phys. stat. sol. (b) **218**, 211 (2000)

Subject classification: 71.30.+h; 72.80.Cw; S5; S5.12

Variable Range Hopping Conduction in Neutron-Transmutation-Doped ^{70}Ge:Ga

K. M. Itoh

Department of Applied Physics and Physico-Informatics, Keio University and PRESTO-JST, 3-14-1, Hiyoshi, Kohoku-Ku, Yokohama 223-8522, Japan
e-mail: kitoh@appi.keio.ac.jp

(Received September 7, 1999)

Variable range hopping conduction in neutron-transmutation-doped, isotopically enriched ^{70}Ge:Ga samples for the temperature range T = 20 to 250 mK has been studied in the critical regime for the metal–insulator transition. Eighteen samples investigated had Ga concentrations in the range $N = 0.942N_c$ to $0.998N_c$, where N_c is the critical Ga concentration for the metal–insulator transition. The low temperature resistivities ϱ of all samples obey the variable range hopping conduction theory of Efros-Shklovskii type with an appropriate temperature dependence in the pre-factor; $\varrho = \varrho_0 T^{-1/3} \exp(T_0/T)^{1/2}$. The critical exponent α of T_0 as a function of N has been determined using the form $T_0 \propto (1 - N/N_c)^{\alpha}$. A clear crossover of α from $\alpha \approx 1$ to 3.5 has been observed at $N \approx 0.99N_c$. The excitation of holes from the lower- to upper-Hubbard bands is the dominant conduction mechanism in the $\alpha \approx 1$ region, while the standard variable range hopping of holes from occupied to empty Ga sites takes place in the very vicinity of the transition where $\alpha \approx 3.5$. The important role of the doping compensation is proposed in connection with the critical behavior of the electrical conductivity.

The nature of the hopping conduction in highly doped crystalline semiconductors has been studied extensively in the past via measurements of the low temperature electronic conductivity [1]. However, a number of systematic studies of the hopping conduction in the critical regime for the metal–insulator transition is rather limited due to the technological difficulty of producing appropriate samples. A precise tuning of the doping concentration (N) around the critical doping concentration (N_c) for the transition at the level of 1% of N_c is extremely difficult even with today's advanced semiconductor-processing technology. For example, the spatial fluctuations of N in widely used melt- (or metallurgically) doped samples is unavoidable due to the effect known as doping striations and segregation during the crystal growth [2].

The present study is the result of an extensive materials science effort to overcome such difficulties and to produce homogeneously doped samples near N_c. Our sample fabrication technique, neutron-transmutation-doping (NTD) of isotopically enriched ^{70}Ge, has been explained in detail in a number of our previous publications [3 to 6]. This technique guarantees macroscopically homogeneous, microscopically random distributions of impurities. It allows us to control N at the step of $0.0002N_c$. The doping compensation ratio, i.e., the amount of donors with respect to that of majority Ga acceptors, can be suppressed down to an extremely small level, typically less than 0.1% [3].

Figure 1 shows the temperature dependence of the resistivity of eighteen samples in the temperature range T = 20 to 250 mK. The concentration of each sample is given in

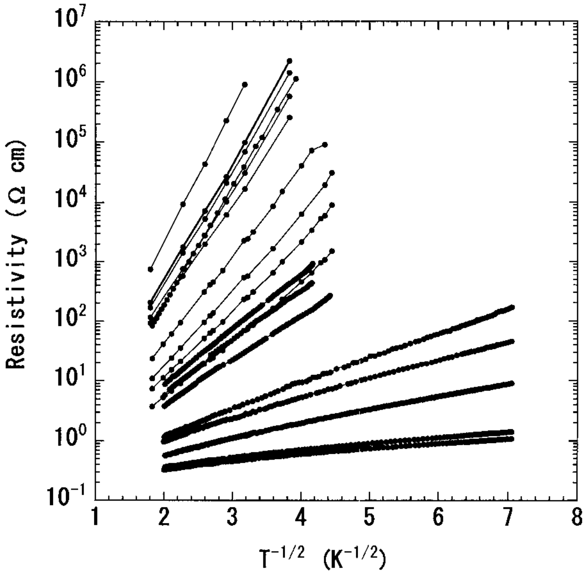

Fig. 1. $\ln \varrho$ vs. $T^{-1/2}$ for 18 insulating samples. The concentration N from top to bottom in the unit of N_c are 0.923, 0.942, 0.947, 0.950, 0.951, 0.956, 0.965, 0.970, 0.980, 0.985, 0.989, 0.989, 0.990, 0.991, 0.993, 0.994, 0.996, and 0.998 ($N_c = 1.860 \times 10^{17}$ cm^{-3})

the figure caption in units of N_c ($N_c = 1.860 \times 10^{17}\text{cm}^{-3}$). In general, the temperature dependence of the resistivity ϱ for variable range hopping conduction is given by

$$\varrho = \varrho_0 T^q \exp{(T_0/T)^p} , \tag{1}$$

where $p = 1/4$ and $1/2$ have been predicted for hopping across [7] and within [8, 9] a parabolic-shaped Coulomb gap in the density of the states, respectively. $q = 0$ is usually assumed since the temperature dependence in the strongly localized regime is determined mainly by the value of p in the exponential term. Note that $\ln \varrho$ is plotted against $T^{-1/2}$ in Fig. 1. Because most of the curves in the low temperature limit appear to form straight lines supporting the relation $\ln \varrho \propto T^{-1/2}$, we shall analyze our data based on the theory of Shklovskii and Efros [8].

The theory of Efros and Shklovskii has been derived from the one-electron hopping scheme in a strongly localized regime. Therefore, some may claim that it is not applicable to hopping in the critical regime, and that theory based on the many-body picture [9] must be employed. Although such a possibility cannot be excluded in the framework of the present study, we shall present an evidence that the hopping picture of Efros and Shklovskii is valid for the critical regime investigated in this work. According to Efros and Shklovskii, T_0 is given by

$$k_B T_0 = \frac{1}{4\pi\epsilon_0} \frac{2.8e^2}{\kappa(N)\,\xi(N)} \,. \tag{2}$$

$\kappa(N)$ and $\xi(N)$ are the dielectric constant and localization length, respectively, given by $\kappa(N) = \kappa_0(1 - N/N_c)^{-s}$ and $\xi(N) = \xi_0(1 - N/N_c)^{-\nu}$ as N approaches N_c from the insulating side. With these relations T_0 becomes

$$k_B T_0 = \frac{2.8e^2}{4\pi\epsilon_0\kappa_0\xi_0} \,(1 - N/N_c)^{\alpha}, \tag{3}$$

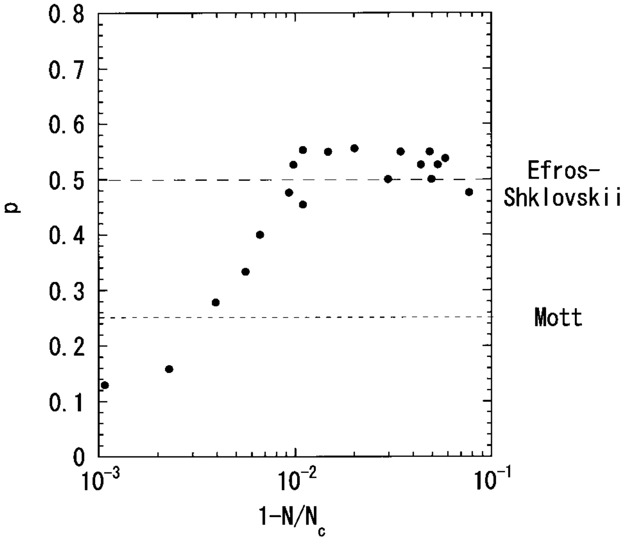

Fig. 2. N dependence of p assuming $q = 0$ in Eq. (1)

where the critical exponent for T_0, i.e., $\alpha = s + \nu$, is determined experimentally in our study. Because the width of the Coulomb gap Δ_{CG} collapses as N approaches to N_c from the insulating side, the hopping energy may eventually become larger than $\varDelta_{CG}$. In this case we expect to observe a crossover of p from 1/2 to 1/4, i.e., transition of variable range hopping from Efros and Shklovskii to Mott types. Observation of such a crossover has been reported for Si:P [10]. Figure 2 shows the N dependence of p found from the calculation of $\mathrm{d}\log\varepsilon/\mathrm{d}\log T$, where $\varepsilon \equiv -\mathrm{d}\log\varrho/\mathrm{d}\log T$ and/or from the direct fitting to Eq. (1) assuming $q = 0$. We see that $p \approx 1/2$ is maintained for N up to $0.99N_c$, and instead of crossing over to 1/4, p decreases monotonically to the value as small as 1/8 as $N \rightarrow N_c$. This unexpected behavior of p turns out to be an artifact caused by setting $q = 0$. We now introduce an appropriate value of q since the temperature dependence of ϱ for $N > 0.99N_c$ is becoming too small to be accounted for using the parameter p only. Recently, we have shown that the temperature dependence of the conductivity σ at the transition N_c is proportional to $T^{1/3}$ [5, 6]. In order to assure the continuity across the transition, it is necessary to set $p = -1/3$, i.e., we plot $\ln\varrho T^{1/3}$ versus $T^{-1/2}$ and $T^{-1/4}$ in Fig. 3a and b, respectively. The result of this analysis is quite convincing; $\ln\varrho T^{1/3}$ of four samples having $N > 0.99N_c$ shows a clear $T^{-1/2}$ dependence of Efros and Shklovskii type (Fig. 3a) while the same plot against $T^{-1/4}$, i.e. the Mott plot, contains $\ln\varrho T^{1/3}$ curves turning upwards. Therefore, for the rest of the paper, we shall assume that the resistivity of all samples is described by Eq. (1) with $p = 1/2$ and $q = -1/3$.

Figure 4a and b show the values of T_0 and ϱ_0 as a function of $1 - N/N_c$, respectively. The critical exponent α of T_0 is found to be $\alpha \approx 1$ and 3.5 for the regions $0.99N_c < N < 0.9N_c$ and $0.999N_c < N < 0.99N_c$, respectively. The solid curve in Fig. 4a represents theoretically estimated T_0 for the $0.99N_c < N < 0.9N_c$ region using Eq. (3) with $\alpha = 1$, $\kappa_0 = \epsilon_h = 16$ (dielectric constant of host Ge), and $\xi_0 = 8$ nm (Bohr radius of Ga acceptors) as was done in one of our previous publications [5]. The agreement between the solid curve and experimentally found T_0 is surprisingly good for the $\alpha = 1$

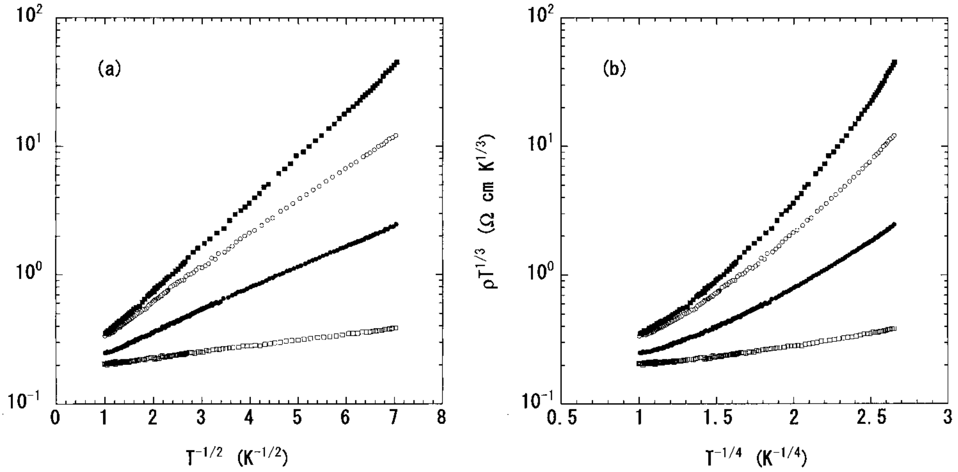

Fig. 3. Resistivity ϱ multiplied by $T^{1/3}$ vs. a) $T^{-1/2}$ and b) $T^{-1/3}$. From top to bottom $N = 0.993N_c$, $0.994N_c$, $0.996N_c$, and $0.998N_c$

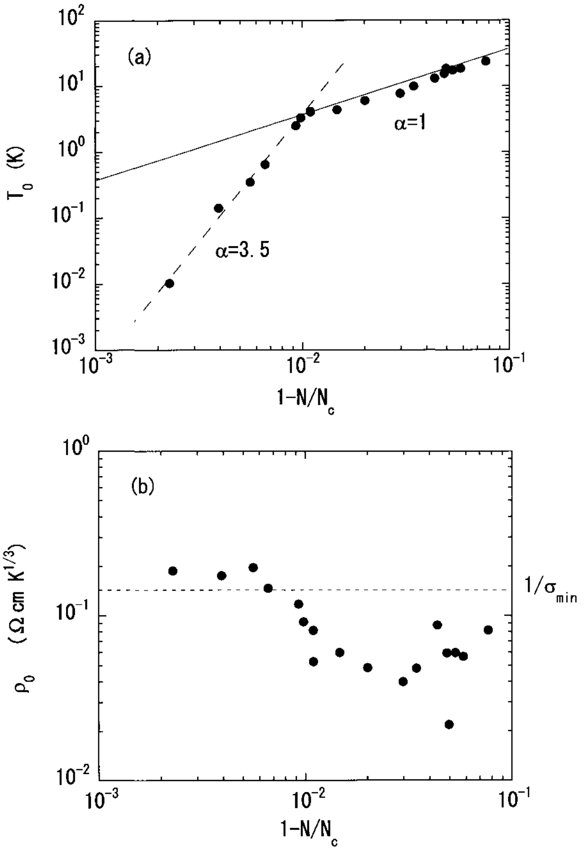

Fig. 4. a) T_0 and b) ϱ_0 as a function of $1 - N/N_c$ determined with $p = 1/2$ and $q = 1/3$ in Eq. (1)

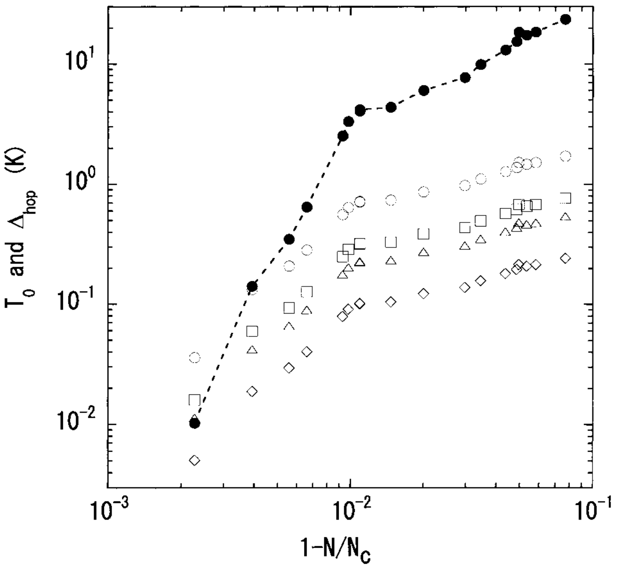

Fig. 5. N dependence of T_0 (●) and the hopping energies Δ_{hop} at T = 10 mK (◇), 50 mK (△), 100 mK (□), and 500 mK (○)

region. However, $\kappa_0 = \epsilon_h - \epsilon_{Ga}$ (where ϵ_{Ga} is the dielectric susceptibility due to a single Ga acceptors) should be employed formally in Eq. (3). Assuming α = 3.5 and ξ_0 = 8 nm for the 0.001 $N_c < N < 0.01N_c$ region, we find $\kappa_0 \approx 2 \times 10^{-4}$ from the numerical fitting of T_0. This value of κ_0 is much more reasonable than $\kappa_0 = 16$ we found in the $\alpha = 1$ region, and it suggests that we are observing Efros-Shklovskii's type of hopping only in the high concentration limit ($N > 0.99N_c$). Because the compensation ratios in our samples are extremely small, hopping of holes from occupied to empty sites is allowed only when overlaps of hole wavefunctions become sufficiently large very close to the transition. The values of ϱ_0 approach the inverse of Mott's minimum metallic conductivity ($1/\sigma_{min}$) as $N \to N_c$ from the insulating side, and become constant at $\varrho_0 \approx 1/\sigma_{min}$ for $N > 0.99N_c$ (Fig. 4b).

Figure 5 shows the N dependence of T_0 and the hopping energy Δ_{hop} in the unit of temperature (K) for T=10, 50, 100, and 500 mK. The relation $\Delta_{hop} = 0.5k_B T^{1/2} T_0^{1/2}$ has been used for the calculation [1]. Since Efros and Shklovskii's theory is valid only when $T_0 > \Delta_{hop}$, the idea is to see if this condition is satisfied for the temperature range we used in the analysis. Figure 5 shows that such a condition is satisfied for all the samples at $T < 500$ mK except for the one closest to the transition ($N = 0.998N_c$). For this sample only the data taken at $T < 100$ mK have been analyzed in the context of Efros and Shklovskii's theory.

Finally, we shall discuss on the results of our analysis with respect to the critical exponent μ of the conductivity of metallic samples defined by $\sigma(0) \propto (N/N_c - 1)^{\mu}$, where $\sigma(0)$ is the conductivity at $T = 0$. In the past we have reported $\mu \approx 0.5$ for the region $N_c < N < 1.4N_c$ in the same series of NTD ^{70}Ge:Ga samples [4, 5]. However, we have found recently an evidence for $\mu \approx 1.2$ only in the small region $0.994N_c < N < 1.004N_c$ using so called finite temperature scaling analysis [6]. Since the insulating samples included in the finite temperature analysis show $\alpha = 3.5$, we have applied Efros and Shklovskii's theory of variable range hopping conduction under a magnetic field [11] to find the critical exponent for the localization length

$\nu \approx 1.2$ and for the dielectric constant $s \approx 2.3$ [12]. Thus, Wegner's scaling law of $\nu = \mu$ for three-dimensional systems is confirmed [12]. So we raised the question, "Why is the critical region showing $\mu \approx 1.2$ and $\alpha \approx 3.5$ so small in our samples?" This question is especially important since $\mu \approx 1$ and $\alpha \approx 3.5$ has been obtained for the wide region of N ($\pm 40\%$ of N_c) in NTD Ge:Ga,As samples of the compensation ratio 0.4. Such observations lead us to think that the width of the $\mu \approx 1$ and $\alpha \approx 3.5$ regions must depend on the compensation ratio, and that only $\mu \approx 0.5$ *and* $\alpha \approx 1$ *are observed in the limit of the zero compensation*. Of course this hypothesis needs to be confirmed in the future by measuring samples whose compensation ratios are controlled systematically.

In summary, we have probed the nature of variable range hopping conduction in the critical regime for the metal–insulator transition using isotopically enriched, neutron-transmutation-doped ^{70}Ge:Ga. Efros and Shklovskii's theory of hopping conduction has been applied successfully for the quantitative analysis of our data. The width of the $\alpha \approx 3.5$ and $\mu \approx 1.2$ region around N_c is determined by the value of the compensation ratio.

Acknowledgements This work has been performed in collaboration with M. Watanabe, Y. Ootuka, and E. E. Haller. We thank T. Ohtsuki, S. Katsumoto, and M. Eto for valuable discussions. The work at Keio was supported in part by a Grant-in-Aid for Scientific Research from the Ministry of Education, Science, Sports, and Culture, Japan.

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