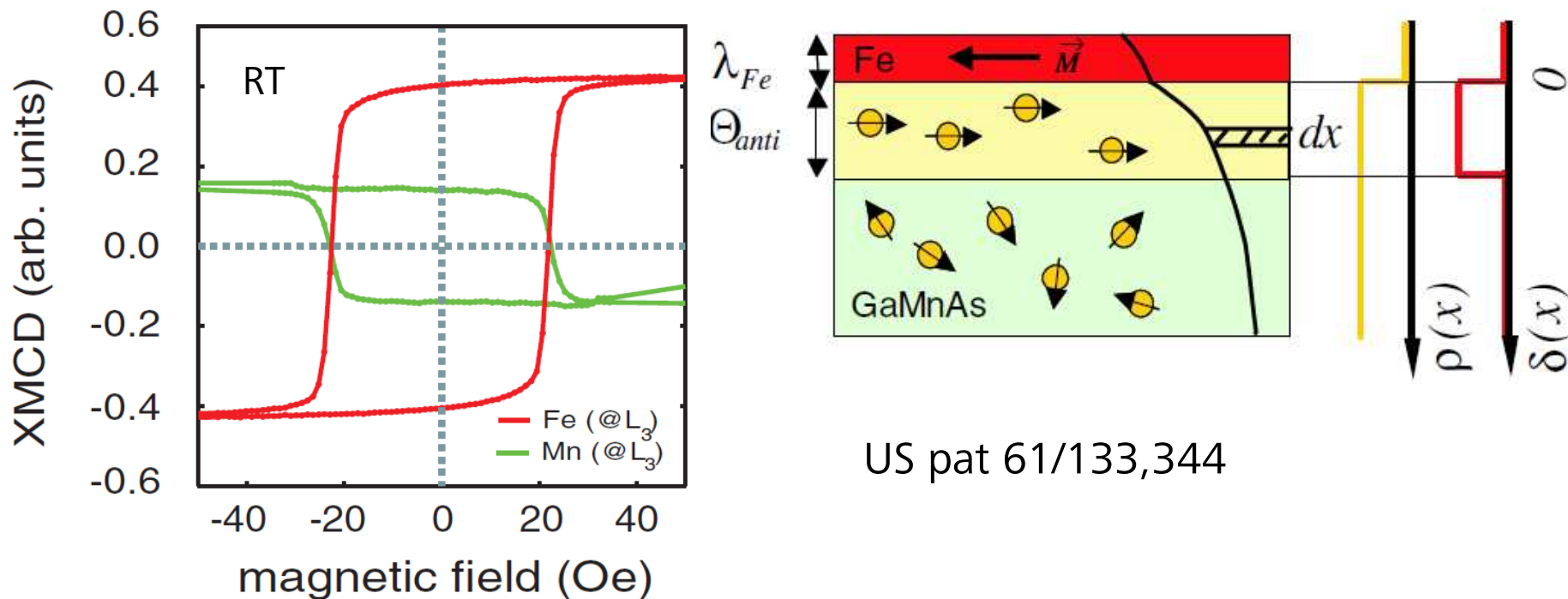


F. Maccherozzi et al.  
PRL **101**, 267201 (2008)

M. Sperl et al.,  
Phys. Rev B **81**, 035211 (2010)



# XMCD: antiferromagnetic coupling



Thin ferromagnetic iron film induces **antiferromagnetic coupling** in (Ga,Mn)As **at room temperature**. Minimum thickness of the coupled (Ga,Mn)As layer is at least 1 nm

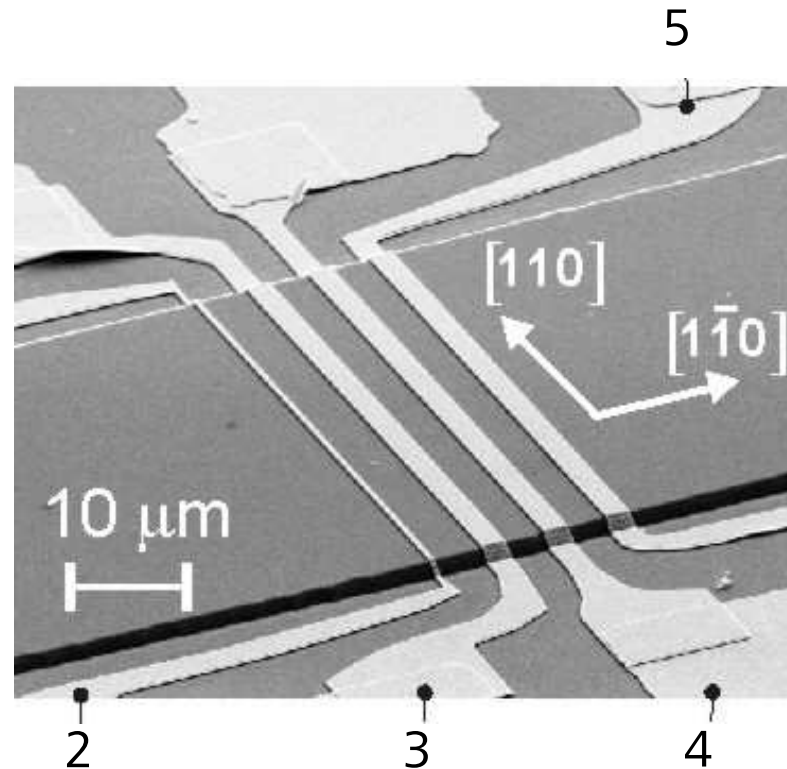
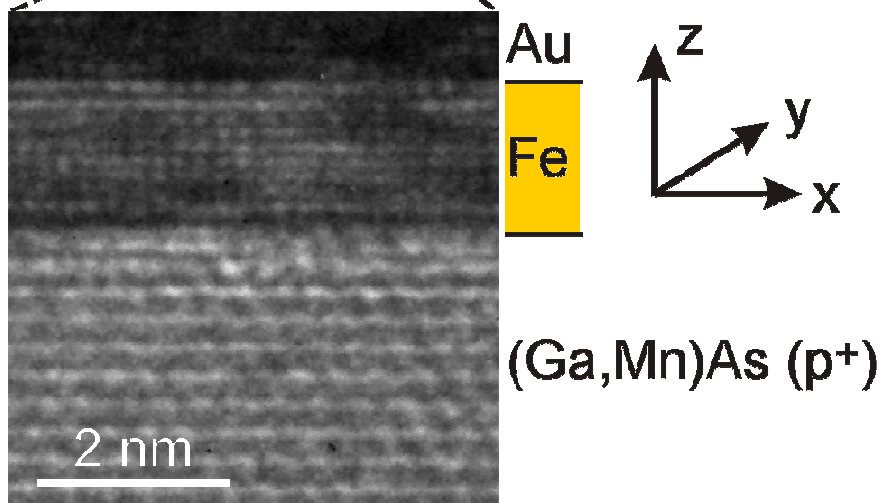
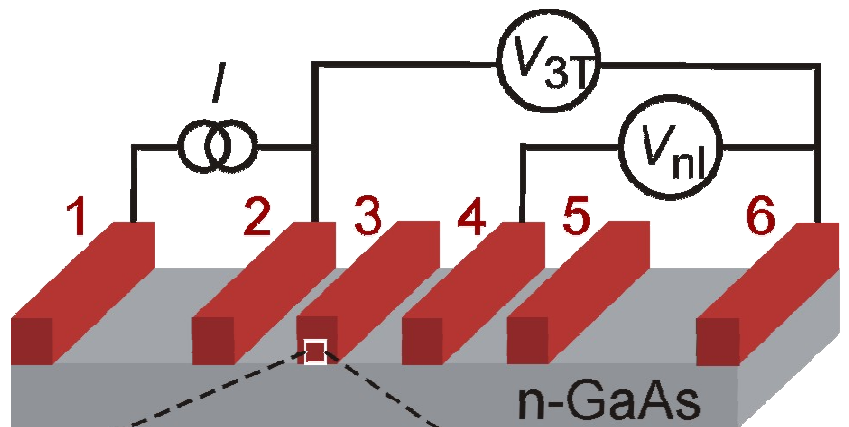
F. Maccherozzi et al. PRL **101**, 267201 (2008)

M. Sperl et al., Phys. Rev B **81**, 035211 (2010)



# Spin injection: Proximity effect enhances $T_c$

Proximity effect: exchange coupling between Fe and Mn at Fe/(Ga,Mn)As interface



- FP-34** C. Song et al.
- WP-30** M. Ciorga et al.
- WP-105** J. Shiogai et al.



# Magnetism and Spin-Orbit Interaction:

**In retrospect**

## **Magnetism**

### Basic concepts

magnetic moments, susceptibility, paramagnetism, ferromagnetism  
exchange interaction, domains, magnetic anisotropy,

### Examples:

detection of (nanoscale) magnetization structure  
using Hall-magnetometry, Lorentz microscopy and MFM

## **Spin-Orbit interaction**

### Some basics

Rashba- and Dresselhaus contribution, SO-interaction and effective  
magnetic field

### Example:

Ferromagnet-Semiconductor Hybrids: TAMR involving epitaxial  
Fe/GaAs interfaces

# **The End**